About "Logic Reasoning" - They were much less curious than some of the 14 years old
pupils who had posed some of the initial questions: The "Why" column tended to be left
aside. They were, nevertheless, more enthusiastic on treating the subject than the English
students.

There is definitely a large way to be run by these students until they reach enough maturity which will allow them to teach properly the facts or to treat properly the activities dealing with number π . However the reason for this state of things might not be entirely their fault and it might very well be the case of certain changes being needed in the way they themselves are taught at the University; changes on methodology of teaching and evaluation more than changes on curricula seems to be the case for expecting better mathematical knowledge by part of the students willing to become mathematics teachers in our schools.

It came finally clear to us that the 14 years old pupil to whom we referred to, at the begining of our research, did not receive, from his teacher, a satisfactory answer to his problem because she herself did not know the answer nor knew how to explain the differences between π and $\sqrt{2}$ to all her 8^{th} grade pupils. We, the authors of this study, will go on experiencing history of mathematics episodes in our lectures and searching for epistemological obstacles in order to alter situations such as the one that we have just reported about π .

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Conflict and Compromise: the Evolution of the Mathematics Curriculum in Nineteenth Century England

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Abstract

This paper explores the social and ideological background which determined the kind of mathematics taught to different groups of people during the early part of the nineteenth century in England. These ideologies arose in and were transmitted through institutions which determined choices and decisions about what was valued as scientific knowledge. The mathematics taught in the universities and in the Public Schools was determined by a classical liberal ideology, whereas the mathematics taught in elementary schools and colleges was driven by a practical ideology of utility, democracy and social justice. The consequences of this conflict can be seen in our current school mathematics curriculum in England. Some observations are also made on the historiographical problems of the history of mathematics education.

1 Introduction

Until recently histories of mathematics paid attention only to the significant developments in mainstream mathematics and nowhere is this more obvious than in the nineteenth century. The reasons are not too difficult to determine; the recent past is more accessible, and the writing of accounts of the work of recently deceased colleagues or editing their papers was a task which coincided with the foundation of new universities and the growth of mathematics departments. Since the interests of the writers focussed almost without exception on their perception of the developments of abstract mainstream mathematics, we have the production of the 'standard' texts¹ which view mathematics as an abstract, almost 'organic' structure and totally ignore the original problem situations and creation processes and the grass-roots teaching, learning and applications in everyday life.²³

This paper is an attempt to give a brief overview of some of the significant social and political movements in the nineteenth century which contributed to the development of mathematics curricula in England; particularly of the mathematics taught outside the universities leading to the formation of and divisions in our contemporary school curriculum.

2 Some remarks on Historiography

2.1 Differing views on the nature of historical enquiry

Traditionally, history has been viewed as a study of carefully delimited aspects of the past employing systematic research in all available sources. The approach can be from a social, political or economic point of view, (BARNES et al. 1996, FAY 1996) and necessarily employs a general philosophy in its interpretation. More recently, history is being regarded as a set of processes and power relations linking the past to the present, where the interpretations of events and facts are critically interrogated, the underlying assumptions are revealed, the status of texts are called into question and where groups of people and their conditions are defined and redefined by those in power.

In a similar manner, we have changes in the way history of mathematics is undertaken. 'Inductivist' history of mathematics is recognised by its tendency to see mathematics as a subject isolated from 'external' influences and as a progression of ideas which are improving and becoming more abstract and general with time, and where the events of the past are seen as instances of steps towards the present more 'perfect' structures. This kind of history tends to interpret the past in terms of modern concepts.

Formerly, the majority of research in the history of mathematics has been carried out by mathematicians, whose interpretation not only utilises technical language, but has employed a style which maintains the genre of the hypothetical-deductive style employed in mathematics itself. In this way it has been seen as an authoritative account of the events in question.

Historians, on the other hand, realise that there are many different kinds of questions about the past, giving rise to many different styles of history. The events, structures and processes

¹For example in the first three quarters of this century like CAJORI (1893), BELL (1945), SCOTT (1958), EVES (1969), BOYER (1968) and KLINE (1972).

