et par savoir. Notre allusion, dans la section 1 aux travaux de DESCARTES, d'une part, et à ceux de l'école française de sociologie des années 20, montrent clairement que ces concepts, comme tous les concepts d'ailleurs, ne sont pas des données en soi.

Il faudra, peut-être, élargir notre concept de culture et aller au delà du problème de qui construit quoi, la culture, le savoir ou l'individu, et, au lieu de voir la culture comme une entité monolithique, la voir, comme le suggère Fay (1996, p. 231), à la manière de zones interactives hétérogènes d'activité, d'opposition et d'agrément où les individus se construisent entre eux et en se construisant construisent le savoir et la culture elle-même qui, à leur tour, construisent ces individus, etc.

Références

AMIOT M. (1967). Le relativisme culturaliste de Michel Foucault, *Les Temps Modernes* (janvier 1967), Vol. 22, N° 248, pp. 1271-1298.

DESCARTES R. (1637/1967). *Discours de la Méthode*, Texte et commentaire par Étienne Gilson, 4e édition, Paris : Librairie Philosophique Vrin.

DURKHEIM É. (1968). *Les formes élémentaires de la vie religieuse*, Paris : Presses Universitaires de France.

ELIAS N. (1991). *La société des individus*, Paris : Fayard.

FOUCAULT M. (1966). *Les mots et les choses*, Paris : Éditions Gallimard.

FOUCAULT M. (1969). *L'archéologie du savoir*, Paris : Éditions Gallimard.

FOUCAULT M. (1971). *L'ordre du discours*, Paris : Gallimard.

FOUCAULT M. (1975). *Surveiller et punir*, Paris : Éditions Gallimard.

Foucault, M. (1984). *Histoire de la sexualité*, Vol. 3 : Le souci de soi, Paris : Gallimard.

FOUCAULT M. (1991). *Remarks on Marx. Conversations with Duccio Trombadori*, New York : Semiotext(e).

FOUCAULT M. (1994). *Dits et écrits*, Paris : Gallimard, Vol. I.

LEBON S. (1967). Un positiviste désespéré : Michel Foucault, *Les Temps Modernes*, Vol. 22, n° 248, pp. 1299-1319.

LÉVY-BRUHL L. (1949). *Les Carnets*, Paris : Presses Universitaires de France.

LÉVY-BRUHL L. (1960). *La mentalité primitive*, Presses Universitaires de France (15e édition; 1ère édition : 1922).

MARKUS G. (1982). *Langage et production*, Paris : Éditions Denoël /Gonthier.

SARTRE J.-P. (1966). Jean-Paul Sartre répond, *L'Arc*, numéro spécial "Sartre Aujourd'hui", Entretien avec Bernard Pingaud, 87-96.

SPENGLER O. (1917/1948). *Le déclin de l'Occident*, Paris : Gallimard, traduit par M. Tazerout de l'édition de 1927.

VINEY W. & KING B.D. (1998). *A history of Psychology*, Boston, London, Toronto, etc. : Allyn and Bacon.

WILDER R.L. (1968). *Evolution of Mathematical Concepts. An Elementary Study*, New York, London, Sydney, Toronto : John Wiley & Son, Inc.

WILDER R.L. (1981). *Mathematics as a cultural system*, Oxford, New York, Toronto, etc. : Pergamon Press.

Using π to Look for Obstacles of Epistemological Origin in University Students

RALHA Elfrida, VAZ Olga
Universidade do Minho (Portugal)

Abstract

We used two specific Archimedean tasks to investigate 4^{th} year mathematics students' approach to mathematical fallacies which, in one form or another, they are prone to be meeting when teaching mathematics in Portuguese secundary schools. We were particularly interested in tracing their understanding of some basic mathematical concepts comparing their performances with other groups.

In selecting the "$\pi = 3$" and the "$\pi = \lim 2\nu R$" problems, we tried to find epistemological obstacles while:

(a) exploring episodes on the history of mathematics;

(b) encompassing a wide range of basic school mathematics (geometric constructions, circumference, length, trigonometry, equation, rational, irrational and transcendent numbers);

(c) reinforcing the links between different areas of mathematics, namely arithmetic, geometry, algebra and calculus;

(d) underlying a very acute question (mathematical proof) with very simple problems;

(e) presenting problems which are not directly treated within the typical material of the University courses;

(f) bringing in geometric (visual) statements into numerical/algebraic arguments and not the other way, more conventional, around;

(g) promoting mathematical creativity;

(h) assessing mathematical reasoning and communication skills.

