NAVIGATION AND SURVEYING : TEACHING GEOMETRY THROUGH THE USE OF OLD INSTRUMENTS

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The workshop included an exhibition of twelve Al boards which illustrated various aspects of old instruments. These boards were set up in the lunch interval, so that early arrivers at the workshop (or late leavers !) could occupy themselves by looking at this material. This exhibition featured one board on the use and development of the cross staff ; one board on the quadrant ; eight boards on sundials in Northumbria ; one board on international sundials ; and one board on using sundials in schools.

Outside the workshop room a table was set in the sun with a vertical shadow stick erected on the top. As participants arrived they were invited to mark with a cross and their initials were they thought the tip of the shadow would be at the end of the session. This was the basis for the first activity on the sun activity worksheet (see appendix : worksheets 1a and 1b). A small prize was offered (a bottle of wine), which was won at the end of the session by Caterina Vicentini, with a banana as a booby prize ! (winner will remain anonymous : is that O.K. Fred ?).

The workshop started with an introduction about the school where I teach. Prudhoe Country High School is a 13 - 18 mixed comprehensive of about 850 pupils in Northumberland, U.K., and most of the work has ben used with Y10 (14/15 years old) or Y12 (16/17 years old). Why use old instruments when teaching geometry ? I do not use them all the time, but try to use them when the time is relevant, or when I feel that pupils would benefit from a different stimulus. I enjoy working with my hands, and it gives me great pleasure to try to recreate these old instruments from, sometimes unclear, diagrams and all those bits of odds and ends that I tend to strore in the garage (which can be useful refuge in times of stress !) knowing that one day they will come in useful. I an also interested in local history and if I can raise awareness of the impact history has had on mathematics (or vice versa), then perhaps my pupils will see the place of mathematics in other subjects more clearly (as advocated in our National Curriculum), and enhance its cross-curricular use. Geometry is not just theoretical : if pupils can see its practical uses then it takes on a more important role for them. The practical aspect of mathematics has been encouraged in the U.K. since the publication of the Cockcroft report over a decade ago, and the work outlined on the worksheets, and done in this workshop gives everybody the opportunity to do some practical work.

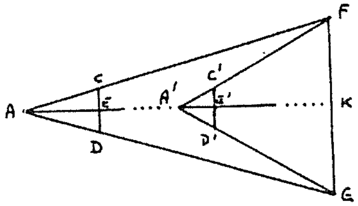
The first instrument we looked at was the cross staff, or Jacob's staff. This was reputedly invented by a Jewish astronomer, Levi Gerson (1288-1324), and was used by Regiomontanus for astronomical observations. During the sixteenth century it was adopted for surveying and astronomical use. Edmond Gunter wrote a book "The description and use of the sector, crosse-staffe & other instruments..." (second edition) in 1636, and some of the pages from this book were used in the exhibition. For a description of its use, and diagrams, see worksheets 3a and 3b in the appendix. Staff and transom can both be marked with scales for distance and angle measurement, and marking the staff and/or transom provides an opportunity for some trigonometry. Michael Coignet in 1581 described the cross staff in the form it was to retain until it became obsolete in the mid eighteenth century. It survived longer as a navigational instrument than as a surveying instrument. He described it as a staff of square cross section, about 1 metre to 1m 30cm in length with transoms whose lengths were in the ratio 1:4:8, or 4cm, 16cm, 32cm. Wood (ebony) was generally used, being easily worked and durable under many conditions. Pictures of instruments in the Science Museum, London formed part of the exhibition in the workshop room.

Four other posters, produced by the Wipple Museum on Practical Mathematics in the Age of Discoveries (featuring Astronomy, Navigation, Surveying and Cartography) were on show as the cross staff features in all these activities.

The cross staff works on the idea that the cross, or transom, subtends a certain angle at the eye depending on its distance from the eye. It has been assumed that this principle has been in use for a long time using different, but simple apparatus. Workshop participants worked through the introductory exercise (appendix 2a and 2b) using the width of a hand, and the similarity or results was remarkable. The accuracy was given to anything between one and four significant figures, and discussion on the degree of accuracy was spontaneous.

We then took the replica cross staff outside to use. In the absence of an uncrossable river, or dangerous enemies we imagined a suitable obstacle between ourselves and two of the posts supporting the walkway cover. A volunteer, Norman, followed the instructions as on worksheets 3a and 3b and declared the required distance as 7.05 metres. On measuring the actual distance we it to be 7.08 metres. A result within 1/2% ! Participants were amazed that what seemed to be a very crude way of measuring a distance could be so accurate. On returning to the workroom, people discussed other similar instruments based on the same principle, and it is hoped that these ideas will find their way into the mathematics classroom.

