LEARNING FROM THE WORLD : THE TEACHING AND LEARNING OF WHOLE NUMBER ARITHMETIC IN THE ICMI STUDY 23

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Résumé
In this presentation we summarize the ongoing process of the ICMI STUDY 23, from the launch of the Discussion Document in April 2014 to the Conference (held in Macau in June 2015). From the very beginning the study appeared as a world wide effort: the ten IPC members were appointed from the five continents, plenary speakers were chosen from three different cultures and professional experiences, participants were selected from as many contexts as possible and panels were constructed with the involvement of people from different cultural backgrounds. The Conference venue was chosen in Macau, as the meeting point of East and West some centuries ago. It is the first study launched by ICMI to address primary school teaching and learning. It is a challenge against the belief in universality of mathematics and didactics of mathematics which seems to be shared by many educators in the West, as if the theory and practice of teaching and learning mathematics might repel contexts and the findings of studies might be applied everywhere.

I - INTRODUCTION

Aim of this paper is to summarize some outcomes of the twenty-third study led by the International Commission on Mathematical Instruction (ICMI). The study address, for the first time, mathematics teaching and learning in primary school (and pre-school as well) for all, taking into account international perspectives including socio-cultural diversity and institutional constraints. Whole number is the core content area, which is regarded as foundational for later mathematics learning; its teaching and learning are thus very important due to larger impact for later mathematics knowing. The study was launched by ICMI at the end of 2012, with the appointment of two co-chairs (Maria G. (Mariolina) Bartolini Bussi and Xuhua Sun) and of the International Program Committee (IPC), which on behalf of ICMI is responsible for conducting the Study. The IPC of the ICMI Study 23 was: Maria G. (Mariolina) Bartolini Bussi, Sun Xuhua, Berinderjeet Kaur, Hamsa Venkat, Jarmila Novotna, Joanne Mulligan, Lieven Verschaffel, Maitree Inprasitha, Sybilla Beckmann, Sarah González, Abraham Arcavi (ICMI Secretary General), Ferdinando Arzarello (ICMI President), Roger E. Howe (ICMI liason).

II - THE PROCESS

1 The Discussion Document

In January 2014 (19-24) the IPC meeting took place in Berlin, at the IMU Secretariat, which generously supported the costs. The IPC members were welcomed by Prof. Dr. Jurgen Sprekels, director of the Weierstrass Institute for Applied Analysis and Stochastic (WIAS, Berlin), and by the

ICMI President Prof Ferdinando Arzarello, who participated in the whole meeting. The meeting in Berlin took place in a productive and collaborative climate: a draft version of the Discussion Document was agreed and the Conference dates and venue were chosen. Five themes (each corresponding to a Working Group in the Conference) were identified :
The why and what of whole number arithmetic
Whole number thinking, learning, and development
Aspects that affect whole number learning
How to teach and assess whole number arithmetic
Whole numbers and connections with other parts of mathematics
Three plenary speakers were invited :
Ma Liping : *The theoretical core of Whole Number Arithmetic*
Brian Butterworth : *Low numeracy: from brain to education*
Hyman Bass : *Quantities, Numbers, Number names, and real number line*
Three plenary panels were identified :
Traditions in whole number arithmetic (chaired by Ferdinando Arzarello);
Special needs in research and instruction in whole number arithmetic (chaired by Lieven Verschaffel);
Whole numbers arithmetic and teacher education (chaired by Jarmila Novotna).
The text of the Discussion Document was widely disseminated at the beginning of April 2014, 15 months before the Conference. In the Discussion Document a special emphasis was given to the importance of cultural diversity and to the effects of this diversity on the early introduction of whole numbers. In order to foster the understanding of the different contexts where authors had developed their studies, each applicant for the Conference was required to include background information about this context. The different contexts will be discussed in the volume of the ICMI Study (Bartolini Bussi and Sun, in preparation).

2 The Conference (University of Macau, June 3-7, 2015)

The review and selection processes took place in December 2014 - January 2015. At the end more than 80 mathematics educators took part in the Conference, including the 10 IPC members, the 3 plenary speakers and some observers.