CONTENTS

1 The Motivation: How does one explain to a 14 years old the difference between Pi and $\sqrt{2}$?

This research of ours started from a question that was posed to us by a 14 years old boy who came home one day very intrigued with what he had just learnt about "irrational numbers".

His mathematics teacher had told him that both Pi (that, until then, he had always treated as being 3.14) and $\sqrt{2}$ were examples of irrational numbers (meaning numbers that could not be represented in the form $\frac{m}{n}$ where both m and n are integers and n is not zero) and from there she had explained that, nevertheless, one might construct geometrically a line segment whose length is $\sqrt{2}$, or $\sqrt{3}$, or .... Having taught, afterwards, these constructions and having ignored Pi, the pupil asked: "*What about Pi?*"

"*Pi will, for the time being, still be seen as* 3.14 *and therefore you may represent it (in an ordered straight line) close to* 3.14 ", the teacher answered.

But, this teacher was a former student at our University and we started wondering about the meaning of such an answer; we were very much interested in knowing weather her mathematics degree had given her a full understanding of Pi, weather she really knew the differences between Pi and other irrational numbers and weather she was fully aware of the importance of explaining these concepts with different emphasis according to each pupil's level and interests.

This research of ours then began with a search on historical data about Pi.

2 Pi: a never-ending story

Through the ages Pi[1] -any circumference and its diameter are directly proportional- has kept alive the interest of many, more or less famous, mathematicians.

About 4000 years ago, while in Southwest England, an ancient people built the world's known oldest circle (Stonehenge), the ancient Babylonians were using $\pi = 3\frac{1}{8}$ (the error is 1.6593×10^{-2}) and the Egyptian stone masons were approximating the area of a circle by an area of a square according to the following rule "shorten the diameter of the circle by $\left(\frac{1}{9}\right)^{th}$ to get the side of the square"[2] from where one gets $\pi = \left(\frac{16}{9}\right)^2$ (the error is 1.8901×10^{-2}).

[1]It is believed that the Welshman William Jones first introduced the symbol in the early 18^{th} century but it was Euler who generalised its use.

[2]By the Egyptian scribe Ahmose in The Rhind Papyrus, problem 48.

Others, about 2500 years ago, were content enough with $\pi = 3$ (the error is 0.14159), such is presented in the Bible (Kings 7:23).

However the first rigorous mathematical calculation of π is ascribed to Archimedes of Syracuse ($\sim$250 BC). Archimedes used a very clever geometrical scheme, based on inscribed and circumscribed polygons, to come to $3\frac{10}{71} < \pi < 3\frac{10}{70}$ (with $\pi = 3\frac{10}{71}$ the error is only 7.4758×10^{-4} while with $\pi = 3\frac{10}{70}$ the error is 1.2645×10^{-3}) following a reasoning based on the method of exhaustion but also presenting two postulates in order to guarantee the "consistency" of his subsequent reasoning, namely:

Postulate 1: The shortest route between two points is that of the segment which joints them.

Postulate 2: For two curves joining two points and convex in the same direction, the greatest length belongs to the curve that contains the other.

All over the world and through many cultures one may easily find examples of people interested in π: Tsu Ch'ung-chih (China, V^{th} century), Al-Kwarizmi (Islamic World, IX^{th} century) or unknown Indian mathematicians (India, XV^{th} century) who found the series

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...$$

with its important special case

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$$

Pedro Nunes (Portugal, XVI^{th} century), Newton (England, $XVII^{th}$ century) or Adam A. Kochanski (Poland, $XVII^{th}$ century) and J. Anastácio da Cunha (Portugal, $XVIII^{th}$ century) were among the ones who were merely using (applying) or fundamentally searching for understanding of such a "mysterious" number; looking for some decimal expansion with repetitions and thus disclosing the possible rationality of π or solving the famous *quadrature of the circle*.

Frenchmen Lambert and Legendre proved, in the late 1700s, that π is irrational (the search for repetition in π's decimal expansion was over) but some others still wondered whether *the quadrature of the circle* was possible or not; in particular whether π might be the root of some algebraic equation with integer coefficients. This search was finally settled in 1882 when the German Lindemann proved that π is transcendental.