We spent a few minutes looking at the geometry of how the instrument works, and the proof of the method on worksheets 3a and 3b is shown here :



$AA' = AK - A'K$

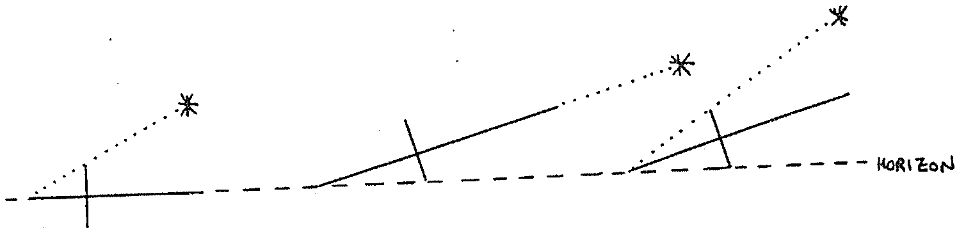
$\Delta\ AFG,\ ACD$ are similar so $\frac{AK}{AE} = \frac{FG}{CD} \Rightarrow AK = \frac{AE.FG}{CD}$

$\Delta\ A'FG,\ A'C'D'$ are similar so $\frac{A'K}{A'E'} = \frac{FG}{C'D'} \Rightarrow A'K = \frac{A'E'.FG}{C'D'}$

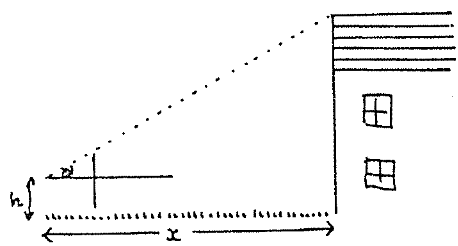
$\therefore AA' = \frac{AE.FG}{CD} - \frac{A'E'.FG}{C'D'} = \frac{FG}{CD}(AE - A'E') \quad \because C'D' = CD$

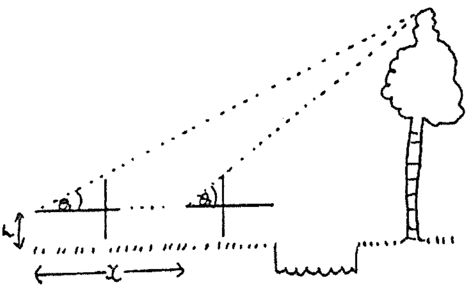
$= \frac{FG}{CD}.CD \quad \therefore \underline{AA' = FG}$

To measure angles, the cross staff can be used in three ways, as illustrated here



This is then the basis for measuring the angles necessary to find heights of buildings or inaccessible trees ! These heights can either be found by scale drawing or trigonometry, depending on the ability of the pupils, as illustrated in the following diagrams :





The second instrument used in the workshop was the sundial. Appendix 2 gives some of the historical background to the sundial. Brief notes on three of Montpellier's dials were given on the blackboards, and these are listed here :

1. Le Peyrou

A large analemmic sundial in the form of an ellipse, major axis 5.36m, minor axis 3.88m. The gnomon is provided by a person standing at the appropriate mark for the time of year. Erected in 1926.

2. Faculty of Medicine

A vertical south facing dial. Made of limestone (?) with an iron gnomon. Height 1.5m, width 1.4m (estimated). This dial is next to the cathedral.

3. Saint-Pierre Cathedral

Go through the faculty of medicine (noting a bust of Cardan) and observe a vertical west facing dial, estimated to be 2 metres square. Too worn to make out any markings.

Information about further dials at the Rectorat, the Hotel of Rodez of Benavent, and the Station Oenologique de l'Hérault was provided at a later date by Marie-Jeanne Plane ; further investigation leads me to believe there may also dials at the Maison Gasquet (once used as a hospital for the sick) and on the Lycee, formerly a Jesuit college.

Participants then made an equatorial dial, as shown in the worksheets 4a and 4b in the appendix. The use of equipment provided by Helix, the geometrical instrument makers, for this workshop was appreciated, and participants were allowed to take the equipment home with them. We then went outside to test the sundials, and use some portable dials, based on old instruments, that can be purchased in France.

A simple sextant made from a Helix angle measurer was demonstrated, and the angle of elevation of the sun taken. The simple use of this instrument was appreciated when we tried to use a real sextant to 'shoot the sun'. This was achieved by using a false horizon, because of the absence of the real one due to surrounding buildings.