²See for example DAUBEN (1985) (pp. 397-400) and WILDER (1950, 1968, 1981).

³There are exceptions; see TAYLOR's work on the Mathematical Practitioners (1964 and 1966) and YELDHAM (1926 and 1936) but there is little attention to social issues.

2.2 Facts and Events

The notion of a 'fact' is ambiguous, since it includes the sense of both 'event', (meaning the 'real' or 'imaginary' status of an occurrence), and 'a statement about an event' (where the concern is with the 'truth' or 'falsity' of an utterance) (WHITE 1995). In this sense, facts are constructed in the documents which refer to the occurrence of the events, not only by interested parties (either contemporary or recent) commenting on the events or the documents, but also by historians interested in giving what they believe is a true account of what really happened. Therefore it is the facts that are subject to revision and further interpretation, and they can even be dismissed given sufficient reasons.

This view allows us to account for the fact that historiographical consensus about any event is very difficult to achieve, since it is always open to revision from another perspective. We not only change our ideas of what the facts of a given matter are, but our notions of what a fact might be, how facts are constructed, and what criteria should be used to asses the adequacy of a given collection of facts in relation to the events which they claim to support. The relation between facts and events is always open to negotiation and reconceptualisation not because events change with time, but because we change our ways of conceptualising them.

2.3 The 'Problematique' for the Social History of Mathematics

Hence the problem is much more difficult in the history of mathematics since what we now choose to identify as mathematics has been perceived differently by people in the past, so the history of mathematics is different for different periods and cultures. In particular, for the history of Mathematics Education, we are faced with a plethora of sources which can be interpreted in various ways. For example, textbooks often have prefaces which state the authoris intention and pedagogical approach; they also may contain advertisements for books by the same or other authors, but the use of the book and the pedagogical methodology in the hands of individual teachers may be a very differnt issue. Contemporary philosophical works on the nature of mathematical knowledge influence the culture at different levels; histories of mathematics of the time tell us differnt stories, and curriculum documents produced by different institutions and interest groups relate the mathematics to the general social, political and economic milleu of the time. This sounds a daunting task, but I believe that as long as we realise that history is an interpretive activity we can cope with this over-abundance of evidence which is incomplete, fragmentary, already interpreted and politically and conceptually biased.

3 The Influences on Education in England in the late Eighteenth Century

The scientific revolution of the seventeenth century had produced significant changes in the popular views about the place of man in the physical universe, and the nature of that universe itself. The views of English philosophers like Locke, Hume, Mill and Bentham, with their theories about the nature of man, of society, and of the acquisition and purpose of knowledge

With the growth of the population of England from six million in 1750 to nine million in 1800, there was a rapid change from urban to industrial communities with all the problems that this brings. With the hopes that industry would improve the conditions of life, the philosophy of utilitarianism was developed in the belief that society should seek to obtain and provide the "greatest good for the greatest number". At the same time, "laissez-faire" economics was promoted, with the purpose of regulating society by allowing the interplay of free forces in economics and society and monitoring their effects with minimal legislative interference. These benefits and forces, it was assumed, would be controlled by educated people making the right judgments about moral, political, and economic issues.

This belief that education was the real key to the improvement of the human condition was very strong. Philosophers emphasised human perfectibility; the idea that man is born without knowledge and becomes what education makes him, and that with the growth of knowledge and better education he is continually improving. Thus, education could do everything in influencing human beliefs, attitudes, morals and conduct. In fact, Locke's views on education were generally accepted by the "philosophes" of the enlightenment, and they appear to have had an influence on early French revolutionary politics from about 1790, and the events in France were regarded with hope and alarm by different sections of the population here in England

From the late 1700s, we see a gradual development of the professionalisation and institutionalisation of mathematics teaching. Set against the considerable changes in social organisation and the economic development of the time, the kind of mathematics, the people who taught it, and the places where it was taught all underwent significant changes.