Yet, the motivation surrounding π's characteristics did not end. With the development of computer technology in the 1950s, π has been computed to millions of digits[3]. Beyond immediate practicality mathematicians keep, nowadays, interested in studying π, for example, in relation to its normality and computer scientists are using π to test the integrity of both hardware and software.

[3]We, nevertheless, know that an approximation of π to 40 digits would be more than enough to, for example, compute the circumference of the Milky Way Galaxy to an error less than the size of a proton.

3 About a method for evaluating π: Archimedes of Syracuse

Archimedes is often considered one of the greatest mathematicians of all times. In his time, he was seen as a "wise man" and a "master". Most of his original works are lost, but his methods and achievements survived in texts, such as the ones by Pappus and Hero of Alexandria.

Archimedes used inscribed and circumscribed polygons, which lengths he easily computed, to be able to estimate π: "squeezing" it between a greater and a smaller value rather than getting only a one-way approximation.

Most of the books on History of Mathematics refer to such calculations, but because they have to rely on interpretations of Archimedes' works, one may find several approaches to the problem from where one gets, at least, two methods attributed to him.

For our particular research we came to choose one of these methods, namely the one explored by Wilbur KNORR (an algorithm to compute ratios)[4].

In his article[5], Wilbur KNORR examined a writing from Hero about the works of Archimedes. There, in particular, he attempted to demonstrate that the values of the ratios concerning the circumference and diameter of a circle, as reported by Hero, must be wrong and possibly only due to a miswriting, because:

> Archimedes could not possibly make such a great mistake, such as the case of using $\frac{211875}{67441} \approx 3.141634$ and $\frac{197888}{62354} \approx 3.173774$, both greater than π.[6]

Starting from hexagons[7] and measuring the perimeter of the circumscribed hexagon Archimedes needed to compute $\sqrt{3}$ and came to choose $\frac{265}{153} < \sqrt{3}$ and $\frac{1351}{780} > \sqrt{3}$, without further explanation[8]. These ratios have an error less than 10^{-4}($\frac{265}{153} \approx 1.7320261$, $\frac{1351}{780} \approx 1.7320518$ and $\sqrt{3} \approx 1.7320508$) which he might have considered sufficient enough to carry on the subsequent calculations.

Let us, therefore, have the hexagon circumscribed to the circle of diameter $2r$ having side $2s$ such as in the figure

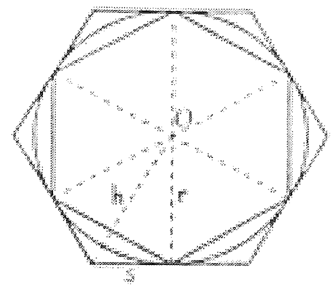
[4]We have also established some comparisons to other method presented by DÖRRIE.

[5]"Archimedes and the measurement of the circle" (1976).

[6]KNORR also refers to works of other Mathematics' historians, who most of them accepted the numbers given by Hero, namely Paul Tannery (~1880), Adriaen Metius (1625) and Ludolph van Ceulen (~1615), trying afterwards to redo the calculations to justify those values.

[7]The choice for starting with hexagons rather than, for example, with triangles or squares may be due, we suppose, to both the fact of a hexagon being clearly (visibly) nearer the circle and being much simpler to construct with ruler and compass alone.

[8]This choice of ratios was taken as if these numbers were well known. This lead to much controversy over the years, from W. W. Rouse Ball in 1908 to Sondheimer and Rogerson in 1981, and the question is not, as far as we know, yet answered. One may, nevertheless, assume that the Babylonian method for evaluating square roots was being used.



$2r$ = diameter circle
$2h$ = diameter circumscribed hexagon

One may consider the right triangle with legs s and r and hypotenuse h so that

$$r^2 + s^2 = h^2,$$

$$\frac{s}{r} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

and

$$\frac{s}{h} = \sin\frac{\pi}{6} = \frac{1}{2}$$

from where we get the numerical values in KNORR's table, such as the one below. In it, such as follows, KNORR presented the values he attributed to Archimedes, namely A, B and C so that $\frac{A}{B}$ and $\frac{A}{C}$ are, respectively, the ratios between the side of the circumscribed hexagon and the diameter of the circle, and the side of the inscribed hexagon and the diameter.

