

----- The use of Mathematics History and Epistemology in Mathematics Education of Teachers -----

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Abstract: The paper is intended to show the presence of the History of Mathematics in Teacher Education in Portuguese Universities and Schools of Education. The topics That are taught at some universities are described. Results of teachers' knowledge concerning the history of Mathematics are shown. The knowledge of prospective mathematics teachers at the university entrance is discussed. One experience with a teaching course on mathematics history is described and results are discussed.

1. The History of Mathematics and Teacher Education

1.1. The Mathematics Teacher

The use of mathematics history on the teaching of mathematics, could be analysed based on teachers' preparation at the university/polytechnics level, teachers' willingness, and training.

At the middle and secondary level, there are a lot of teachers that are not specifically qualified to teach maths. This means that some of them have a university degree, but not in maths (engineering and economy are the most common) and a professional diploma, and some of them don't have any specific degree. The mathematics teachers for middle schools (5 th and 6 th grade, age 10/11) belong, mainly, to the 1st group.

Teachers with maths degree are mainly teaching 10th, 11th and 12th grades (age 14/15 - 17/18).

If we assume that mathematics teachers start working as professionals at the age of 24 and reach retirement only at the age of 70 years old, the mean age will be 47 years old. It implies that their mathematics preparation at university starts on the late fifties or early sixties. That was the age of " modern mathematics " in Portuguese Universities and the mathematics courses were intended to prepare mathematicians and not to give any special attention to the preparation of teachers.

According to some published materials (gazeta da Matemática nº 68-69, Set/Dez 1957 and Gazeta da Matemática nº 100, Julho/Dez 1965) the recommendations from international meetings (Recomendação nº 43 da Conferencia Internacional da Instrução Pública, e Conclusões e Recomendações do Simpósio Internacional sobre o Ensino Escolar da Matemática) stated that the mathematics courses (1956-1965) must consider:

- Mathematical Logic and Set Theory
- Algebra
- Topology
- Geometry (axiomatics and its implications)
- Analysis
- Mathematical Theory of Probability and Statistics
- History of Mathematical Thinking

In general these topics have been stressed, in University Courses, until today.

Concerning specif preparation for teaching only in 1972, a diploma in the teaching of Mathematics was created. Its curriculum includes courses in didactics, methodology and psychology of learning. Mathematics History is, in general, a semester course.

For a long period (about 1952 - 1974) the secondary school curriculum has been devoted to basic skills in calculus, geometry algebra and analysis, and is supposed to prepare

pupils to attend university courses. From 1974 - 1992 the secondary school mathematics curriculum was modified (set theory and algebric structures were include) but general objectives were simplified and the scope of contents was reduced. In both periods, some courses on the didactics of mathematics have taken place but only a few teachers (teachers' supervisors and inspectors) have attended them. The opportunity to attend seminars, summer courses, or short term courses is also difficult for teachers. On the other hand there wasn't a strong educational policy for implementing in service teachers' courses to give teachers expertise on contents such as mathematics history and geometry.

1.2. Prospective Teachers Training Courses at Universities and Polytechnics

These are some examples of the courses that are given at the university level. It's duration could vary between one or two semesters. In what concerns the preparation given to prospective teachers, at the Higher Schools of Education (Polytechnics), it must be said that only some aspects of the history of mathematics are included in courses dealing with the didactics of mathematics.

. University of Coimbra

Course Title: "The History of Mathematical Thinking"

main aspects

- . The study of Portuguese Mathematicians and the development of mathematical knowledge:
 - . Pedro Nunes (XVI century) and the solution of equations and geometric problems
 - . Anastácio da Cunha (1744- 1787) and the study of mathematical sequences and others concepts developed in his book " Principia Mathematica" .
 - . The study of written materials (textbooks, conferences, articles) by Portuguese mathematicians, such as José Sebastião e Silva (1914- 1972) and Bento de Jesus Caraça (1901- 1948).
- (These authors have related the teaching and learning of Mathematics with the use of mathematics history in the development of mathematical activities for pupils).

The contribution of educational reforms for the development of the study of mathematics since 1773.

ressources: original texts and selected Bibliography

. University of Minho - Braga

Course Title: "Mathematics History"

main aspects

- Mathematics History Methods
- Egyptian and Babylonian Mathematics
- Greek Mathematics : the Pythagoreans, the Eleatic school, the Aristotle's influence upon Mathematics, Plato's Philosophy of Mathematics, Euclid's Elements .
- Euclid , the Theory of Parallels and the birth of Non - Euclidean Geometry,
- Hilbert's " Foundations of Geometry"
- The history of Mathematics in Portugal

ressources: original texts , selected Bibliography.

1.3. In service Teachers training courses

Mainly in service teachers' training courses are devoted to give a professional diploma. The courses are on the Psychology of Learning, Curriculum Development, School Management and Didactics of Mathematics.

The specific content or areas of study are established by the didactics course mentors. In general, this study is devoted to scientific knowledge and to the development of instructional activities based on the secondary curriculum for secondary school education.

