USING HISTORICAL ARITHMETIC BOOKS IN TEACHING MATHEMATICS TO LOW-ATTAINERS

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What can the history of mathematics mean to you when you are teaching mathematics to pupils from 12 to 16 years whose intellectual capabilities range from below average to average ?

During eleven years I was a mathematics teacher at a Dutch MAVO-school. Many of my pupils had learning problems. They couldn't concentrate very well, were not interested in school and certainly not in mathematics. For many of my pupils mathematics was an obscure system of tricks invented by some sadist who definitely wanted to poison their youth. How could the history of mathematics change their opinion ? A "photo of Pythagoras" should only act as a target to work off their frustrations : "Aha, he is the one !" Whereafter they all would draw beards, moustaches and worse on his noble face. I am not talking about this creative activity when I claim that the history of mathematics can colour your lessons.

In each mathematics lesson my pupils asked me at least once : "Why should we learn this ? What can we do with this ?" At the moment they could see how they could apply the learned algorithms and solution methods they became (a little bit) interested in mathematics. Practical problems and practical situations were a good means to motivate them. They drew graphics of their heartbeat during a sports tournament and measured packings from all sorts of products in the supermarket. They enjoyed it and at the same time they did a lot of mathematics.

In this period I started my research on 16th century Dutch arithmetic books. I read many old arithmetical manuscripts and printed books and I discovered many problems that seemed very suitable for the use in the modern mathematics lesson. I even found practical experiments in my sources. For instance a method to calculate the height of a tower with the help of a mirror. I tried out some of these problems in my classes and it was a big success. My pupils were amazed when they discovered that 400 or more years ago people struggled with comparable problems as they were doing nowadays.

Besides translations and transcriptions I gave my pupils photocopies of the original Dutch handwriting, they tried to decipher and to imitate it. They liked it very much and they felt as if they could shake hands with their ancestors. I asked my pupils to compare their own solution methods with the old ones and they were surprised by the similarities as well as by the differences. For instance it seemed incredible to them that you can solve mathematical problems without mathematical symbols or notation. My pupils were not interesed in names, dates or portraits of dead mathematicians. They wanted to see the problems on which their historical colleagues had worked.

Each 6 or 8 weeks I treated my pupils to a historical problem. Perhaps I could have tried to do it more often but then I had the risk that the novelty value would wear off. And besides that you must realize that using historical sources means work ! It is more than making a photocopy of your source and giving it to your pupils. Finding a suitable piece of historical material is only step one Next you must ask yourself : Where can I use it, with which pupils on which moment ? Which aid can I give to my pupils to solve the problem ? Do I want my pupils to solve it in a specific way ? Can I connect more questions to the given problem ? Do I confront my pupils only with the historical problem or with the historical solutionmethod too ? Shall I ask my pupils to study the historical solution method to see why it works and to compare it with their own modern solution method ? Using historical sources in the classroom needs a lot of preparation. I believe and I have experienced that it is all worth while.

During the summer university in Montpellier the participants of my workshop have prepared pieces of 16th century arithmetic books for the use in the classroom. They worked on problems, had vivid discussions in small groupes and produced interesting material and practical ideas. It was a pleasure (and very easy) for me to do this workshop with such hard working people. Their enthusiasm and dedication has convinced me again. The best way to let your pupils experience that mathematics is the result of people working on problems, is to let them work on these problems. Let them jump into the original sources and they will work with red cheeks. This is the way in which the history of mathematics may colour my lessons.

On the next pages you can find the material that I have used during my workshop.

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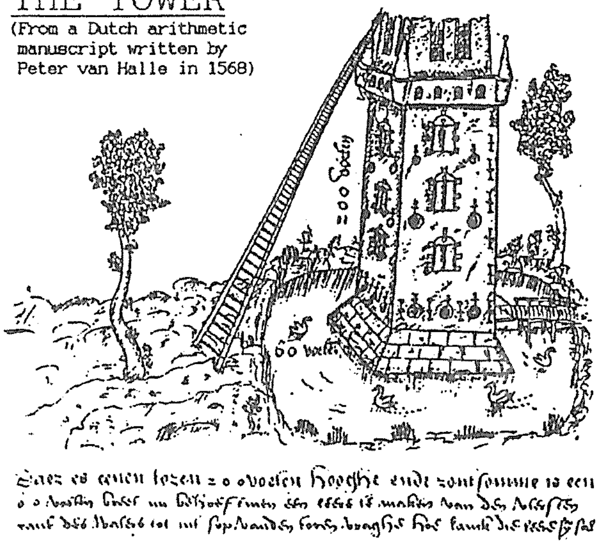
Montpellier, 19–23 July 1993, workshop by Marjolein Kool