n	A	B	C	n	A'	B'	C'
6	153	265	306	**6**	780	1351	1560
12	153	571	$591\frac{1}{8}$	**12**	780	2911	$3013\frac{3}{4}$
24	153	$1162\frac{1}{8}$	$1172\frac{1}{8}$	**24**	780	$5924\frac{3}{4}$	
					240	1823	$1828\frac{9}{11}$
48	153	$2334\frac{1}{4}$	$2339\frac{1}{4}$	**48**	240	$3661\frac{9}{11}$	
					66	1007	$1009\frac{1}{6}$
96	153	$4673\frac{1}{2}$		**96**	66	$2016\frac{1}{6}$	$2017\frac{1}{4}$

Archimedes was therefore searching for a hexagon so that the ratio between the side and the diameter was $\frac{153}{265}$, the first value used for $\frac{1}{\sqrt{3}}$. We may then suppose that he started with a circle of radius 265, which led to a circumscribed hexagon of side 2×153, so that $\frac{A}{B} = \frac{153}{265}$. In addition, we may verify that $\frac{A}{C} = \frac{153}{306} = \frac{1}{2}$ is the ratio between the side and the diameter of the inscribed hexagon, where $C^2 = A^2 + B^2$.

Afterwards Archimedes demonstrated that, for a right triangle with legs A and B and hypotenuse C, if we bisect the vertex angle, we obtain a new right triangle whose legs satisfy the ratio $(B + C)/A$.

At this stage, Archimedes abandoned the initial geometric problem, in favour of a construction of an effective algorithm, keeping A as the side of the circumscribed hexagon and adjusting the diameter to ensure the relations[9]

[9]This "technique" of adjusting the size of the diameter for proceeding with the calculations was also used by Chinese and Japanese mathematicians dealing with the same problem.

$$A_n^2 + B_n^2 = C_n^2$$

and

$$B_{2n} = B_n + C_n.$$

Archimedes repeats this process with both initial values for $\frac{1}{\sqrt{3}}$, making five doublings, and finally used the circumscribed 96-gon with $A = 153$ and $B = 4673\frac{1}{2}$, and the inscribed 96-gon with $A = 66$ and $C = 2017\frac{1}{4}$, leading to

$$\pi < 96 \times \frac{153}{4673\frac{1}{2}} = \frac{29376}{9347} = 3 + \frac{1335}{9347} \approx 3.142826575$$

and

$$\pi > 96 \times \frac{66}{2017\frac{1}{4}} = \frac{25344}{8069} = 3 + \frac{1137}{8069} \approx 3.140909654.$$

The error is less than 2×10^{-3}, which is quite a good approximation, as we still use it nowadays.

In the end, Archimedes searched for simpler values of π, suited for practical calculations. Using $\pi < 3 + \frac{1335}{9347}$, we may notice that $\frac{9347}{1335} \approx 7.00149 > 7$ and subsequently one gets $\frac{1335}{9347} < \frac{1}{7}$, from where we eventually come with the well known $\pi < 3 + \frac{1}{7}$.

Similarly, for $\pi > 3 + \frac{1137}{8069}$ we have $\frac{8069}{1137} \approx 7.09674 < 7.1$ and therefore $\frac{1137}{8069} > \frac{10}{71}$ with suitable errors[10].

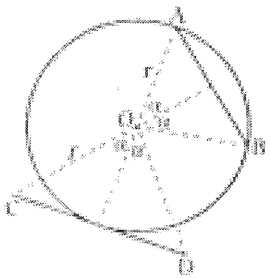
This process of using polygons with an increasing number of sides, creating a sequence that approximates the circumference, supported by the well known method of exhaustion, lead to real techniques of integration (concept of infinitesimals) allowing Archimedes to calculate areas, volumes and surface areas[11], which meant dealing with numerical results and thus developing the geometrical approaches presented in Euclid's Elements.

Basing his explanation in quite a powerful axiomatic method -which started from the two axioms we saw earlier-Archimedes calculates the circumference (C) by squeezing it between the perimeter of an inscribed n-gon (P_i), and the perimeter of a circumscribed n-gon (P_e) knowing that while Postulate 1 justified $C > P_i$, Postulate 2 served as a justification for $C < P_e$ from where we may summarize some of this data.

[10]We may also observe that, using the initial hexagons, we would obtain $\pi < 6 \times \frac{153}{265} \approx 3.46415$ and $\pi > 6 \times \frac{780}{1560} = 3$.