In the last two years, due to the curriculum reform, some universities and professional unions have given short courses (1 day or 1 week) on mathematics history.

2. The teaching of the Mathematics History at secondary school level, teacher's knowledge, teacher's resources and pupils' knowledge

2. 1. The New National Mathematics curricula for Primary, Middle and Secondary Schools

The new Portuguese Mathematics curricula for primary and secondary education states, concerning methodological aspects to meet general objectives, that teachers must:

- develop activities concerning the growth of human knowledge ;
- teach concepts according to the way they were developed by the human being (genetic approach)
- organize instructional activities that develop the student knowledge about the contribution of mathematics to the comprehension and solution of human problems over time (through the centuries);
- establish the relationship between mathematics historical steps and the growth of humanity;

Maths curricula also give examples about the way that some aspects of Mathematics history must be developed:

- "- when exploring the Pythagorean theorem it would be useful to refer some aspects of Mathematics History (the contribution of Greeks and Eghipciens, the twelve nodes rope episode, the concept of proof in Mathematics History , etc);
- pupils must do some group work on projects dealing with equations and the way solutions are found over time (in Ancient Greece, in The Mathematics of the XVI century and using the work of Pedro Nunes (Portuguese Mathematician 1492 - 1577), the use of symbols and their contribution to the development of equations' solution, and their application and use.)
- Historical aspects of Trigonometry (ex: how Eratosthenes found the earth sphere radius)
- Historical aspects of geometry (the understanding of axioms, theorems and proof)

These are some of the demands teachers have by now, but with the lack of materials, resources, and knowledge about how to develop learning activities, teachers very soon will forget this kind of methodological demands.

2.2. Resources

Secondary schools teachers very often rely on the mathematics textbook to develop their teaching activities. Knowing this, textbook authors start in 1991 introducing some historical aspects in textbooks, mainly called "curiosities ". But teachers need more specific information for their own preparation and for the construction of learning materials. Having this in mind, Silva (1993) has prepared a bibliography list with 68 references, and comments that there are only four good books written in Portuguese.

A survey on the mathematics textbooks that have been used for the last 12 years, show that about 70% have little episodes or biographies and the last 30% don't have any reference to historical events or to mathematicians biographies.

2.3. Teachers knowledge about concepts : an exemple " The number π "

The introduction to the concept of π starts at grade five when perimeter and area of the circle are studied. Further explorations concerning the number π as a real number are done on 8th /9 th grade. However teachers are not very aware of the dimension of number π use and applications. This statement is supported by a questionnaire applied to 29 teachers at the start of a one day course on " the number π ".

The questionnaire results are presented in the following table:

The number π is associated with:	% of correct answers	% of incorrect answers	% of blank answers
1. the perimeter of the circle	100 %	0%	0%
2.the computing of areas and volumes	100 %	0%	0%
3.an infinite product of rational numbers	56%	12%	32%
4.trigonometric functions and infinite series	42%	17%	41%
5.the limit value of the serie of reciprocals of the odd integers with alternating signs	15 %	52%	33%
6.the sum of the reciprocals of the squares of the positive integers	4 %	42%	54%
7.the area under the Gaussian curve	37%	21%	42%
8.the probability of a lattice point , chosen at random, to be visible from the origin	23%	31%	46%
9.to the probability of two integers , chosen at random, beeing relatively prime	8%	37%	55%

2.4. Students' knowledge

Based on the survey about mathematics history on textbooks, it's hypothesized that some knowledge about mathematic events, concepts and mathematicians' biographies had been acquired by students at the university entrance.

To develop the study, a questionnaire was modeled after the historic references found on the textbooks studied. The preliminary results show that sentences like:

- . Claudius Ptolemy (85 - 165) and René Descartes (1596 - 1650) divided the glory of having created the Cartesian Geometry;
- . the use of straightedge and compasses starts with Arabs studies in astronomy and navigation in the IX century;
- . the Indu - Arabic numeral system was developed by the ancient Greeks.

The better results, among the 14 questions, obtained were those described in the table which correspond to the questions:

Questions	1	2	3	4	5	6
% of correct answers	28 %	50 %	40 %	32 %	40 %	32 %

1. - The mathematics language

- . arises from the common language
- . only establishes relationships between numbers
- . results from abstract thinking upon real life problems

2. - The Analytic geometry allows:
 - the arithmetization of geometry
 - the use of coordinates to represent a point on a plane
 - to describe geometric figures by the use of equations.
3. - The learning of the Roman numbers:
 - as a curiosity
 - to understand ancient documents and inscriptions
 - as path towards the number systems comprehension.
4. - The name of Pythagoras is connected with:
 - the proof of geometric properties of the right triangle
 - the dependence of musical intervals upon numerical ratios
 - the discovery of mathematic formulas that gives the distance between the earth and the moon.
5. - To the Greek mathematician Euclid is ascribed:
 - the writing of the "Elements".
 - the first mathematician that was paid for his work
 - the study of prime numbers properties and the algorithm to find the greatest common divisor between two numbers.