One of my themes is that the isolation from industrial growth and technical improvement of the universities with their classical traditions was maintained and even increased, and the need for the practical applications of science and mathematics was answered by other sources and other institutions. In very broad terms, the division between the old universities and the other institutions where mathematics was taught was closely linked to the class system and the established English intellectual and social attitudes of the time. While the upper classes and the established church wanted to preserve the status quo, radical philosophy was tolerated and even fashionable among the privileged, but among the workers it was seen as sedition. This division was closely linked to the class system and the established English intellectual and social attitudes of the time.⁴

4 Early Traditions in English Mathematics Teaching

In mathematics teaching two traditions can be identified from the sixteenth century. The "Liberal" tradition was based on translations of Billingsley's Euclid (1570) evolving into a formal style which continued until Playfair's Euclid of 1792 became the standard for the next hundred years.

The other "Vocational" tradition is based on Robert RECORDE's "Pathway to Knowledge" where the principles of geometry were set out so they might

⁴For some general background to this period and its social, economic and political detail see SIMON (1960) and HOBSBAWN (1968).

In this way geometry and arithmetic became popularised in many self-help books where detailed explanations and exhortations to the student accompanied the examples. (FAUVEL 1989).

John Dee's "Preface" to Billingsley's Euclid contains a comprehensive description of the "Mathematical Arts" showing their universal usefulness and giving reasons for studying mathematics at all levels. In Elizabethan times mathematics (which included astronomy and astrology) was seen as the key to knowledge and the mysteries of the universe. This tradition continued (TAYLOR 1964, 1966) and in the 1780s at Woolwich Bonnycastle was writing alternative treatments of geometry intended for students with different aspirations, and Hutton's "Course in Mathematics" of 1798 was the practical text for the artillery and engineer cadets.

5 The English Radicals: Science as a Foundation for Education

From the 1790s onwards working people began to read the radical press, attend lectures, and learn by participation in political discussion. Organisations supporting these activities were called "corresponding societies", and provided organised and disciplined opportunities for study. "The Rights of Man" (PAINE 1798), was an attack on the established social order and its exploitation of the poor and working classes. The radicals saw the Church as the main obstacle to political reform in its reinforcement of the strong social stratification, and they replaced this indoctrination with rational education through their own schools, aiming to inform people of the reasons for their condition and the state of society and industry, and placing instruction within the reach of everyone. Teaching methods encouraged self-confidence, and the capacity for clear self expression, and the organisers realised the importance of combining systematic education with mass political agitation. Books and newssheets were shared: an individual would take a book home, read a passage and prepare a talk for the next meeting; the book would then be passed on, and the process repeated. Many subjects, including some elementary mathematics were learnt in this way. As a result, men and women became informed and critical leaders of the new working class movement, able to master and comprehend some of the most advanced political thinking. This was recognised as a threat by the establishment, and in 1799 an Act of Parliament was passed "... for the more effectual suppression of societies for seditious purposes..." (SIMON 1960, p. 183).

By 1817 there were popular demands for a rational secular education for all. PAINE had demanded the teaching of science which was directly applicable, to be regarded as the cornerstone of a rationalist philosophy. These demands were of great concern, and in 1817 a House of Lords Secret (sic) Committee reported on the unprecedented circulation of

publications of the most seditious and inflammatory nature, marked with a peculiar character of irreligion and blasphemy, and tending not only to overturn the existing form of government and

⁵A "plat" was a plan used for building or surveying; a map or chart for finding direction or navigation; or an expalnatory table or diagram.

⁶It is interesting to note that the editions of Euclid up to Leek and Serle (1661) all contained Dee's "Preface" but after this it disappears from further editions. Thus Euclid clearly becomes part of the "Liberal" tradition.

⁷BONNYCASTLE (1810 p. ii) carries a list of practical texts of arithmetic, geometry, mensuration, astronomy, plane and spherical trigonometry, some having gone through numerous editions.