Reading the time from a sundial involves estimation, graph work and simple calculations to take into account the longitude of the position of the dial, so not only geometrical work but many other basic mathematical skills are involved, as was appreciated in the workshop.

Unfortunately we ran out of time at this point, but there are many other instruments that it is hoped teachers will utilise in the classroom. It is hoped that participants and their pupils get as much pleasure out of doing similar work in the classroom as I have with my pupils.

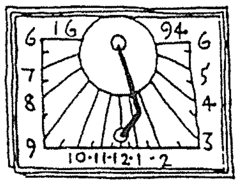
REFERENCES

Make A Sundial — British Sundial Society

Surveying Instruments, history and classroom use — E.R. Kiely

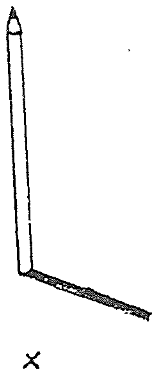
History of Mathematics — D.E. Smith

Sundials, their theory and construction — A.E. Waugh



TEACHING GEOMETRY THROUGH THE USE OF OLD INSTRUMENTS

Some sun activities for the 1st European Summer University
Montpellier 19-23 July 1993



The first thing you need to experience is how the sun appears to move. Make a pointer (a pencil stuck on a piece of Blu-tack, a bent piece of card, etc.), and place it on a piece of paper in a sunny position. Mark where the tip of the shadow lies. Now put a mark where you think the tip of the shadow will be an hour later. Most people are quite surprised when they see how far the shadow has rotated. If a group of you do this, you can make it into a competition by seeing who is closest at the end of a given period.

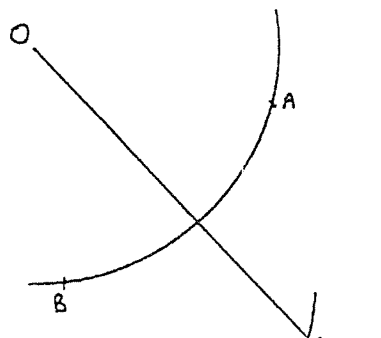
Use this idea to find due south, a follows :

1. At some point in the morning (couple of hours before noon) mark the tip of the shadow on the paper at A, say.

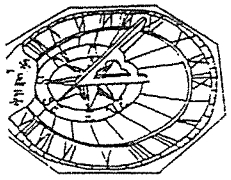
2. Draw an arc, centre O at the base of the pointer, radius OA.

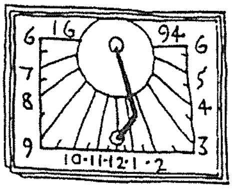
3. Later in the day mark where the shadow next meets this arc. Call this point B.

4. Use a pair of compasses to bisect angle AOB. ON then lies due north/south, whit N north of O.



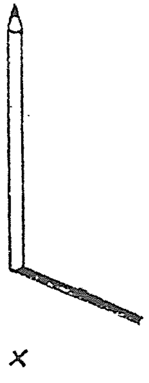
If you have already made a sundial then you can use the direction you have just found to align the gnomon along this line, making sure that the gnomon points to the north. Then you can use the dial to find the time by reading the time indicated by de shadow, making the necessary adjustements for longitude, the time of year, and any daylight saving alteration that operates in the country in which it is so be used.





L'ENSEIGNEMENT DE LA GÉOMÉTRIE À TRAVERS DES INSTRUMENTS ANCIENS

Des activités solaires pour la première université estivale
à Montpellier du 19 au 23 juillet 1993



La première chose à noter est la façon dont le soleil semble bouger. Faites une baguette (un crayon enfoncé dans une boule de Blu-Tack, une carte courbée, etc.) et placez-le sur une feuille de papier au soleil. Marquez le bout de l'ombre. Maintenant, marquez le point où vous estimez que le bout de l'ombre sera dans une heure. La plupart des gens sont plutôt étonnés quand ils voient jusqu'où l'ombre arrive. Si on le fait en groupe on peut en faire une compétition pour voir qui l'a estimé au plus près à la fin d'une heure.

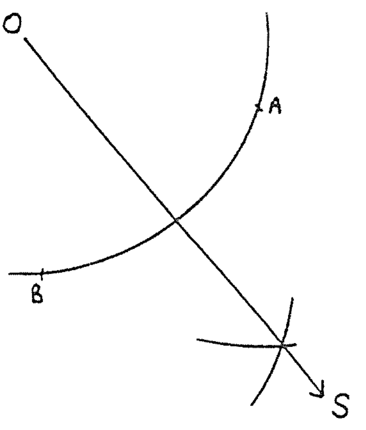
Utilisez cette idée pour trouver le sud :

1. A un moment donné, le matin (deux ou trois heures avant midi), marquez le bout de l'ombre sur la feuille de papier à A, par exemple.