Thanks to generous support from the University of Macau, for the first time, this ICMI study was able to invite observers from non-affluent countries. The choice was to privilege CANP (Capacity & Networking Project, The Mathematical Sciences and Education in the Developing World) that is the major development focus of the international bodies of mathematicians and mathematics educators.

Hence, one representative for each of the following project was invited with a generous financial support of the University of Macau and ICMI:

- CANP1, Edi Math (Mali, 2011, with participants from across Sub-Saharan Africa)[10];

- CANP2, Central America and the Caribbean (Costa Rica, 2012, with participants from Latin America and the Caribbean);

- CANP3, South East Asia (Cambodia, 2013, with participants from ASEAN);

- CANP4, East Africa (Tanzania, 2014, with participants from Tanzania, Kenya, Uganda and Rwanda);

- CANP5, Andrean Region (Peru, to be held in 2016, with participants from Peru, Ecuador, Paraguay, Bolivia and Amazonian Brazil).

Some policy makers and observers from the Chinese area were also invited to join the Conference.

[10] At the end the representative of CANP1 was not able to come because of visa problems.

The proceedings were carefully edited by Xuhua Sun, Berinderjeet Kaur and Jarmila Novotna and can be downloaded free[11].

III - SNAPSHOTS FROM THE CONFERENCE

1 Languages

The possibility to meet so many colleagues from different regions and with different linguistic backgrounds made the Conference, and especially the working groups a unique occasion to reflect of language differences in whole number arithmetic. It is naively shared that the teaching and learning of whole numbers is the same all over the world. Yet this statement is very quickly challenged when mathematics educators compare the different wording of whole numbers and of arithmetic operations in the different countries. It is not necessary to go out of Europe to find examples of number wording which may hint at or hinder arithmetic meaning. For instance, the reading of 184 is substantially different in English, French and German

English :	one hundred eighty four
French :	cent quatre-vingt quatre (with the memory of a base twenty system)
German :	hundred vier und achtzig (with an inversion of tens and units similar to Dutch and Danish ones)

Learning to tell numbers is usually very demanding and requires the mastery of a lot of words and of some conventions. On the contrary, the Chinese wording is completely regular:

一百八十四

Fig. 1. Yī bǎi bā shí sì: *one hundreds eight tens four (ones)*

In the above cases everyday language and school language are the same (if one does not consider the case of immigrant pupils or minorities), hence there is no conflict between the school and the cultural identity of learners. But there are cases where the school language is different from everyday language. This is a common feature of countries with a colonial past, where the school system and the school language have been modeled on the Western traditions and languages. In the Conference there were reports about the situation in Algerie (Azrou, 2015) and in Tanzania (Sarungi, personal communication). In these cases the choice of different languages in everday life and in school is related also to the construction of cultural identity (Barwell et al., in press).

2 Artefacts

The discussion on artefacts in whole number arithmetic started from the very beginning, in the panel on Tradition that was held on the first day. Two contributions were given by Bartolini Bussi (2015) and Sun (2015) to compare / contrast two different popular approaches to whole numbers practiced in the West and in China. The two approaches may be represented by two different artefacts: the *number line* (where numbers identify positions and jumps from one position to another) and the *suànpán* (the Chinese bead abacus, where numbers are related to counting units by means of the place value conventions). Historic-epistemological analysis shows that the fortune of either approach is strongly related to the deep values of Western and Chinese mathematics, with emphasis on continuous vs. discrete quantities (see Jaworski et al.,in press, for a summary). Hence the number line and the suànpán are cultural artefacts that reveal valuable information about the society that made or used them and, when continuity between tradition and today's

[11] The proceedings are available at http://www.umac.mo/fed/ICMI23/index.html

practices is maintained, foster the students' cultural awareness of the role mathematics played in their society, that is their cultural identity.

Fig. 2. A floor number line

Fig. 3. The customized suànpán for the participants in the Conference

Many different artefacts were discussed in the Conference. They were taken from cultural traditions (as explained above) or designed by means of Information and Communication Technologies. An interesting artefact (with both physical and virtual realization) was presented as a result of an international cooperation between France and Italy: it is a duo of artefacts (*pascaline* and *e-pascaline*,), constituted by a mechanical arithmetic machine (inspired by the instrument designed by Pascal in the 17th century) and its digital counterpart.

