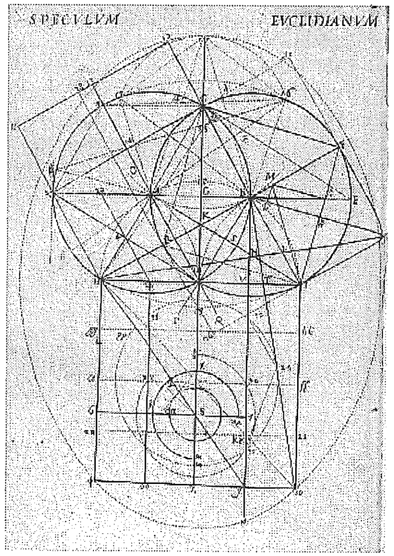

History of mathematics in a preservice program and some results

PHILIPPOU George, CHRISTOU Constantinos
University of Cyprus (Cyprus)

Abstract

We refer to a preservice mathematics program, based on the history of mathematics, which we have developed and implemented during the past nine years (see PHILIPPOU & CHRISTOU 1998). First, we set the stage for the theoretical perspectives, next we elaborate on specific dimensions of the program by giving examples and describing the ways in which they were meant to function, and finally, we present the evaluation results concerning the effectiveness of the program. The efficacy beliefs[1] of the first graduates of the program, with respect to teaching of mathematics, were found to be significantly better than the respective efficacy beliefs of teachers who graduated from other programs.

[1] "Efficacy beliefs" as defined by BANDURA (1997) means "teachers beliefs in their ability to produce a desired effect" (see section 2.1) below.

1 Theoretical background

1.1 Primary preservice mathematics education

Primary school teachers often lack the connected conceptual understanding which might be defined in terms of two constructs: the nature of mathematics and the teachers' mental organization of mathematical knowledge. The former depends upon the ever-growing content of mathematics as a discipline and the latter refers to the organization of teachers' knowledge, how it is acquired, structured, and retrieved. The modern mathematics curriculum has increased demands on the part of the teachers, who are expected to select worthwhile mathematical tasks, to orchestrate classroom discourse, to seek connections that facilitate students, to deepen their mathematical understandings, to help students use technology, and to assess progress (SWAFFORD 1995).

The debate on preservice teacher mathematical education continues, though some of the earlier established concepts and perspectives have been well grounded. BROWN & BORKO (1992), for example, mention Shulmans' seven domains of cognitive schemata from which teachers draw (subject matter, specific and general pedagogical content, other related matter, the curriculum, the learners, and educational aims). LAPPAN & THEULE-LUBIENSKI (1994) focus on a shorter list of three domains that should be considered in the teacher program namely, knowledge of content, knowledge of pedagogy, and knowledge of students. The same outhors consider the integration of knowledge in these three domains with beliefs, as the main concern in the education of professional teachers.

COONEY (1994) refers to content knowledge, pedagogy and psychology of learning as the three requirements for a teacher to be an *adaptive agent* in the classroom. He suggests that a certain orientation toward knowledge and change is also necessary, since the lenses through which we see our world influence what we do. Knowledge of the content of a discipline, knowledge of the teaching of the same subject and one's orientation toward these types of knowledge are different entities. An implication for teacher education is that the development of meaning with respect to teaching any subject is fundamentally connected to the meaning one assigns to learning and teaching this subject. Prospective teachers need to learn a content in a way that is consistent with the way they are expected to teach that content.

Much of the existing practice in schools is based on the assumption that knowledge is manufactured elsewhere, acquisition is tested by another statutory agency, and the teacher is seen as simply to act as a mediator; he or she is only trusted to "deliver" the package (BURTON 1992). The new conception of teachers' education assumes that teachers need to develop a conceptual base and the orientation to become pedagogically powerful. According to COONEY (1994) this means that teachers have: a) to develop a knowledge of mathematics that permits the teaching of mathematics from a constructivist perspective, b) to develop expertise in identifying and analyzing the constraints they face in teaching and in dealing with those constraints, c) to gain experience in assessing a student's understanding of mathematics, and d) to offer teachers opportunities to translate their knowledge of mathematics into viable teaching strategies. There is a growing consensus that constructivist epistemology could be the basis for preparing teachers to teach in reform-oriented ways. According to the perspective, learning is regarded as an ongoing process of individuals trying to make sense and construct meaning, based on their experiences and interactions with the environment. In a project, driven by constructivist views, ZASLAVSKY & LEIKIN (1999) refer to goals such as enhancing teachers' mathematical

and pedagogical knowledge, offering teachers opportunities to experience alternative ways of learning challenging mathematics, fostering teachers' sensitivity toward students' feelings and mathematical understanding, and promoting teachers' ability to reflect on their learning and teaching experiences.