2. Dessinez un arc, entre O à la base de la baguette, radius OA.

3. Plus tard dans la journée, marquez le point là où l'ombre rencontre de nouveau l'arc. Appelons ce point B.

4. Utilisez un compas pour couper en deux parties égales l'angle AOB. ON, donc, se trouve directement dans une ligne du nord au sud, avec N au nord d'O.

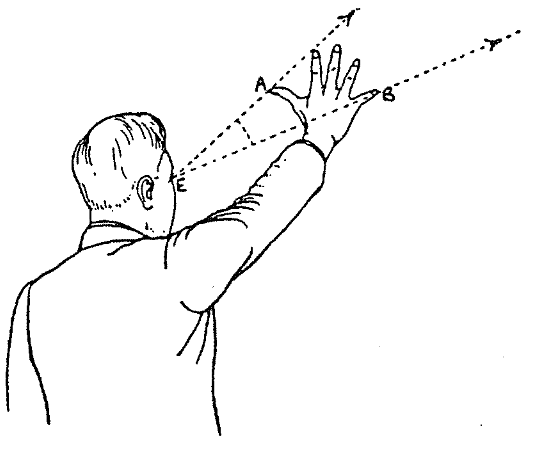


Si vous avez déjà fait au cadran solaire, vous pouvez utiliser la direction que vous avez trouvée pour aligner le style le long de cette ligne, en vous assurant que le style est orienté vers le nord. Ensuite vous allez pouvoir utiliser le cadran pour lire l'heure en lisant l'heure indiquée par l'ombre, en le réglant pour la longitude, le mois de l'année, et quelque heure artificielle qui existe dans le pays d'utilisation.

TEACHING GEOMETRY THROUGH THE USE OF OLD INSTRUMENTS

An introductory exercise to the cross staff
1st European Summer University, Montpellier 19-23 July 1993

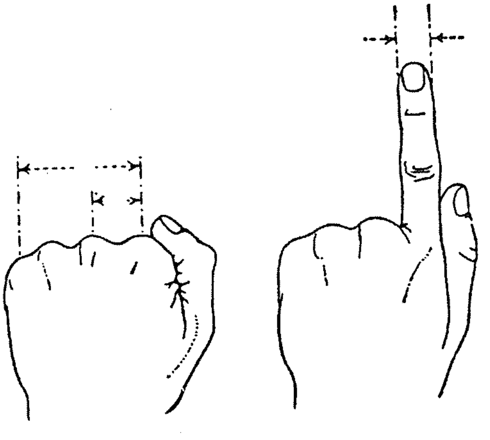
1. With the help of a partner measure the distance from your eye to your hand, held at arm's length (the diagram below should help).



2. Measure the span of your hand (AB).

3. Draw an appropriate right angled triangle and calculate the angle AEB subtended at your eye by the span of your hand.

4. Repeat the experiment to find the angles subtended at your eye be the distances shown below.



5. Compare your individual results with the class average.

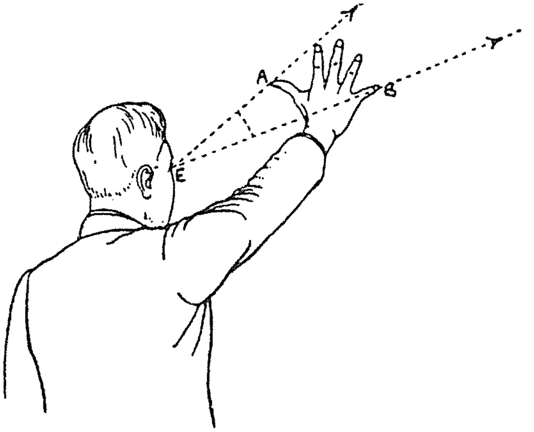
6. How accurate do you think these angular distance are ?

7. Use these body measurements of angular distances to estimate some angles of elevation.

L'ENSEIGNEMENT DE LA GÉOMÉTRIE À TRAVERS DES INSTRUMENTS ANCIENS

Un exercice comme introduction au baculum
Première université estivale de Montpellier 19-23 Juillet 1993

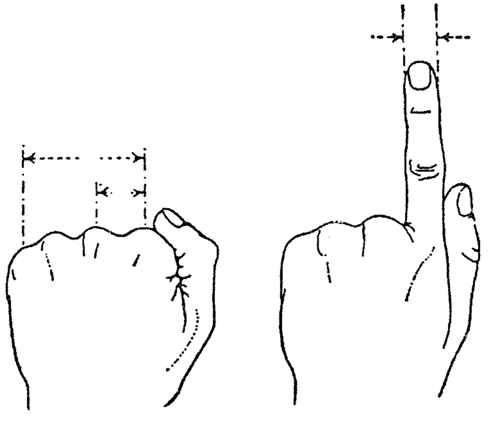
1. Avec l'aide d'un partenaire, mesurez la distance de votre oeil à votre main, le bras tendu (voir ci-dessous).