« The pascaline is an arithmetic machine composed of gears analogous to the famous machine, called Pascaline, invented by the French mathematician Blaise Pascal in 1642. It is a crucial tool in the history of European mathematics because it represents the first example of addition performed independently of the human intellect. […] It provides a symbolic representation of the whole numbers from 0 to 999 and enables arithmetic operations to be performed. Each of the five wheels has ten teeth. The digits from 0 to 9 are written on the lower yellow wheels, which display units, tens and hundreds from the right to the left. When the units wheel (respectively the tens wheel) turns a complete rotation clockwise, the right upper wheel (respectively the left upper wheel) makes the tens wheel (respectively the hundreds wheel) go one step forward. […] We have designed the e-pascaline, a digital version of the pascaline, to build a complementary duo of artefacts in which each component adds value to the other […]. The e-pascaline is not a simulation of the pascaline, as close as possible to the model, but is rather a separate artefact which is close enough to the physical one to enable students to transfer some schemes of use, but also different enough (in appearance or in behavior) to reduce components that have inadequate semiotic potential for mathematics learning » (Soury-Lavergne and Maschietto, 2015, pp. 372-3).

Fig. 4. The duo: pascaline (right) and e-pascaline (left)

In the working group, the case of this duo (pascaline and e-pascaline) was a prompt towards a discussion of the relationships between "real" and "virtual" manipulatives, to avoid the naive belief that a concrete manipulative may be substituted with a "virtual" copy without any consequence on the learning processes (see, for instance, the presentation of *The National Library of Virtual Manipulatives* in USA[12]). In the Conference, we collected several beautiful examples of "virtual" artefact (e. g. Kortenkamp[13], Sinclair[14] & Coles, 2015) and started the discussion about the consistency of them with the epistemological analysis of mathematical meanings.

3 Western and Chinese mathematics: the visit to the Ricci Institute

The comparison / contrast between the number line and the suànpán approaches hints at the differences between Western and Chinese mathematics. The Conference was held in the right place to encourage this discussion as Macau was the site of the Jesuit mission to China and Far East. Matteo Ricci was a pioneer of cultural relations between China and the West, and his appreciation of Chinese cultural and moral values enabled him to make China known to the West and the West to China.

The social program of the Conference offered the possibility to visit the Macau Ricci Institute and to listen to general presentation of the Jesuit activity in China and to a scientific presentation of Ricci's activity in "translating" the first six books of Euclid's Elements, given by Siu Man Keung from Honk Kong University (Siu, 2011). Matteo Ricci translated into Chinese Euclid's Elements (in the Clavius version), with the help of a collaborator (XU Guang-qi). The translated text was published in 1607 and was given the title *The Source of Quantity.*

幾何原本

Fig. 5. Jĭhéyuán běn

[12] website : www.nvlm.usu.edu

[13] http://www.facebook.com/PlaceValueChart

[14] website : touchcounts.ca

Euclid's Elements were different from traditional Chinese mathematics. Siu (2011) offers beautiful examples of the different thinking styles, the Euclid's one and the traditional Chinese one. They indicate a kind of incompatibility between each other so that it would be unnatural to force one into the mould of the other. However they also indicate an admirable attempt of Matteo Ricci and XU Guang-qi to synthesize Western and Chinese mathematics (Siu, 2011).

A well-known example is Pythagora's theorem or *gōu gǔ* (Dai and Cheung, 2015) that was known also by ancient Chinese mathematicians. Euclid's proof in the Elements is the 47 propositions of the first book of Elements (Heath, 1908, put in the order where it is possible to proof it drawing on the previous propositions in a deductive chain. On the contrary, the classical Chinese texts contain

« Multiple proofs to theorems and multiple approachese to the same problem so as to help the students when they need to apply the same principle to other similar problems. [..] [The different proofs] highlight the importance of applying known results (in this case the area of rectangles) to deduce some unknown facts (the Pythagora's theorem). Offering two proofs of the same problem help reinforce the understanding of the problem by looking into it from different perspectives. Students can be inspired by this way to actively look for other alternative approaches when they encounter mathematical problems in the future » (Dai and Cheung, 2015, p. 18-20).

