

----- ED509: a course in history and psychology for second year university mathematics students -----

David Nelson
School of Education, University of Manchester
England

Abstract

The decline in the quality and quantity of mathematics graduates entering the teaching profession has been a matter of acute concern to mathematicians in the U.K. in recent years. In 1990 the Mathematics Department of Manchester University requested a 2 semester course in Mathematics Education for inclusion as an option in the second year of their three year degree course. The author responded with a course which for 1 semester discusses the early history of mathematics and for the second semester examines psychological studies of (i) learning mathematics (ii) mathematical activity. This paper discusses the genesis of the course, its content, assessment, and the work and opinions of the students who have taken it.

Introduction

Mathematics students at English universities who wish to become teachers usually take a 1 year Postgraduate Certificate of Education (PGCE) after completing their 3 year degree course. The need to maintain an adequate flow of mathematics graduates into school teaching has been a matter of concern in recent decades: see (1), (2), (4), (6), (7). For example, the Cockcroft Committee reported that in 1979 only one third of mathematics teaching in secondary schools was being done by mathematics graduates.

In the 70's and 80's mathematics graduates had many job opportunities other than teaching. The majority went into commerce or industry and the percentage entering teacher training fell from 32% in 1962 to 7.5% in 1984. Moreover there was growing concern about quality: "Not only are insufficient numbers coming forward but the quality of those who do enter teacher training, as judged by degree results, is low in mathematics and physics in relation to other subjects" (7).

At Manchester the situation was slightly better than average -10% entering teacher training in 1984, some of whom were very well qualified. Nevertheless we too were concerned and decided to investigate the attitude of our mathematics students to teaching as a career.

There appeared to have been few such studies at the time. The Cockcroft Committee (1982) took its evidence on attitudes from the Association of Graduate Career Services. Later, in 1986 the Department of Education and Science commissioned a study and obtained a qualitative investigation based on 35 interviews with 110 students in the final year of mathematics or physics degrees at 6 universities and 3 polytechnics (3).

In 1987, after careful discussion and with the energetic co-operation of the Mathematics Departments at Manchester University and UMIST (University of Manchester Institute of Science and Technology) we put a 10-item questionnaire to all third year mathematics students and 157 out of 204 completed it. The same questionnaire was given to third year students in 1988 and 138 out of 207 responded. Overall the response rate was 72%.

In both years about 8% stated that teaching was their first choice of career, describing it as a rewarding, challenging occupation. The 272 (92%) who did not put teaching as first choice were asked to rank various reasons for not putting teaching first. The following table shows how important each reason seemed to the students.

Reason	Rank 1: frequency	Rank 1-3: frequency
pay levels too low	81	185
poor work conditions in school	31	106
low status of teaching	17	106
too hard a job	17	38
mathematically undemanding	12	37
little job satisfaction	35	65
too time consuming	3	19
little career prospects	33	148
other	24	37

(19 students failed to complete this item)

So this table shows that 31 students gave 'poor work conditions in school' as their principal reason for not wishing to teach and 106 gave it as one of their 3 main reasons. The antiquity of this complaint is borne out by the comment on Roman Education by Juvenal who

"...represents it as one of the hardships of a teacher's life that he has to breathe air in his schoolroom poisoned by the smoke of the many lanterns which the pupils brought before dawn." (8)

The main reason given under "other" was "unsuited to working with children" (given rank 1 by 21 students).

These findings are in line with those of the qualitative study mentioned earlier where the negative image of teaching was characterised by "The low salaries, poor status, unattractive work, remoteness from the degree subject and other negative perceptions..." (3)

Two responses to teacher shortage

It has been said that 'ignorance breeds fear and fear breeds resentment' and it was partly a desire to provide more information about teaching that led to two developments at Manchester University.

The first was the establishment of a scheme, in partnership with 12 schools, which enabled undergraduates in mathematics (and later physics) to visit a local school regularly to observe and assist the teaching of mathematics. The scheme began in 1987 and continues to run successfully. Students who have been in recent contact with life in schools and enthusiastic teaching feel they are better placed to make informed decisions about their careers and this has helped recruitment to teaching.

The second initiative grew around the hypothesis that an academic course could also show students mathematics education is a skilled, intellectually interesting and challenging occupation. So it was decided to introduce a 2-semester optional course of mathematics education into the second year of the degree and provide a demanding rigorous course which bore comparison with the students' other courses. For one should always remember the remark reported by Polya "The mathematics department offers us tough steak we cannot chew and the school of education vapid soup with no meat in it." (5) Thus the course ED509 Mathematics and Education was launched in 1990 and allowed students to devote one sixth of their second year to the study of mathematics education.

The course structure

We assumed that mathematics students who intended to teach would probably take this course but there would be others who did not want to teach but wished to include educational studies in their degree. We also knew that some students, but not all, would be participating in the

school experience scheme mentioned above.

We took into account the fact that two areas which tended to excite our PGCE students were the history of mathematics and the psychology of learning mathematics and solving problems. These students often chose to base their main mathematics education project in one of these areas. Moreover, informal discussions with former students and local teachers confirmed that they would have welcomed foundation courses in history and psychology as part of their degree.

Thus in 1990 we offered the following course which gives a 12 week semester to each area. It attracts about 10 students each year.