2. Mesurez l'empan de votre main (AB).

3. Dessinez le triangle rectangle qui en résulte et calculez l'angle AEB soustendu à votre oeil par l'empan de votre main.

4. Refaites l'expérience pour trouver les angles soustendus à votre oeil par les distances indiquées ci-dessous.



5. Comparez vos résultats avec le moyen pour la classe.

6. A quel point trouverez-vous ces distances exactes ?

7. Utilisez ces mesures corporelles des angles pour estimer quelques angles d'évaluation.

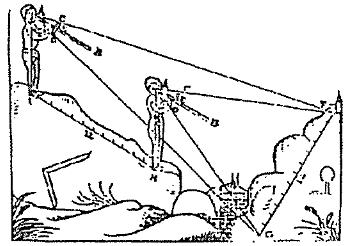
The Cross Staff

The three pieces are best cut out of thick card. This can be done by photocopying or sticking this sheet onto card before cutting it out. For a bigger cross staff photoenlarge initially.

Cut out the slots in the transoms, and after folding the staff lengthways the appropriate transom can be slid along the staff.

Stand at some point I, equidistant from F and G. The cross staff is held horizontally, and the transom CD is moved so that the points F and G are just visible from I.

Now move the transom towards you through a distance equal to its length. Walk forward along the line IH until you come to the position H where you can just see F and G once again. If you measure the distance IH you have also obtained the distance FG.



THE BACULUM, OR CROSS-STAFF

From Oronce Fine's *De re & praxi geometrica*, Paris, 1556, showing the methods of measuring distances

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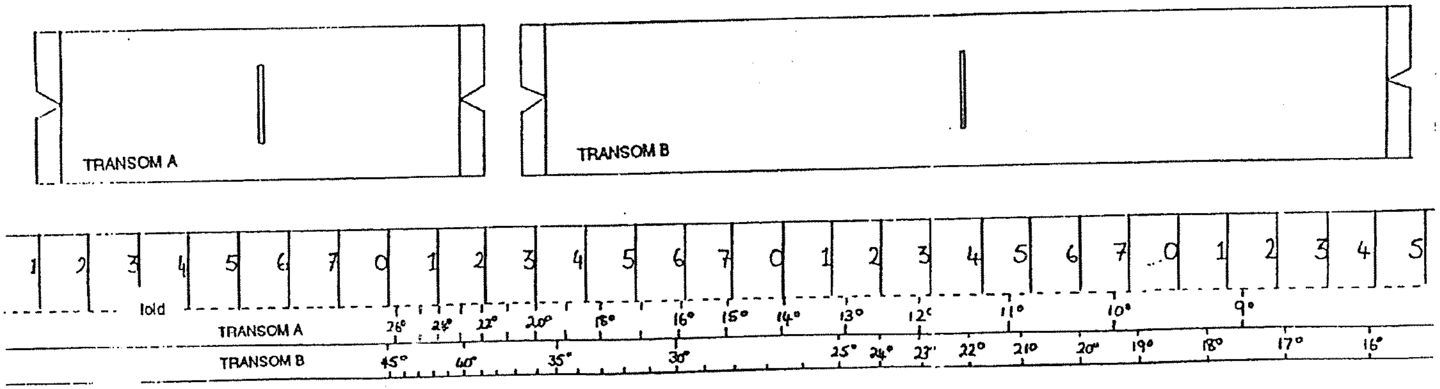


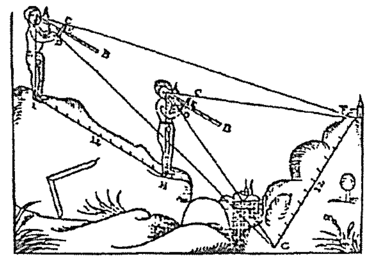
Le Baculum

Il vaut mieux découper les trois parties dans une carte épaisse. Ceci peut se faire ou en photocopiant ou en collant cette feuille sur de la carte avant de la découper. Pour un baculum plus grand, l'aggrandir avant de commencer avec la photocopieuse.