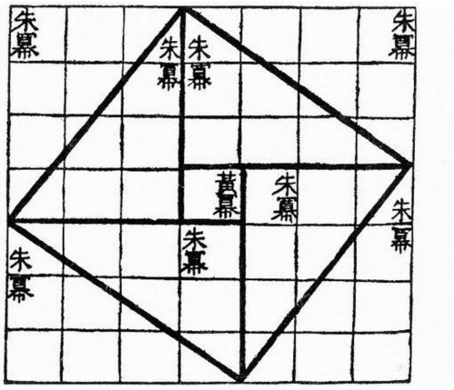

Fig. 6. The image of a classical Chinese proof of *gōu gǔ*, dating back to the first century BCE [15]

Chinese geometrical proofs are full of hints towards measuring, dissecting and recomposing and using visual proofs. Another feature of traditional Chinese mathematical teaching is the use of colors: colors in the proofs of Pythagora's theorem appeared around 200 BC (about a millenium before color-printing techniques were available in China), by labeling each portion (in dissection) with the name of the color. This was a very useful teaching aid. It is worthwhile to mention that, in the West, the first attempt to use colors for educational advantage is credited to Byrne's edition of the first six books of Euclid's Elements (1847[16]).

4 Primary school teacher education and development

4.1 The Chinese model of « open classes »

In the Chinese region, especially in the urban areas, it is likely that primary school mathematics teachers are specialists; hence they deepen, in the pre-service education at universities, some issues about mathematical contents. The teacher development is further carried out in the school system, by means of an original interpretation of the model of the Japanese lesson study (Inprasitha et al. 2015), that in this paper is mentioned as « open classroom » or « to observe classroom ».

15 http://www.maa.org/press/periodicals/convergence/mathematical-treasures-zhoubi-suanjing

16 https://www.math.ubc.ca/~cass/Euclid/byrne.html

观摩课

Fig. 7. Guān mó kè : *to observe classroom*

There are different types of open classrooms (Sun, Teo and Chan, 2015):

Open classes for outside audience

➢ open-classes to publicly demonstrate new education ideas (e.g. new curriculum/textbook use and expert-level classroom instruction demonstrations),

➢ open-classes for *research* purpose (e.g. research lessons for new thought, same content-different-approaches)

➢ open-classes for *evaluation* purpose (e.g. recruitment, teacher-promotion, and teaching competitions).

The second type above (*same content – different approaches*) is connected to the ancient Chinese tradition of looking at the same problem from different perspectives (mentioned above). We shall reconsider it below.

Open-classes for internal audience

➢ *single-cycle* open-classes for mentor-mentee training

➢ *multiple-cycle* open-classes for mentor-mentee training, supported by the school-based mentor-mentee programme: co-planning, co-designing, co-teaching, co-reflecting.

The Chinese model of open classes has exploited the Chinese conception of teaching as a public activity with norms and structures that favour a *collaborative spirit*; has exerted a major influence in the *professional development* of teachers in China for many years; has played a major role in fostering *learning communities* within Chinese schools; has proven to be an effective way to induct new and inexperienced teachers into the teaching profession.

The basic idea is to design and to test a lesson in order to improve it. This way of working in the school system (with little support if any from the University, in contrast with the Japanese lesson study) is related to the cultural identity of the Chinese teachers. We may quote an example from (Trouche, 2015), in the report of a visit to a class of prospective teachers in Shanghai.

« A la question « pouves-vous citer un mot chinois, relatif à l'einseignement des mathématiques, dont la traduction pose problème ?, la résponse des estudiants fut tout de suite le mot (fig. 8) qui demande, selon les étudiants, pou ètre « traduit », tout un developpement : « le processus de préparer une leçon sur le temps long, avec la volonté de la faire de mieuz en mieux, avec la conviction que c'est un processus sans fin […] et que c'est une responsabilité essentielle de l'eisegnant, qui doit se nourrir des interactions avec ses élèves et ses collègues » (Trouche, 2015)

磨課

Fig. 8. mó kè : *to clean the lesson*

Although the literal translation seems very simple, the true meaning, according to the Chinese students, refer to the complex process of multiple cycles, that takes place in the open classes.