The Stamp Act (1817) required the registration and taxing of all newspapers and journals, and as a result, the radical newspapers were forced underground.⁸

Richard Carlile's "Address to Men of Science" (1821) also demanded a curriculum which contained reading, writing, the use of figures, elements of astronomy, geography, natural history and chemistry so that children may

at an early period of life form correct notions of organised and inert matter, instead of torturing their minds with metaphysical and incomprehensible dogmas about religion. (Carlile 1821 p. 22)

He believed that science, best studied by observation and experiment, was the key to knowledge and freedom, and promoted a materialist psychology, and demanding social and moral education by example.

6 The Schools

In the mid eighteenth century some grammar schools existed, but few taught any mathematics; perhaps the first two books of Euclid, and some simple arithmetic. Any other kind of education was locally organised, usually by well-meaning clergymen and public benefactors. Some clergymen took private pupils and this tradition continued well into the next century.

By the late 1780s, to counter the radical political literature that was freely circulating, Sunday Schools were established for the poor, their major purpose being to indoctrinate pupils in the principles of religion and the duties of their state in life. Here, if you were lucky, it was possible to learn reading, writing, elementary arithmetic, and the catechism. However, due to the teachers' concern for the health and welfare of their pupils they unwittingly 'created thought in the unthinking masses'. (SIMON 1960, p. 183).

In the 1830s we begin to see the establishment of the English Public School system. The amount of mathematics and science taught in these schools was very variable and schools like Eton, Harrow and Rugby⁹ did not appoint mathematics masters until challenged by some of the newly founded institutions. Substantial reforms were made to preserve the establishment, ¹⁰ by requiring these schools to provide an appropriate education for politicians, civil servants, the clergy, the army and the administrators of the Empire. Since most of the schoolmasters had been educated at Oxford or Cambridge, it was no surprise that the 'Liberal' ethos prevailed, and the theorems of Euclid were regarded as part of the corpus of classical literature.

From 1766 the "Lunar Society" held informal monthly meetings in Birmingham.¹¹ This was typical of a number of "Literary and Philosophical" Societies whose members were forward looking scientists or innovators with interests in practical applications of the new ideas of natural philosophy. Later, more radical interests developed, and they also began to encourage social and political education intending to prepare their sons for their place as leaders of the new industries.

The Private or 'Dissenting' Academies were the places where Non-conformists could be educated. The earliest of these was Warrington Academy, founded in 1757, and the subjects taught had obvious practical applications. Manchester College of Arts and Sciences, founded in 1783, taught sciences and practical arts on four evenings a week. Its syllabus contained classical languages, grammar and rhetoric, mathematics (including trigonometry), mechanics, natural philosophy, (including astronomy and chemistry) English composition, French, commercial and economic geography, history, politics, writing, drawing, book-keeping and shorthand. Subjects like these became the standard curriculum, and most of the important cities of this time developed similar educational institutions. There was a great demand for applied science, and "mixed mathematics". In 1786, the Manchester Academy was established, providing full-time education for students, and a permanent mathematical tutor was appointed in 1787.

The Literary and Philosophical Societies also supported the development of Mechanics Institutes, which became another focus for working class self education. They introduced science, literature and the arts; deliberately excluded politics and religion, and provided lectures, evening and day classes, and libraries. There was a substantial demand for reading scientific (and clandestinely also political) texts and reading rooms and loan systems were established. The curriculum was based on what was "useful" to workers, and lectures were related to practical applications and local engineering and manufacturing problems. Advanced classes were given in a selection of subjects like Grammar, French, Latin; Science, Chemistry, Electricity; Mixed Mathematics, Algebra and Mensuration. Provision for science also meant that collections of apparatus began to be built up, and lecturers established courses, developed curricula, and wrote texts. (INKESTER 1975, Royle, 1971).