THE TOWER

(From a Dutch arithmetic manuscript written by Peter van Halle in 1568)



Daer es eenen toren 200 voeten hooghe ende rontsomme is een 60 voeten breet. Nu behoeft eenen een leere te maken van den veersten cant des waters tot int sop van den toren. Vraghe hoe lanck die leere sijn sal.

TRANSLATION:
There is a tower 200 feet high and around the tower is a canal with a breadth of 60 feet. Now somebody needs to make a ladder over the water to the top of the tower. The question is: how long should that ladder be?

If you find a historical problem that seems suitable for the use in the classroom, you must try to answer the following questions:

1. *Context: Which pupils would you confront with this problem, of which age and which level? Which mathematical subject would you like to practice with this problem? What will be your aim and what prevous knowledge do you expect?*

2. *Aid: If your pupils are not able to solve the problem, or if you want them to solve the problem in a particular way, which questions would you ask to help them or to send them in a particular direction?*

3. *Application or extension: Can you suggest a further task connected with the preceding problem that can be used as an extra exercise of application?*

To show you what I mean, I have worked out the problem of the tower:

1. Context: In the second form my pupils (about 14 years old) learned the Pythagorean theorem. After they have had some experience with problems about triangles and applications in modern situations I gave them this 16th century problem.

2. Aid: To solve problems with the Pythagorean theorem my pupils have learned to use a diagram. To remind them to do this and to help them with the problem of the tower I ask them:
   Fill in the diagram first:

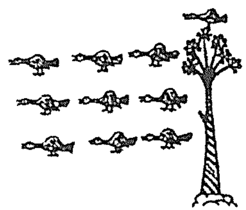
	x	x^2
Breadth of the canal		
Height of the tower		
+		
Length of the ladder		

3. Application or extension:
   On the backside of the tower (not visible in the picture) there is another ladder over the canal with a length of 80 feet. This one reaches just to the window-sill of the first window in the tower. What is the distance between the window-sill and the waterlevel?

GOD BLESS YOU 1

This problem and the next one are variations of the same problem.

(From the <u>Columbia-Algorismus</u>, an Italian codex from the 14th century, once part of the library from B.Boncompagni, now in the Columbia University Library in New York. Ed. Kurt Vogel, 1977.)

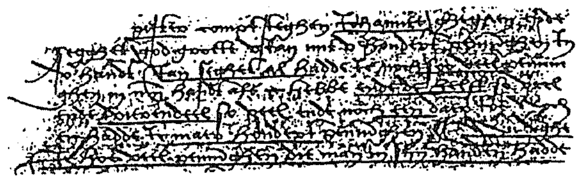


There was a pigeon sitting on a tree. Some other pigeons passed this tree and our first pigeon said: "God bless you, you twenty-five pigeons!". "No", said the other pigeons, "our number is not twenty-five! If you take our number twice and add the half of our number and also the fourth part of our number and a fourth then you will have twenty-five together." Question: How many pigeons passed the tree with our first pigeon?

* *As you can see, the picture shows the right answer. How, do you expect, will pupils react to this picture? How would you use the picture in your lesson?*

GOD BLESS YOU 2

(From a Dutch arithmetic manuscript written by Christianus van Varenbraken in 1532. Ed.Marjolein Kool, 1988.)



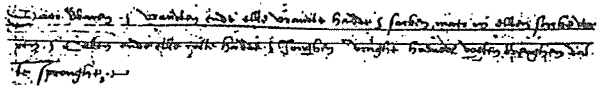
Pieter compt teghen Johannes ghegaen ende segghet god groete v Jan mit v hondert penninghen in v handt. Jan seghet: al hadde ic noch soo veel penninghen in mijn handt als ic hebbe ende de helft so veel ende tvierendeel so veel ende noch een daer toe, noch en hadde ic maer hondert penninghen. Nu vraghe ic hoeveel penninghen die man in sijn handen hadde.