Découper les fentes dans les traverses, et après avoir plié le bâton longitudinalement, la traverse choisie peut être glissée le long du bâton.

Mettez-vous debout à un point I, équidistant de F et de G. Le baculum se tient horizontalement, et la traverse CD se déplace de façon que les points F et G sont juste visibles de I.

Maintenant avancez la traverse vers vous d'une distance qui correspond à sa longeur. Avancez le long de la ligne IH jusqu'à ce que vous arrivez à la position H où vous pouvez tout juste voir F et G de nouveau. Si vous mesurez la distance IH vous avez aussi trouvé la distance FG.



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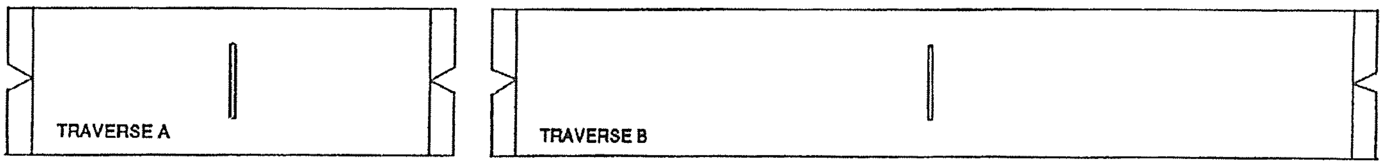
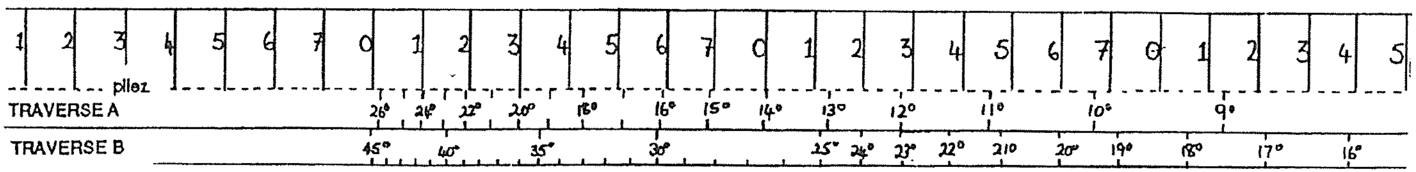
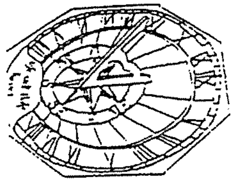
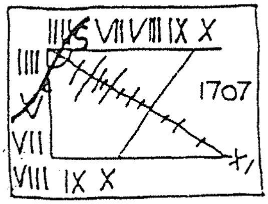








TEACHING GEOMETRY THROUGH THE USE OF OLD INSTRUMENTS

An equatorial dial for the 1st European Summer University
Montpellier 19-23 July 1993

1. Mark a 1cm margin at the top and bottom of your card.

2. Mark the centres of the lines just drawn, and at the bottom centre draw two parallel lines 2mm apart as shown.

3. Place the centre of the Helix angle measurer at the top centre, draw round it, and mark every 15° as shown.

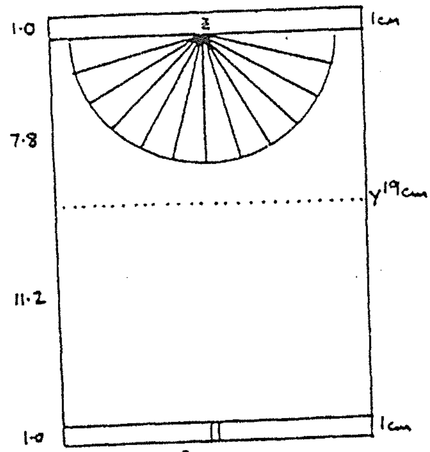
4. Turn the card over and repeat, so that the hour lines coincide on both sides of the card.

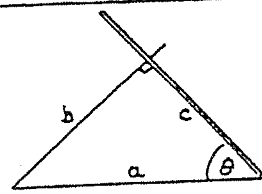
5. The line XY has to be drawn so that when folded, the straw is at an angle O, equal to the angle of latitude of the place where it is to be used.

6. In Montpellier this angle is 43.7°, so the calculations to find the position of XY are shown opposite.

7. Push the straw through the card at the centre Z, and anchor it by folding the card along XY and tucking the tongue of card into the straw. The lenght of the straw below the card is calculated as shown.