Note: Each question was considered correct when all three statements were signed properly (true or false).

The results obtained by this exploratory study , suggest that prospective teachers need :

- to be prepared to face mathematics not only as a structured scientific knowledge but also as human science and to know the cultural connections of mathematics in the sense that mathematics doesn't develop in a vacuum.
- to develop a better appreciation and understanding of mathematical problems and procedures.
- to have knowledge about the history of the mathematic subjects they are supposed to teach.
- to develop their ability to communicate their own personal enjoyment of mathematics.

3. A course on History and Didactics of Mathematics for prospective teacher (High School of Education - Castelo Branco)

Population

Prospective teachers in the last year of curricular studies of a Maths/Science degree. Students develop teaching practices at pre-secondary schools at the same time.

Aims

- To construct a basic knowledge about the development of mathematical thinking with respect to numbers, numeral systems, early computing, fractions, and geometry.
- to foster an understanding of how mathematics is used and why it is needed in society
- to develop an understanding of the nature of mathematics
- to give a deep insight into mathematical reasoning: hypothesizing, proving, generalizing, etc.
- to develop teaching and learning skills, based on the study of specific aspects of mathematics history, using computer devices and audio-visual technics.

Course organization

1. Student's work

Prospective teachers choose a topic from the curriculum syllaby of the 5th or 6th grade (Ex: rational numbers; numbers and properties; quadrilaterals), and are expected:

- (a) to begin an on-going exploration of the literature which is relevant to have a good knowledge of the history of mathematics related to the subject they choose .
- (b) to devise a plan for a learning unit which must connect mathematics history and learning activities and which also includes assessment devices.
- (c) to study films, cartoons, articles and software and other everyday materials to devise learning materials for classroom use.
- (d) to devise a sequence of lessons for the learning unit with a detailed description of learning activities that must incorporate the use of instructional aids such as computers, videotapes and manipulative materials (ex: cuisenaire rods, geoboard) and with mathematics history.

2. Time schedule

3/4 of a school year, 3 hours a week.

3. Methodological aspects

To perform (a), (b), and (c), lectures are given on the nature of mathematics as a subject, on research into the learning of mathematics, and on maths history. Textbooks, journal articles, videotapes and software are explored in group work. To present their conclusions, groups must give small written reports. They can also use posters or other kinds of visual aids.

Part (d) is an individual work. Its structure and contents are discussed with the teacher, and each activity is carefully analyzed. A tutorial work is developed.

Results

Each student used a different approach in developing their final work. These will be summarized by topic, major characteristics of the work produced, resources and prospective teachers attitudes developed.

① Topic ② Number of references	Major characteristics of the work	Connecting historical events with lessons' plans materials	Student commitment
① Fractions (concept, equivalence, ordering and algorithms) ② 3	. historical narrative (primitive counting and Egyptian use of fractions) . Kieren levels of instruction	Learning activities don't have any relationship with historical events	Not very strong
① Angles and triangles ② 2	Historical narrative (Pythagorean and Egyptian procedures)	Learning activities don't have any relationship with historical events	Not very strong
① Perimeter and Area ② 4	Historical narrative (From primitive concepts to Hilbert)	Learning activities don't have any relationship with historical events	Not very strong
① Polygons ② 4	Historical narrative (Euclidean constructions)	Historical episodes are used in lessons' plans	Strong
① Statistics ② 5	Historical narrative (From Roman census to XVII century)	Learning activities don't have any relationship with historical events	Strong

① Integers and their properties ② 2	Historical narrative (The use of numbers in ancient China, India, Greece and its development until XVII century)	Historical episodes are used in lessons' plans	Not very strong
① Problem solving for broadcasting programs ② 8	Historical narrative dealing with the context or the content of the problem	Each program was planned with three issues - the problem, the historical connection and the discussion of the problem solution	Very, very strong
① Instructional Aids: the compass, the ruler, the straightedge ② 5	Historical narrative dealing with the use of the compass, ruler and straightedge by Greeks and the famous unsolved problems	The constructed piece of software enables the exploitation of the historical context and the understanding of the use of these tools	Very, very strong
① Exploring textbooks for secondary level ② 15	A survey on historic events presented and the way they are explored	Problem solving activities based on the arithmetic / algorithms progress through the centuries (Egyptian, Russian and Fibonacci computation, numbers series)	Very strong
① Exploring the number π using a video cassette ② 4	Construction of a framework of historical events, their selection and translation	Learning activities and instructional materials were constructed according to selected historical episodes	Very, very strong

Materials

- .Students questionnaires developed by the author
- .Teachers questionnaires developed by the author
- .Textbooks on mathematics history, school manuals, journal articles
- .The videotape on "the story of π ", program guide and workbook, written by Tom M. Apostol.
- Project Mathematics, California Institute of Tecnology.
- .Individual student plans developed during the course.