<u>TRANSLATION</u>:
Pieter met Johannes and said: "God bless you, Jan, with your 100 pennies in your hand." Jan said: "If I should take the amount of my pennies twice and add the half of my pennies and also the fourth of my pennies and one penny, then I should have 100 pennies." The question is: How many pennies has Jan?

Picture from: Adam Riese, "Rechenung auff der linihen vnd federn...", 1522. ed. Stefan Deschauer, 1992.

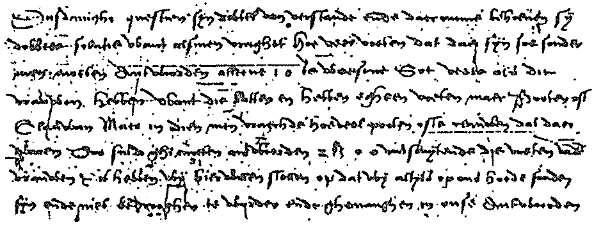
5 WOMEN

(From a Dutch arithmetic manuscript written by Peter van Halle in 1568)



Daer waren 5 vrauwen ende elke vrauwe hadde 5 sacken maer in elken sacke waren 5 catten ende elke catte hadde 5 jonghen. Vraghe hoeveel voeten brenghen dat te spronghe?

TRANSLATION:
There were 5 women and each woman had 5 bags. In each bag were 5 cats and each cat had 5 young ones. Question: How many feet were there?



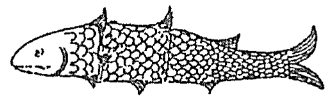
Dusdanighe questien sijn dobbel van verstande ende daeromme behoeven sij dobbele solutie, want alsmen vraghet hoeveel voeten dat daer sijn, soe soudemen moeten antwoorden: alleene 10, te weetene soe veele als die vrauwen hebben, want die katten en hebben egheen voeten maer pooten oft clauwen. Maer indien men vraechde hoeveel pooten ofte clauwen dat daer waren soo suldi ghi moeten antwoorden 2500 vutwijsende die voeten vanden vrauwen. Dit hebben wij hier willen stellen opdat wij altijts op ons hoede souden sijn ende niet bedrooghen te worden ende ghevanghen in onse antwoorden.

TRANSLATION:
Such questions have a double interpretation and that is why they need a double solution. If you ask how many feet there are, you must say: only 10, because women have feet and cats don't. They have paws or claws. And if you ask how many paws or claws there are, you must say 2500 excluding the feet of the women. We have written this here to teach that we must always have our wits about us so that we are not misled and become trapped in our answering.

* *The answer 2500 is wrong. How would you use that in the classroom?*

THE FISH

(From an Italian arithmetic manuscript written by Paolo Dagomari in the 14th century. Ed.G.Arrighi, 1964.)



There is a big fish swimming in a lake. Its head is one-third of the whole. Its trunk is one-fourth of the whole and its tail is 9 feet. How long is this fish?

Om te vijnden hooghde met een speghele.

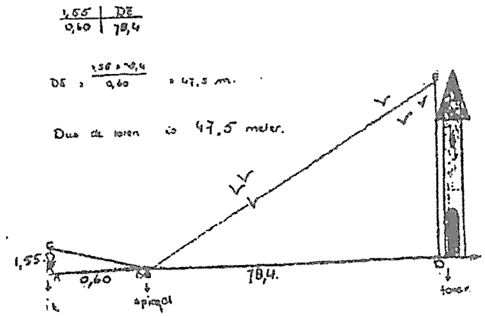
TO FIND THE HEIGHT WITH A MIRROR

How can you find the height of a tower if you don't have the opportunity to climb to the top? Master Adriaen vander Gucht describes in his arithmetic book of 1569 how you can do that.

Legt een platte speghele op de eerde/ zo dat ghi ziet de hooghde/ en multipliceert de distancie van u voeten tot u ooghen metter distancie van den speghele totter tooren die divideert met de distancie van u tot de speghele

Place a mirror on the ground. Look into it and try to see the top of the tower. Multiply the distance between your feet and your eyes by the distance between the mirror and the tower and divide the result by the distance between you and the mirror.

1. Go outside and look for a tower or another high building or tree in the neighbourhood. Try to calculate the height of this object with the method of Master Adriaen.
2. Why does he use the distance between your feet and your eyes and not your complete height?
3. Make a sketch of the situation.
4. Prove that there are two similar triangles.
5. How would you calculate the height of your object?
6. What is the difference between your calculationmethod and the method of Master Adriaen?