8. Point the straw north, and you're ready to tell the time !





$a+b=19$ $\quad b = a\sin\theta$

so $a + a\sin 43.7° = 19$

$1.69a = 19$

$a = 11.2$, and by substitution

$b = 7.8$

By Pythagoras, $a^2 = b^2 + c^2$

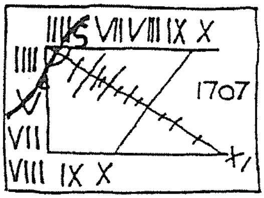
$11.2^2 = 7.8^2 + c^2$

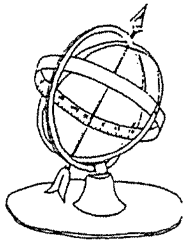
$c^2 = 11.2^2 - 7.8^2 = 66.0$

$c = 8.1$

To find the time from the dial, first estimate the time shown by the shadow, then :

1. Make the adjustment for the time of year from the graph.
2. Add (or subtract) 4 minutes for every degree you are west (east) of Greenwich.
3. Add. 1 hour in British Summer Time, or two hours in France.





L'ENSEIGNEMENT DE LA GÉOMÉTRIE À TRAVERS DES INSTRUMENTS ANCIENS

Un cadran équatrorial pour la première université estivale
à Montpellier du 19 au 23 juillet 1993

1. Dessinez une marge d'un centimètre en haut et en bas de votre carte.

2. Marquez le centre des lignes que vous venez de dessiner, en bas, au centre dessinez deux lignes parallèles à deux centimètres de distance -voir à côté.

3. Placez le centre du rapporteur Helix en haut au centre, tracez sa forme, et marquez un point tous les 15°, comme indiqué ci-contre.

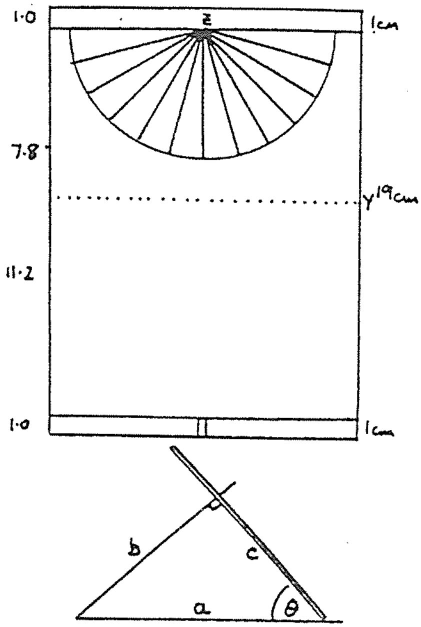
4. Retournez la carte, et répétez le processus afin que les lignes d'une heure coincident des deux côtés de la carte.

5. La ligne XY doit être dessinée afin que, une fois pliée, la paille se trouve à l'angle O, égal à l'angle de latitude de l'endroit où l'on s'en servira.

6. A Montpellier, cet angle est de 43.7° ; donc les calculs pour trouver la position de XY sont donnés ci-contre.

7. Poussez la paille à travers la carte au centre Z et fixez-la en pliant la carte le long de XY et rentrant la languette de carte dans la paille. La longueur de la paille en dessous de la carte est calculée ci-contre.

8. Orientez la paille vers le nord, et vous êtes prêt à lire l'heure !



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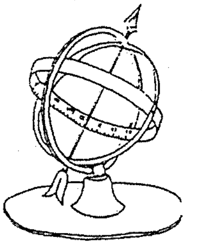
$11.2^2 = 7.8^2+c^2$
$c^2 = 11.2^2 - 7.8^2 = 66.0$
$c = 8.1$

Pour trouvez l'heure à partir du cadran, premièrement estimez l'heure indiquée par l'ombre, puis :

1. Réglez-le selon la saison de l'année en utilisant la graphique.
2. Ajouter (ou soustraire) 4 minutes pour chaque degré plus à l'ouest (ou à l'est) de Greenwich.
3. Ajouter une heure en été en Grande-Bretagne, ou deux heures en France.



FIRST EUROPEAN SUMMER UNIVERSITY
History and Epistemology in Mathematics Education

Montpellier 19-23 July 1993

Background to the sundial

The principle of the sundial was known to the Chinese as early as 2500 BC., and sundials were widely used by the Greeks and Romans. In AD.606 the Pope is said to have ordered that sundials be placed on churches and this was probably the beginning of the long association of churches with sundials and clocks. The Saxon dial on Escomb church is thought to be the oldest dial in situ in the U.K., dating from the seventh century AD., though the Saxion dial on Bewcastle Cross also claims this honour ! Some larger examples, containing information that allow us to date them more accurately, can be found in Yorkshire. The dial at St. Gregory's Kirkdale dates from about 1060 AD..