4.2 The open class at the Hou Kong primary school

The experience of a research open-class was offered to the participants by the Hou Kong primary school in Macau, with two lessons, one about addition and one about subtraction. In this paper we report only the lesson about addition.

Research open-classes are a standard way of working in the Hou Kong School, where a very active Mathematics Research Group is established with the habit of multiple cycle for co-planning, co-designing, co-teaching, co-reflecting. The two of us observed two consecutive lessons of the same multiple cycle with the same teaching plan, the same teacher and different first grade classes. It was evident that some « small » but relevant changes had been designed and realized in order to have a smoother functioning and a better use of time. Between the two lessons (a ten days interval) meetings of the Hou Kong Mathematics Research Group have been realized in order to analyse and criticize the teaching plan and the videos of the lesson (co-reflecting). The Conference participants observed the second lesson only. The teaching plan was distributed in advance.

The structure of the lesson followed the classical scheme used in mainland China (Wang, 2013), with an additional short practice about the mental calculus skill (about 6 minutes in total, at the beginning and at the end of the lesson). The importance of speed in mental calculus is stated in the Standards (in both Macau and mainland China) and is realized controlling the speed with a timer (fig. 9).

Fig. 9. The timer

The core of the lesson was about different ways of calculating two digits addition with grouping and regrouping (grouping tens). The lesson was realized in the sport hall, with a classroom in the middle (with desks, a whiteboard, an interactive board) and seats for about 60 observers from the Conference and 20 observers from the school (including the members of the Mathematics Research Group, the principal, the English teachers for helping in translation). The whole lesson was video-recorded in order to be analysed again by the Hou Kong Mathematics Research Group. A short session of discussion was realized immediately after the lesson with the Conference participants.

The total time of the lesson was 40 minutes, but the core problem (the situation of the day) was carried out in about 15 minutes, as planned. A short summary of the general teaching plan and of the situation of the day follows.

4.3 The general teaching plan

Lesson Plan
School: Hou Kong Primary School
Grade one: Class A
Teacher: Miss Amanda
Subject: Mathematics
Date: 5th June, 2015
Time: 40 minutes
Teaching Topics: Addition within two digits numbers and one digit number (with regrouping)
Students' previous knowledge:
1. Addition and subtraction up to 20 (11+2=13, 13-2=11)
2. Addition up to 20 (with regrouping) (9+8=17, 7+5=12)
3. Subtraction up to 20 (with decomposition) 11-9=2, 12-8=4)
4. Addition and subtraction within tens (40+20=60, 70-50=20)(without regrouping and decomposition)
5. Addition within tens and one digit number (20+4=24, 60+9=69)(without regrouping and decomposition)
6. Addition with two digits numbers plus one digit number (without regrouping) (25+2=27)

7. Addition within two places (without regrouping) (25+20=4)

Learning objectives:

Knowledge acquired: Conceptualize and perform addition within two digits numbers and one digit number with regrouping in oral calculation.

Skill developed: Develop and train critical thinking and language skills.

Civic education:

1. Foster the spirit of co-operation and self-learning.
2. Experience the relationship of mathematics in daily life.

Common learning difficulties:

Pupils may have difficulties in moving the composition of numbers. The teacher can use the concept of « making 10 » to help them understand the composition of numbers.

Teaching aids:

Overhead projector (OHP), Multimedia, candies (boxes) (and other cards for further exercises).

The candy boxes are very interesting artefacts. Each small group of four pupils receives 4 candy boxes with 2 boxes of 10 candies, 1 box of 4 candies (and 6 empty places) and 1 box with 9 candies (and 1 empty place). In each box, the candies are put in 2 lines with 5 places each.