Marlies Franken (15)

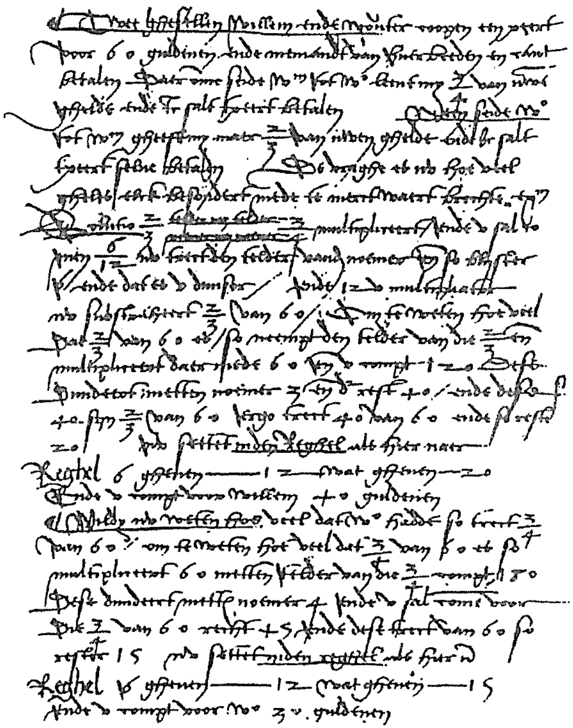
In this previous example, I let you work on a problem from the 16th century. As you have experienced, I gave you more than just the problem. I also showed you the 16th century solutionmethod and asked you to compare the old solutionmethod with the modern one. It shows you that sometimes you can do more with the historical sources than just cutting out problems.

You can use historical sources on different levels:
1. *Give the problem to solve it in a modern way.*
2. *Give the problem and the historical solutionmethod to solve it in two ways.*
3. *Like 2 with an extra question: try to understand and explain the old solutionmethod describe differences between the old and the modern method.*

* *Look at the following example, on which level would you use this material?*
Prepare a worksheet for pupils based on this material.

TO BUY A HORSE

From a Dutch arithmetic manuscript written in 1532 by Christianus van Varenbraken.



TRANSLATION:

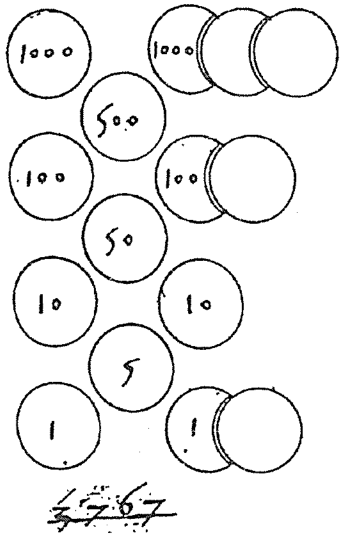
Two friends, Willem and Wouter, want to buy a horse for 60 guilders. But Willem doesn't have enough money to buy the horse on his own and Wouter doesn't either. Willem says to Wouter: "Give me 3/4 of your money and I can buy the horse." "No", Wouter says to Willem, "give me 2/3 of your money and I will buy the horse." The question is: How much money do these friends have?

Solution: Multiply 2/3 by 3/4 and you will find 6/12. Subtract the numerator from the denominator and you will find 6. That is your divisor and 12 is your multiplier. Take 2/3 of 60 by multiplying the numerator of 2/3 by 60. That gives 120. And dividing this by the denominator 3, you will find 40. So 40 is 2/3 of 60. Subtract 40 from 60, you will find 20. Place this in the rule.
RULE: If 6 give ——— 12 ——— what will give 20?
(Multiply 12 with 20 and divide the product by 6)
And you will find for Willem 40 guilders*).
If you want to know how much Wouter has, you must take 3/4 of 60 by multiplying 60 by the numerator of 3/4, that gives 180 and dividing this by the denominator 4, and you will find: 3/4 of 60 is 45. Subtract 45 from 60 and you will find 15. Put this in the rule.
RULE: If 6 give ——— 12 ——— what will give 15?
(Multiply 12 with 15 and divide the product by 6)
And you will find for Wouter 30 guilders*).

*) The author has mixed up the names of Wouter and Willem in the solution of this problem.
The correct final result is:
Wouter has 40 guilders and Willem has 30 guilders.