Sundials rely on the casting by the sun of a shadow, of a simple rod or other structure called the *gnomon* (which projects from the surface), onto a calibrated background called the *dial plate*. It is the leading edge of the shadow, cast by the *style* (the name given to the part of the gnomon that casts the edge of the shadow) that is read from the dial plate to find the time. The angle of style to the horizontal must be the same as the latitude in which the dial is set up. The latitude of Montpellier is 43° 40' North.

The measurement of time is based on the rotation of the earth. The interval between successive apparent crossings of the sun across the imaginary line drawn through the north and south poles and that place (called the *meridian*) is known as the *apparent solar day*. This is because the shadow of the style on the dial plate depends on the position of the sun as it appears in the sky. Since the earth's orbit around the sun is an ellipse rather than a circle, and its axis is not perpendicular to the orbit's plane, the apparent solar day varies in length up to 31 minutes at the extremes of any year. Since this is not practicable for time keeping by clocks and watches, we average out these variations to produce *mean time* as in Greenwich Mean Time. This means that a correction factor needs to be applied to apparent solar time, which is that measured by a sundial. It was the introduction of the railways, and their need for a standard time for the whole country, that led to the introduction of *Greenwich Mean Time* in 1880.

To adjust a sundial's time to G.M.T. then, we have to apply the appropriate correction (called the *equation of time*) which is obtained from a table, graph, or other chart ; adjust for the longitude (add 4 minutes for each degree of longitude West of the Greenwich meridian. For every degree East of Greenwich subtract 4 minutes. Montpellier is 3° 50' East, so about 15 minutes needs to be subtracted) ; and during the summer remember to add one hour for British Summer Time ! (or two in France)

G.M.T. Correction Factor (Equation of Time) in minutes (C.F.)

Date		C.F.	Date		C.F.	Date		C.F.	Date		C.F.
Jan	2	+ 4	Apr	1	+ 4	Aug	17	+ 4	Nov	11	-16
Jan	4	+ 5	Apr	5	+ 3	Aug	22	+ 3	Nov	17	-15
Jan	7	+ 6	Apr	8	+ 2	Aug	26	+ 2	Nov	22	-14
Jan	9	+ 7	Apr	12	+ 1	Aug	29	+ 1	Nov	25	-13
Jan	11	+ 8	Apr	15	0	Sep	1	0	Nov	29	-12
Jan	14	+ 9	Apr	20	- 1	Sep	5	- 1	Dec	1	-11
Jan	17	+10	Apr	25	- 2	Sep	8	- 2	Dec	4	-10
Jan	20	+11	May	2	- 3	Sep	11	- 3	Dec	6	- 9
Jan	24	+12	May	15	- 4	Sep	13	- 4	Dec	9	- 8
Jan	28	+13	May	28	- 3	Sep	16	- 5	Dec	11	- 7
Feb	3	+14	Jun	4	- 2	Sep	19	- 6	Dec	13	- 6
Feb	20	+14	Jun	10	- 1	Sep	22	- 7	Dec	15	- 5
Feb	27	+13	Jun	14	0	Sep	25	- 8	Dec	17	- 4
Mar	1	+12	Jun	20	+ 1	Sep	28	- 9	Dec	19	- 3
Mar	8	+11	Jun	24	+ 2	Oct	1	-10	Dec	21	- 2
Mar	12	+10	Jun	29	+ 3	Oct	4	-11	Dec	23	- 1
Mar	16	+ 9	Jul	4	+ 4	Oct	7	-12	Dec	25	0
Mar	19	+ 8	Jul	10	+ 5	Oct	11	-13	Dec	27	+ 1
Mar	23	+ 7	Jul	19	+ 6	Oct	15	-14	Dec	29	+ 2
Mar	26	+ 6	Aug	4	+ 6	Oct	20	-15	Dec	31	+ 3
Mar	29	+ 5	Aug	12	+ 5	Oct	27	-16			

Carpe diem !

The British Sundial Society exists to help and inform with all matters to do with sundials. If you wish to obtain further details, contact Robert Sylvester, Barncroft, Grizebeck, Kirkby-in-Furness, Cumbria LA 17 7XJ.

"Make a Sundial" is available for £5.75, which includes p+p, from Mrs. J. Walker, 31, Longtown Rd., Little Sandhurst, Camberley, Surrey GU17 8QG.

I am always interested to heart of any projects with which you may be involved, and can be contacted at 12, Annaside Mews, Leadgate, Consett, Co. Durham DH8 6HL Great Britain.