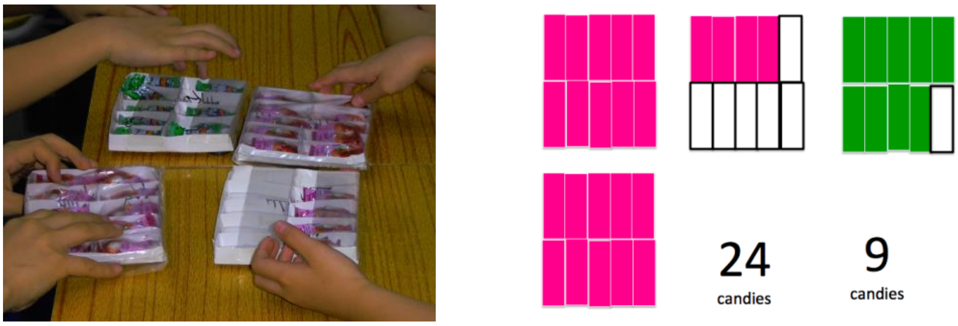

Fig. 10. The candy boxes

The empty spaces in the candy boxes have been arranged to foster different strategies to « make 10 », that is to get full boxes by moving some candies. The different colors in the boxes representing the first and the second addendum allow noticing what candies have been moved.

A scheme of the candy boxes has been prepared also in the interactive white board: it was useful to represent in a different way the actions and operations made by pupils with the concrete boxes.

4.4 The detailled teaching plan for the situation of the day

Situation setting	a. Teacher gives a situation to the class. «There are many guests in our school today. So, Miss Amanda prepares some food for them. Class, can you help me count the food as fast as you can? »	15 minutes
Problem solving	a. Pupils work in groups. b. Provide some candies to each group and let them count.	
Group counting and sharing *(communication, conceptualizing, inquiring)*	a. T invites some group to report their finding about how to count the candies altogether. b. Give comments to the groups and use the multimedia to show three different ways to count the candies. **The first way of putting candies altogether** There are 24 candies on the left, and then there are 9 candies on the right. Next, encourage pupils to investigate and move 4	

	candies on the left and 6 candies on the right to « making 10 ». Finally, 30 candies plus 3 candies equals 33 candies altogether. **The second way of putting candies altogether** There are 24 candies on the left, and then there are 9 candies on the right. Next, encourage pupils to investigate and move 1 candy on the left and 9 candies on the right to « making 10 ». Finally, 23 candies plus 10 candies equals 33 candies altogether. **The third way of putting candies altogether** There are 24 candies on the left, and then there are 9 candies on the right. Next, encourage pupils to investigate and move 4 candies on the left plus 9 candies on the right equals 13 candies. Then, there is a « making 10 » in 13. Finally, 20 candies plus 13 candies equals 33 candies altogether.	

4.5 The process

The small group work is realized by pupils four by four, simply turning a little their chairs to face the two pupils in the back line. During the small group work, the teacher walks in the classroom to encourage pupils to count by « making 10 » in different ways. At the end, she calls the representatives of three groups to show their solutions on the magnetic whiteboard, explaining verbally their process. Three summary forms (already prepared by the teacher, drawing on the teaching plan analysis) are hung by the teacher in the whiteboard below the concrete solutions of the pupils (fig. 11).

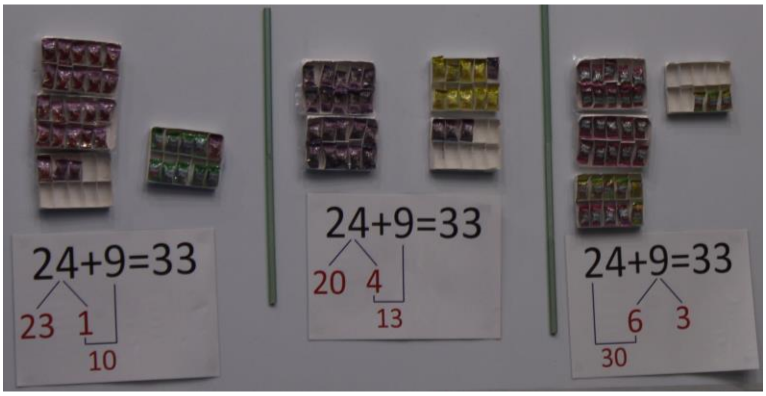

Fig. 11. One Problem Multiple Solutions (OPMS)

This problem is consistent with different issues in the Chinese culture of teaching. It may be defined (Sun, 2011) as One Problem Multiple Solutions (OPMS), within the general category of additive variation problems, as the same problem (24+9) is to be solved in different ways. Looking at the three solutions together foster pupils' attitude to look at the same problem according to different perspectives (see the general discussion in the section 3 above). Moreover, the general idea of « making 10 » refers clearly to the operations with the suànpán.