[11]The concept of infinitesimal would only be reinvented in the 18^{th} century by Leibnitz and Newton, 2000 years later.

$s_i = \overline{AB} = 2r\sin\alpha$
$s_e = \overline{CD} = 2r\tan\alpha,$
$P_i = ns_i = 2nr\sin\alpha$
$P_e = ns_e = 2nr\tan\alpha.$



The centre angle is $\alpha = \frac{\pi}{n}$, we may therefore easily compute the following values:

Figure	n	$\alpha = 180^o/n$	$\sin\alpha$	$\tan\alpha$	P_i	π	P_e	π
Hexagon	6	30^o	0.5	0.5773	$6r$	3	$6.9276r$	3.4638
12-gon	12	15^o	0.2558	0.2679	$6.2099r$	3.10495	$6.4296r$	3.2148
24-gon	24	7.5^o	0.1305	0.1316	$6.2652r$	3.1326	$6.3168r$	3.1584
48-gon	48	3.75^o	0.06540	0.0655	$6.2784r$	3.1392	$6.2880r$	3.1440
96-gon	96	1.875^o	0.03272	0.03274	$6.2821r$	**3.14105**	$6.2854r$	**3.1427**

Archimedes' method, by its nature, allowed us to explore some hints about the mathematical potential of π alone: one may easily be dealing with arithmetic and geometry but also with calculus and algebra or numerical analysis and probability together with intuition and deduction. Therefore we found it possible to be adapted to distinct purposes namely projects involving, according to each group's previous mathematical knowledge, pupils or students and teachers or lecturers.

These facts together with the initial motivation for explaining to a 14 years old some data related to π and also our own curiosity on foreseeing mathematics teachers' reactions to π problems, lead us to posing some questionnaires to a group of future teachers of mathematics at a University in Portugal which we then compared, in some cases, with a similar group of students in England and to a group of in-training Portuguese teachers of mathematics.

4 Some data about teaching Mathematics in Portugal

World-wide growth in higher education has risen from 13 millions in the 1960s to 90 millions nowadays. In Portugal the ratio is even higher, namely 24000 (in the 1960s) to 335000 in the last academic year.

On the one hand, Portuguese universities are, at present, basically facing three types of problems:

- the ones reported in most developed countries such is the case of a huge increase in student numbers, a big diversity of institutions and a large variety of curricula and courses;

- the ones reported in some countries such is the case of the implementation of *numeri clausi* for entrance to the University system of education in general and others dealing specifically with mathematics degrees; such is the case of the belief that researchers are, by definition, good lecturers regardless their didactical preparation or even their motivation for teaching and learning. An almost institutionalised routine of assessment practice based on individual written tests appealing to memorisation and algorithmatisation techniques for evaluating the students' performance on basic or more advanced courses such as calculus, algebra, and geometry or differential equations, Galois theory and topology but never evaluating their performance in mathematics nor employing different assessment instruments which, as a whole, most of the students are expected to be teaching and to be using at secondary schools, in a quite near future, and that they do not seem to be able to "unify" nor to identify;

- the ones that, as far as we know, are not typically identified elsewhere; such is the case of most of our mathematics' students being (for many years) women, a large number of our mathematics lecturers being (quite recently) women and most of our mathematics degrees being (traditionally) teaching degrees (preparing, explicitly or not, future secondary school teachers).

On the other hand, we have been listening, in Portugal and abroad, to researchers willing to achieve success in mathematics courses through changing mathematics *curricula*: "complaining" or presenting the justification of undergraduate mathematics curriculum being topics fundamentally developed in the 18^{th} and 19^{th} centuries as opposed to the very new areas of mathematical practice (neuroscience, finances or telecommunications, for example).

5 The methodology of our research

In dealing with mathematics and teaching one gets used to listen to students using words such us "panic", "worry", "anxiety" or simply "error", "boring" and "difficult" more than to "success", "interesting", "easy" or "relevance". We, the authors of this study, were very much interested in underlying some acute questions with quite simple problems.

Besides, most of our students' experience of mathematics seems quite different from the Greek's motivation for studying it (because they wanted to); nowadays, getting a job is, in Portugal at least, one of the most relevant issues for our University students and, because we have been running short of mathematics' teachers for quite some time, mathematics teaching degrees[12] appear, in Portugal, as some of the most wanted degrees.