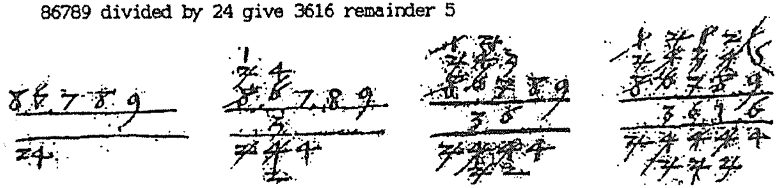
TO CALCULATE WITH COINS

In the Dutch arithmetic manuscript of Christianus van Varenbraken written in 1532, the medieval method of calculating with coins is explained. It worked as follows: Before you start you lay down a vertical line of coins. These are your "layers". They indicate the value of the coins that are placed on the right side of the layers. The first layer indicates 1, the second 10, the third 100, etc. The fields between the layers stand for 5, 50, 500 etc. In this way you can indicate numbers and calculate. (In original there were no numbers on coins)



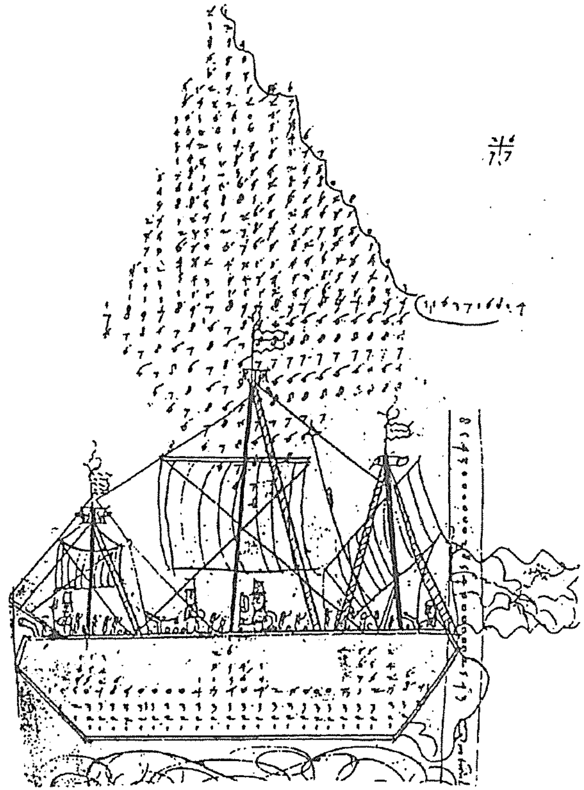
DIVISIONS

EXAMPLE 1
This division is from a Dutch arithmetic manuscript written in 1532 by Christianus van Varenbraken.
You can see here in four steps:
86789 divided by 24 give 3616 remainder 5



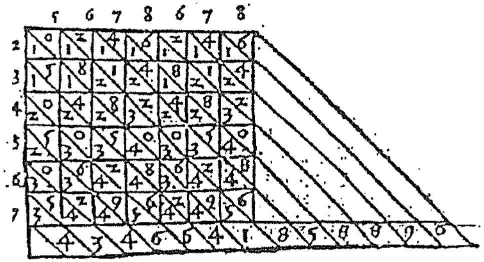
EXAMPLE 2 AND 3

These two divisions are from a Dutch arithmetic manuscript written by Cornelis Pijck in 1584.



THREE MULTIPLICATIONS

EXAMPLE 1
This multiplication is from a Dutch arithmetic book written by Adriaen vander Gucht in 1569.
You can see here: 765432 x 5678678 = 4346641858896