8 Military and Naval Schools

Schools of navigation had grown up in the major ports for merchants and traders, and military and naval academies provided an education for the entrants to the army and navy. Woolwich Academy, where BONNYCASTLE and Hutton taught, was founded in 1741, and the teachers there were familiar with contemporary continental texts. In 1837 the syllabus consisted of arithmetic: fractions, roots and powers, proportion, interest, permutations and combinations; algebra: arithmetic and geometric progressions, logarithms, simple, quadratic and cubic equations;

⁸A significant figure in all this was James Mill (not to be confused with J.S.Mill the philosopher) who was educated at Edinburgh university and came to London in 1802 as a journalist. His political and educational theory can be found in the *Westminster Review* particularly during 1824 - 1826.

⁹Leading the reform was Dr. Thomas Arnold, appointed as Headmaster of Rugby in 1828, who reformed the school and created the ideal school for the Victorian upper middle class.

¹⁰The schools concerned at this time are the so-called 'nine greats': Charterhouse, Eton, Harrow, Merchant Taylors, Rugby, Shrewsbury, St Pauls, Westminster, and Winchester.

¹¹Among its early members were Boulton, Watt, Priestly, Galton and Erasmus Darwin (the grandfather of Charles Darwin).

¹²Oxford and Cambridge would only admit those who were prepared to acknowledge the King as head of the Church of England.

 $^{^{13}}$ That is, practical geometry, measurement, arithmetic and sometimes fluxions. NICHOLSON (1823) is a typical text in this genre. See also COOK (1981).

¹⁴The Prospectus of the Sheffield Mechanics Institute (1832) states;

[&]quot;The object of this Institute is to supply, at a cheap rate, to the classes of the community, those advantages of instruction, in the various branches of Science and Art which are of practical application to their diversified avocations and pursuits." (INKESTER 1975).

geometry: plane trigonometry, mensuration, surveying, conic sections; dynamics, projectiles, hydrostatics, hydraulics and fluxions. The syllabus was eventually updated to include the calculus, and other more recent aspects of applied mathematics, and a system of open competitive examinations. (RICE 1996, p. 404).

The Royal Naval Academy, founded at Portsmouth in 1722, (renamed the Royal Naval College in 1806) transferred to Greenwich in 1873. After undergoing similar problems and reorganisations to its military counterpart, from 1885 the Academy taught ballistics for gunnery and torpedo officers, mechanics and heat for engineers, and dynamics for ship construction. Thus it was that by the end of the century clear, practically focused and vocationally relevant courses had evolved for the training of military and naval personnel.

9 The Education of Girls and Women

Sometimes girls attended elementary school, but generally were only taught the most elementary skills. During the eighteenth century a few boarding schools for girls were set up which taught mathematics, science and astronomy, and by the end of the century some women were pursuing their own studies by corresponding with scientists. (Harris, 1997 p. 37). However, it was not until late nineteenth century that mathematics became firmly established in the curriculum of girls schools.¹⁵

No women were admitted to Oxford or Cambridge before the beginning of this century; the Victorian attitude to the mental capabilities of women, and their low social status, together ensured that any opportunities for further education were severely limited. However, this was to change slowly with the publication of the "Educational Times" in 1847, where subjects like the importance of women in society, and the qualities of women's minds were intelligently discussed. The College of Preceptors, founded in 1846, played a major role in supporting women, and from the 1860s we find a growing movement for the elimination of sex differences in education, particularly in mathematics and science. From the mid nineteenth century, higher education for women began to develop. Queens College¹⁶ was founded in London in 1848, the Ladies College Bedford Square in 1849, and by 1878 University College became the first co-educational institution where women and men were examined together.