We basically posed two problems -The Archimedean Tasks (version one and version two)- to a group of 49 mathematics students (30 women and 19 men) who were all using some writing material as well as a ruler and a pair of compasses. By the time this investigation took place, these students, organised in groups (4 or 5 per group[13]) had already succeeded in the first two academic years of their degree; which means that they had been through all the basic math courses as well as on more advanced ones such as Hyperbolic Geometry, Numerical Analysis or Abstract Algebra. Each problem (version one and version two) was introduced to the students

[12]Mathematics Teaching degrees in Portugal are 5 years degrees. The 5^{th} year is, nevertheless, spent doing in-training service in public schools consisting of teaching normal classes of, more or less, 30 pupils.

[13]There was one group composed only by women, one group composed only by men and the others were mixed gender groups.

in a two-stage format (parts i) and ii)), which meant that part ii) was only made available to them after they had decided to finish part i) and to hand it over to the lecturers. This way, we tried to investigate the students' approach to some mathematical fallacies (in the first part-i)) related to π which, in a form or another, they are prone to be meeting in a near future and, (in the second part-ii)) and afterwards, we tried to assess their reaction to the information that these facts were wrong.

We also tried to create an informal environment of discussing the problems by means of a Mathematics Workshop which took place in a reflection-weekend (away from the University Campus) organised by these students. We have explicitly asked for intuitive as well as deductive reasoning presented in a two-column (How and Why) format answer sheet. The problems were:

The Archimedean Task -Version One

1. i) *Draw a circle and, with a pair of compasses opened to its radius, step round the circle.*

- **a)** *How many points have you marked until you get to your starting point on the circle?*
- **b)** *Will you then be able to conclude that the circumference is six times the length of the radius?*

1. ii) **c)** *How would you explain to a high school's pupil who had conducted such reasoning and come out with $\pi = 3$ that his conclusion is incorrect?*

- **d)** *Which is your own definition of π?*

The Archimedean Task -Version Two

2. i) *The perimeter of a regular polygon inscribed in a circle of radius R might be known from: $2\nu R$, with $\nu = n \times \sin\frac{180^o}{n}$ and n representing the number of sides of the polygon.*

- **a)** *Is this a mathematically valid statement?*
- **b)** *Show that the sequence $45 \times \sin\frac{180}{45}, 90 \times \sin\frac{180}{90}, 180 \times \sin\frac{180}{180}, 360 \times \sin\frac{180}{360}, \ldots$ seems to have a limit.*

2. ii) **c)** *Knowing that $\sin 0.5^o \approx 0.0087265$, $\sin 1^o \approx 0.017452$, $\sin 2^o \approx 0.034899$, $\sin 4^o \approx 0.069756$ how would you conjecture an approximation (with two decimal places) to that limit?*

- **d)** *How would you explain to a high school's pupil, who had come out with this approximation, which is the exact value for that limit?*

6 How do Mathematics students (studying to become mathematics teachers), in Portugal, deal with π: Some typical answers

- *π is a conventional number that is difficult to define; it is transcendental.*
- *π is an irrational number,....*
- *Knowing that the circumference of the circle is equal to the perimeter of the hexagon....*

- *$\pi = 3$ because we can round off π to 3, for after the 3 comes a 1.*
- *From $2\pi r = 2\nu r$, we have $\pi = \nu \Rightarrow \lim \nu = \lim \pi = \pi$.*
- *...the value of ν has to be π.*
- *The value for the limit is an infinite expansion, so we can not determine its exact value.*
- *π is not algebraic and therefore it is not constructible.*

7 Conclusions

Through these Archimedean tasks, we have, as far as we hope, given evidence of having:

(a) sucessfully explored history of mathematics episodes in classes of University students;

(b) encompassed a wide range of basic and fundamental mathematics (geometric constructions, circumference, length, trigonometry, equation, rational, irrational and transcendent numbers);

(c) reinforced the links between different areas of mathematics, namely arithmetic, geometry, algebra and calculus;

(d) underlined a very acute question (mathematical proof) with very simple problems;

(e) presented problems which are not directly treated within the typical material of the University courses;

(f) brought in geometric (visual) statements into numerical/algebraic arguments and not the other way, more conventional, around.

About "*having been able to promote the students' mathematical creativity*", we may only say that they worked on the proposed tasks intrinsically interested for several hours, and kept asking questions on the theme for some time after the session was over, due to calendar's obligations. Besides the diversity of possible answers was attested by the endless list of solutions presented by the students and that we might, in another occasion, be able to present in some more detail.