EXAMPLE 2
Multiplication of Russian Peasants
Multiply 37 x 47

	37	47
/	18	94
	9	188
/	4	376
/	2	752
	1	1504

37 x 47 = 1504 + 188 + 47

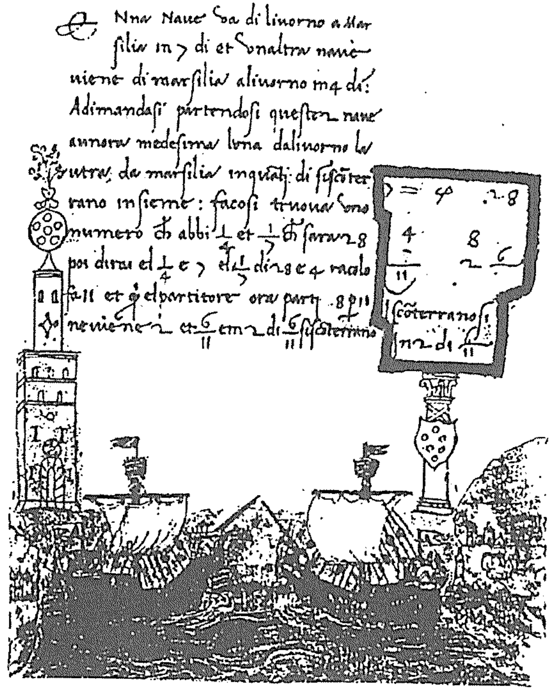
EXAMPLE 3
Egyptian multiplication
Multiply 37 x 47

	1	47
/	2	94
	4	188
/	8	376
/	16	752
	32	1504

37 x 47 = 1504 + 188 + 47

TWO SHIPS

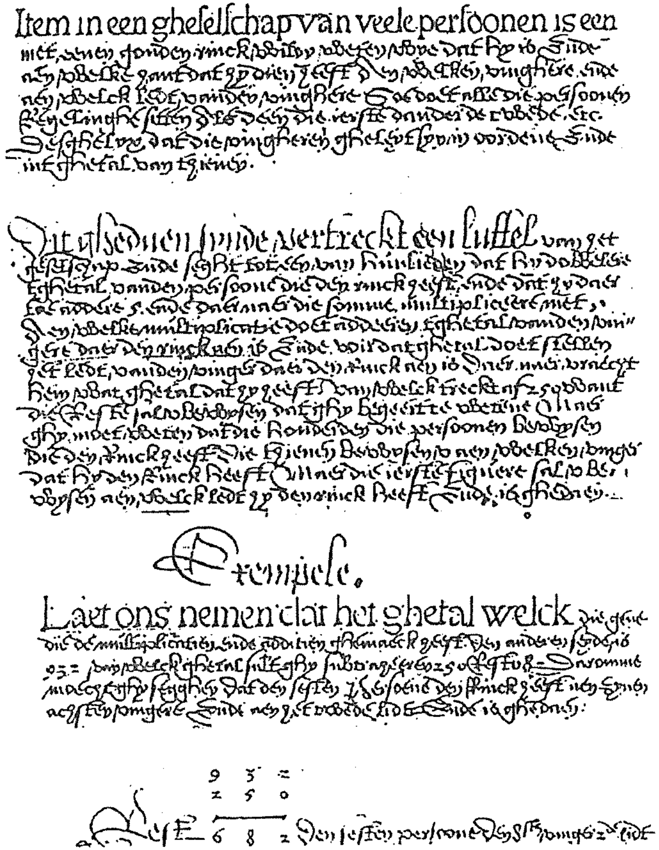
From: Filippo Calandri, Trattato di aritmetica, Florence, 1491. Ed. G. Arrighi. Florence, 1969.



A ship goes from Marseille to Liverno in 7 days and another ship goes from Liverno to Marseille in 4 days. After how many days will they pass eachother?
Solution: In 28 days the first ship can do the travel 4 times. The other ship can do it 7 times in 28 days.
11 times in 28 days make one time in 2 days 6/11.

THE RING

(From a Dutch arithmetic manuscript written by Peter van Halle in 1568)



TRANSLATION

In a group of many people there is a golden ring. If you want to know who this ring has and on which hand and on which finger and on which part of his finger, you must let the people sit on a line. The first one is the first, the next second etc. Look if the fingers are all in the right order and in the amount of ten.

If you have done this, you must leave the group for a little while and when you come back you must ask one of the people in the group to double the number of the person that has the ring and to add 5 to the result and to multiply this with 5, then add the number of the finger on which the ring is and then place the number of the part of the finger before this result (on the right hand). Then ask this person for the final result. Subtract 250, because the remainder will show you what you want to know. And you must know that the hundreds show the number of the person that has the ring, the tens show on which finger he has the ring and the first figure shows the part of the finger on which the ring is.

Example: Imagine that the person of the group gives you as a final result 932. Subtract 250 from this number. You will find 682. That is why you may say that the sixth person has the ring, on his eight finger, on the second part.

	932	
	250	
Remainder	682	the sixth person, eighth finger, second part

LITERATURE

Where can you find historical problems to use in the classroom if you don't have access to the original sources?

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