First semester History of Mathematics

1. *Early History*
Babylon and Egypt, Greece and Alexandria, India and China, Arabia and Europe to 1202.
2. *Selected Topics*
 - (i) The general solution of the cubic in the sixteenth century.
 - (ii) The development of Non-Euclidean Geometry.

Second semester Mathematics and Psychology

1. *Problem Solving*
Gestaltist, Associationist and Information Processing approaches. Heuristics and expertise.
2. *Psychology of Learning Mathematics*
The work of Piaget, Skemp, Krutetskii and others, and its relation to the National Curriculum for School Mathematics.

The students receive two one-hour lectures and one examples class or tutorial each week.

In the first semester, 8 weeks are given to Early History and 2 weeks to each Special Topic. The emphasis on Early History is deliberate. The mathematics of this period is not yet too technical for students and they can reflect on a number of questions we believe to be important in the education of a teacher: (i) How did mathematics begin? (ii) What has been the purpose of mathematics? (iii) How has mathematics evolved in Europe and in non-European cultures? (iv) Who were the key figures in its development?

The two special topics were selected because they involve problems of major importance which have ancient origins and yet take the student at least into nineteenth century mathematics. They are taught by Peter Eccles and Mike Prest of the Mathematics Department.

The psychology component is divided into Problem Solving (6 weeks) - the classical approaches to general problem solving, the work of Polya and Schoenfeld with respect to mathematics - and Psychology of Learning Mathematics (6 weeks) - essentially an introduction to some constructivist thinkers, cognition, learning theory, symbolisation and studies of children's mathematical abilities. The component is taught with some reference to the National Curriculum for England and Wales and to current teaching practice in our schools e.g. with respect to problem solving.

A sheet of examples is given out at the start of each week and discussed in the examples class, after which the solutions are provided. Here are three problems taken from sheets for the Early History, Problem Solving and Psychology components respectively.

- Ex.1. Problem 31: of the Rhind Papyrus. "A quantity, its $\frac{2}{3}$, its $\frac{1}{2}$ and its $\frac{1}{7}$ added together become 33. What is the quantity?" Solve by the method of false position.
- Ex.2. Find the volume of the unit sphere in 4-space.
- Ex.3. Give examples from your own learning experience when new knowledge was (i) assimilated into an existing schema and (ii) accommodated by extending or adapting a schema.

Examination arrangements

To allow students to follow up their interests, 40% of the final work is for a historical essay and for a problem solving project. Historical topics range from individuals: Brahmagupta, Leonardo of Pisa, Lobachevski to subjects such as Paradoxes of the Infinite, the Peruvian quipu and Algebraic Notation. For many students this is their first opportunity to write about mathematics.

For the psychology project, students normally make a detailed study of subjects in a problem solving situation, sometimes school children but more often undergraduates and lecturers (maybe asking them to attempt a problem such as Ex.2. above). Though some of these projects are replications or variants of old experiments, many break new ground for, as Graham Hitch has pointed out, the students have the mathematical sophistication to explore areas of mathematical behaviour which most experimental psychologists do not reach.

To complete the assessment the students take a 3 hour written examination at the end of the course. This counts for 60% of the total and the mark usually matches the coursework mark quite closely.

Course Evaluations

The course was evaluated by the students of 91-92 and 92-93. In each year the course was rated as very interesting, slightly easier than mathematics courses, and appeared to be valued intrinsically rather than as a preparation for teaching. A number of students said they took the course because of an interest in history (or psychology) and were surprised to find the psychology (or history) so interesting. When asked whether they would prefer a course devoted entirely to history (or psychology) the majority felt the present mixture should be maintained - "the psychology complements the history." This perhaps highlights a key feature, namely, that the first half is primarily concerned with how a culture builds mathematical knowledge and solves problems whereas in the second half the emphasis is on how the individual and the child in particular develops knowledge and solves problems.

Concluding Remarks

The course has had an encouraging reception from students and a number of them have decided to enter teaching as a result of it. In 1993, in response to requests from the Mathematics Department we launched another course, ED510, which continues the programme into the first and second semesters of the third year of the degree.

I am grateful to Brian Hartley, John Reade, John Walters and Chris Williams for generous assistance with the 1987,88 surveys and to Peter Eccles, Mike Prest and Graham Hitch for helping teach and examine the course.

References

- (1) Department of Education and Science (1963) "Report of the Committee on Higher Education" (The Robbins Report) London: HMSO
- (2) Department of Education and Science (1982) Mathematics Counts (The Cockcroft Report) London: HMSO
- (3) Department of Education and Science (1986) Report by Social and Community Planning Research, internal document.
- (4) Howson, A.G. (1987) Challenges and Change Bulletin of the Institute of Mathematics and Its Applications, 23, pp 177-83.
- (5) Polya, G. (1965) Mathematical Discovery, Volume 2 New York: Wiley.
- (6) Smithers, A. and Robinson, P. (1988), The Shortage of Mathematics and Physics Teachers, University of Manchester: Department of Education.
- (7) Straker, N. (1987) The Decline of School Teaching as a Career Destination for Mathematics Graduates, Teaching Mathematics and Its Applications, 6, pp151-6.
- (8) Wilkins, A.S. (1905), Roman Education, Cambridge: Cambridge University Press.