About "*assessing mathematical reasoning and the students' understanding of the concepts involved in the tasks*", we may only summarize our feelings with one word: SHOCK!! Namely:

- About "*Basic Concepts*" - They had no idea, whatsoever, about a definition for π: *It is a number...* Most of the times they treated it as if it was a primitive term or a very abstract concept. The Portuguese students showed no evidence of having any instinct or intuition for this number (as opposed to the English students we also interviewed on the subject).
- About "*Knowing vs. Understanding*" - They did not have a "set" of fundamental results which they really understood; they simply had a quite large memorised "list" of results which they thought they knew and imagined that it might be applicable to the problems: *It is algebraic means it is constructible.* They misused theorems, ignored axioms and mistreated possible applications (as opposed to a less large "set" of results remembered by the English students).

- About "*Logic Reasoning*" - They were much less curious than some of the 14 years old pupils who had posed some of the initial questions: The "Why" column tended to be left aside. They were, nevertheless, more enthusiastic on treating the subject than the English students.

There is definitely a large way to be run by these students until they reach enough maturity which will allow them to teach properly the facts or to treat properly the activities dealing with number π. However the reason for this state of things might not be entirely their fault and it might very well be the case of certain changes being needed in the way they themselves are taught at the University; changes on methodology of teaching and evaluation more than changes on curricula seems to be the case for expecting better mathematical knowledge by part of the students willing to become mathematics teachers in our schools.

It came finally clear to us that the 14 years old pupil to whom we referred to, at the begining of our research, did not receive, from his teacher, a satisfactory answer to his problem because she herself did not know the answer nor knew how to explain the differences between π and $\sqrt{2}$ to all her 8^{th} grade pupils. We, the authors of this study, will go on experiencing history of mathematics episodes in our lectures and searching for epistemological obstacles in order to alter situations such as the one that we have just reported about π.

8 Bibliography

BAILEY, D.H. *The Computation of Pi to 29360000 decimal digits using Borweins' quartically convergent algorithm.* Mathematics of Computation 42, 283-296, 1988.

BROUSSEAU, G. *Theory of Didactical Situations in Mathematics.* Mathematics Education Library, Kluwer Academic Publishers, 1997.

D'ALEMBERT. *Différentiel.* Encyclopédie ou Dictionnaire Raisonné des Sciences, vol. 10. Geneva, 1770.

DELAHAYE, Jean-Paul. *Le Fascinant Nombre Pi.* Pour la Science, 1997.

DÖRRIE, Heinrich. *100 Great Problems of Elementary Mathematics: their history and solution.* Dover Publications, 1965.

HEATH, Thomas L. *The Thirteen books of Euclid' ELEMENTS.* Dover Publications, 1956.

HEILBRON, J.L. *Geometry Civilised, History, Culture and Technique.* Claredon Press, 1998.

KLEINER, I. & AVITAL, S. *The Relativity of Mathematics.* Mathematics Teacher, vol. 77, number 7, 1984.

KNORR, W.R. *Archimedes and the measurement of the circle: a new interpretation*, Archive for History of Exact Sciences, 1976, 15, 115-140.

LAUBENBACHER, R. & PENGELLEY, D. *Mathematical Expeditions.* Springer-Verlag, 1998.

MARTIN, G.E. *Geometric Constructions.* Springer-Verlag, 1998.

MIKAMI, I. *The Development of Mathematics in China and Japan.* Chelsea Publishing Company, New York.

PIERPORT, James. *Mathematical Rigor, Past and Present.* Bulletin of the American Mathematical Society, 34, 1928.

Conflict and Compromise : the Evolution of the Mathematics Curriculum in Nineteenth Century England

ROGERS Leo

University of Surrey (England)

Abstract

This paper explores the social and ideological background which determined the kind of mathematics taught to different groups of people during the early part of the nineteenth century in England. These ideologies arose in and were transmitted through institutions which determined choices and decisions about what was valued as scientific knowledge. The mathematics taught in the universities and in the Public Schools was determined by a classical liberal ideology, whereas the mathematics taught in elementary schools and colleges was driven by a practical ideology of utility, democracy and social justice. The consequences of this conflict can be seen in our current school mathematics curriculum in England. Some observations are also made on the historiographical problems of the history of mathematics education.