10 Changes in the Universities

In 1826 University College was founded with the support of those who were excluded from Oxford and Cambridge, liberal politicians, and Jeremy Bentham, the humanist philosopher. In 1828, after demands to provide a religious foundation in London, King's College was founded. In 1828 DE MORGAN was appointed the first professor of mathematics at University College. He was a thoughtful, idealistic and energetic educator whose text books and pedagogical writings show a deep concern for the problems of learning and teaching. His motives for writing On the Study and Difficulties of Mathematics (1831) are to help 'tutorless' students, with the areas of elementary mathematics which give most difficulty, describing their nature without emphasising routine operations. DE MORGAN takes the view that mathematics is a necessary

¹⁵Even then, mathematics was not regarded as a subject really suitable for girls neither in the 'liberal' sense nor in the 'vocational' sense. (see Harris 1997 particularly chapters 3 and 4).

Meanwhile WHEWELL at Cambridge, aimed to place mathematics in the curriculum of every student of the university, reinforcing the "Liberal" view:

I believe that the mathematical study to which men are led by our present requisitions has an effect, and a very beneficial effect on their minds: but I conceive that the benefit of this effect would be greatly increased, if the mathematics thus communicated were such as to dissipate the impression, that academical reasoning is applicable only to such abstractions as space and number. (1836, p. 44).

As the century progressed, university mathematicians seemed less inclined to spend their time educating the masses; growing professionalism motivated more 'pure' mathematical interests and since, from 1850, Cambridge required a knowledge of Euclid for its entrance exams, other universities followed suit.¹⁸

11 The Ideological and Pedagogical Divide

By the end of the nineteenth century "laissez-faire" economics had given rise to a large number of industrial enterprises each requiring ever more specialised training. The Mechanics Institutes were one way to cater for this need, and they helped to develop ideas of economics, of the idealist possibilities of science and technology to improve everyday life, and an acute awareness of the need for appropriate training and new teaching methods. A considerable amount of their work was experimental and practical, and the mathematics required to make the machinery work efficiently was being developed alongside the craft skills of manufacture. Thus it became obvious that the traditional mathematical diet was quite inappropriate to the needs of the new industrial community and advocates of practical mathematics were designing new courses and writing new textbooks. Prominent among these was Perry, ¹⁹ a significant figure in the reform of mathematics teaching at this time. Reforms in school, however well-intentioned, were hampered by schoolmasters educated in the Oxbridge tradition, and a lack of interest from the universities.²⁰

The products of industry shown in the Exhibition of 1851 were based more on the freelance initiatives of innovators than any government sponsored organisation, and it later became clear to government that economic advantage rested not only on technical education but also a good primary education. The 1870 act ensured that education up to age thirteen was available to all, and while the attempts to devise differentiated schooling on a class based system had failed, these attitudes prevailed in the secondary, technical and grammar schools that evolved. It is here that we see the ideological divide; the establishment provided for its own in continuing the liberal tradition in the Public Schools and using mathematics to control the 'gateway' to Universities where 'pure' mathematics flourished, at the same time invoking the utility of 'vocational' mathematics to train the industrial workforce in technical schools and colleges.

¹⁶DE MORGAN taught at Queens college, but only for a year, apparently feeling that the ladies were not of a sufficiently high standard; and as a member of the London Mathematical Society, showed no interest in the attempts to reform the teaching of school geometry. (RICE 1996).

¹⁷The SDUK was founded in 1826 by Henry Brougham and other liberal politicians as an alternative to the radical press, and through its publications intended to give a 'suitable direction' to working class thinking. The Differential and Integral Calculus (1842) was originally published in the Penny Cyclopaedia in forty two weekly parts.

¹⁸Further discussion of the development of the mathematics curriculum and its pedagogy in the latter part of the century can be found in PRICE (1983).

¹⁹Perry was an engineer and his syllabus provided a new paradigm which came from outside the school tradition. (DSA 1899).

²⁰Cayley as chief examiner for Cambridge entrance insisted on keeping Euclid.

Looking at the more recent past, this conflict has been compounded by issues involving pedagogy as well as style and content, but the expectations of the two ideologies can be detected in the nature and mode of presentation of the curriculum materials of today. Dowling (1998) locates these ideologies in a detailed analysis of contemporary school texts; in this brief presentation I am attempting to show their social and historical roots.

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