4.6 A comment from a French participant

A comment of a French participant is reported.
Les classes ouvertes, une expérience singulière.

Nous avons aussi eu l'occasion d'assister à une « classe ouverte » dans une école. Lorsqu'on nous l'a proposée, nous n'imaginions pas que nous serions environ cinquante personnes à suivre tous ensemble une leçon sur l'addition pour une vingtaine d'élèves de 6 ans. Mais si ! Installés en cercle dans la salle de gymnastique, les bureaux d'élèves et les tableaux au milieu, nous avons pu observer les travaux de l'enseignante, très dynamique, et des élèves en uniforme attentifs et impliqués, le tout en chinois évidemment. Sans pouvoir comprendre les détails de l'interaction, nous avons bien saisi l'organisation très construite et précise de la séance, l'utilisation d'outils pédagogiques variés comme des boites de 10 bonbons […] et la forte interaction de l'enseignante avec les élèves collectivement, individuellement ou lors du travail de groupe. Au premier regard l'aspect très programmé de la leçon, ne semble pas permettre une adaptation à la diversité des apprentissages et semble laisser certains élèves en difficulté. Il pourrait même, les solutions des calculs étaient déjà préparés et cachés, conduire les élèves à concevoir le travail mathématique comme la recherche de la réponse déjà prévue par l'enseignant. Cette préparation très réglée d'une leçon s'appuie cependant sur la mutualisation de l'expertise des enseignants d'une école, ou d'une circonscription, sur le long terme, et on peut aussi considérer qu'elle outille l'enseignant pour lui permettre d'ajuster la diversité des outils pédagogiques dont elle dispose, dans le feu de l'action. Cela invite à réfléchir à la question de la formation des enseignants et des ressources nécessaires pour construire un enseignement progressif et cohérent en ce qui concerne par exemple l'institutionnalisation des connaissances construites au cours d'une leçon, la formulation des conclusions et les traces écrites au tableau. Cela offrirait surement aux élèves une cohérence au cours de l'année et d'une année à l'autre, d'un enseignant à l'autre, favorable aux apprentissages (Soury-Lavergne, 2015).

IV - CONCLUDING REMARKS

We have selected some examples from the Conference which have the potential to address cultural relativism in mathematics education: is really the teaching and learning of whole numbers the same all over the world? Why is the number line so popular in the West and not in China? Are there different thinking styles in Chinese and Western traditions? How is primary school mathematics teacher education and development approached in different part of the world?

As participants in other ICMI studies, we believe that this study has some peculiar features:

- The preparation of a context form, to be filled by each participant, to give the background information of the study and/or its theoretical statements,
- The invitation to submit video-clips with papers, to exploit the effectiveness of visual data in the age of web communication,
- The participation of IPC members as authors and not only as organisers and co-leaders of working groups,
- The scientific support offered to authors in the revision of their papers,
- The economic support offered to authors from the University of Macau,
- The supported participation of CANP observers,
- The involvement of both the IMU President (Prof. Shigefumi Mori) and the ICMI President (Prof. Ferdinando Arzarello) in the preparation of the Conference.
- The possibility of observing open classes to come in touch with the Chinese model of teacher education and development,
- The possibility of visiting Macau Ricci Institute to discuss about the most famous attempt to make the Western and the Chinese Mathematics meet each other.

This collective international effort led us to the Macau Conference, as a product of the fruitful cooperation between mathematicians and mathematics educators, when, for the first time in the history of ICMI, the issue of whole numbers arithmetic in primary school was addressed.

We firmly believe that meeting other cultures is a way to get a deeper understanding of one's own.

« This is not about comparative philosophy, about paralleling different conceptions, but about a philosophical dialogue in which every thought, when coming towards the other, questions itself about its own unthought. » (F. Jullien, 2006).

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