KRAINER (1999) proposed a model for teachers' professional practice focusing on *action, reflection, autonomy*, and *networking*. Action refers to attitudes towards and competence in experimental, constructive and goal directed work. Reflection refers to attitudes and competence in self-critical work that reflects systematically on one's own actions. Autonomy refers to attitudes and competence in self-initiated, self-organized and self-determined work. Networking refers to attitudes and competence in communicative and co-operative work with increasingly public relevance. Each of the two pairs "*action-reflection*" and "*autonomy-networking*" expresses both contrast and unity, and can be seen as complementary dimensions, which have to be kept in a certain balance. The interplay between these dimensions is of great importance. In general, more reflection contributes to a higher quality of actions and a higher quality of action and autonomy promotes the quality of reflection and networking. Reflecting on others and their own teaching practices engages teachers in thinking about good teaching and the meaning that teaching has for students. Experience shows that teachers' practice is usually characterized by a lot of action and autonomy and less reflection and networking (KRAINER 1999).

Professional autonomy is equivalent to self-regulation and to one's ability to make decisions without having to be told by others. It involves the ability to exercise judgement, to make decisions by careful consideration of relevant variables, to distinguish between appropriate and inappropriate actions on the basis of well-specified criteria and standards of behavior. Professional autonomy develops as teachers construct their ideas about the content of a discipline and how it can be taught to others (CASTLE & AICHELE 1994). This means that the new experiences should develop the students' ability to challenge the old and foster new dispositions, to apply mathematical methods and symbolism, to view mathematics as a study of patterns and relationships and to enhance self-confidence. During preservice courses, students begin to envision their future role as the organizers of learning activities and develop their first models for teaching mathematics. At this stage, what the students learn is tidily connected to how they learn it; the way mathematics is taught in a preservice program is more important than the content per se. A historical and cultural approach is proposed as potentially capable of opening new perspectives and meanings on the nature of mathematics and its teaching.

1.2 History of mathematics in teacher education: Why and how?

The "law of recapitulation" received strong support for about a century. The assumption that ontogenesis recapitulates phylogenesis was behind Piaget and Garcia's interpretation of the relationship between historical and psychologic al developments in terms of the mechanisms mediating the transition through the three stages through which an idea develops (intraoperational, interoperational, and transoperational). In recent years, however, the recapitulation law is under serious criticism. The major argument derives form the emphasis on socio-cultural influences. To retrace the mental development of the race is neither possible nor desirable, because "key aspects of mental functioning can be understood within the social context in which they are embedded" (RADFORD 1997, p. 28).

JONES (1989) has seen an exaggeration in the recapitulation law; he argued though that history of mathematics coupled with up-to-date knowledge of mathematics is a significant tool in the hands of the teacher who teaches whys. The critique of the recapitul ationistic parallelism has resulted in a more realistic claim, which might be condensed in the argument that "when scrutinized, the phylogeny and the ontogeny of mathematics will reveal more than marginal similarities" (SFARD 1995, p. 15). In conclusion, we accept that similar recurrent phenomena can be retraced throughout the historical development and the individual reconstruction of knowledge. Epistemological obstacles may not be strictly intrinsic to knowledge, but we have enough evidence that students and adults frequently face learning difficulties similar to those encountered during the genesis and development of a mathematical idea. The *similarity principle* provides the ground for non-naïve use of history to facilitate learning. A commendable approach is to use history as an epistemological workshop that could change the learners' conceptions and transform the practice of teaching mathematics (BARBIN 1996; RADFORD 1997).

Several possible ways have been proposed to answer the question of how to incorporate history in teaching. AVITAL (1995), in order to break the image of mathematics as boring and difficult, draws attention to the human side of mathematics by exposing students to anecdotes and exciting stories from the lives of great mathematicians. BARBIN (1996) focuses on the potential role of problems as a means to bring to the fore the process of the construction and rectification of knowledge. Following the process of genesis of mathematics, we can foresee possible learning difficulties and create a climate of search and investigation through problems from history.

2 Teacher education and teachers' beliefs

Beliefs are conceived as the personal judgments and views that constitute one's subjective knowledge about self and the environment. Beliefs and attitudes are organized around an object or situations, predisposing one to respond in a favorable or unfavorable way; they are contextual, experientially formed, and emerge during action. RICHARDSON (1995) identifies beliefs as the teacher's own theories, which are sets of interrelated conceptual frameworks connected with action, a kind of *knowledge-in-action*. A teacher's knowledge is translated into practice through the filter of his/her own beliefs about mathematics and its teaching and learning (SWAFFORD 1995). In general, beliefs drive action while experience and reflection on action can modify beliefs, i.e., actions and beliefs interact with each other.

A preservice program needs to consider the structure of beliefs the students bring to teacher education and provide experiences that will help students overcome common myths and misconceptions about mathematics, its teaching and learning. Students should transform unexamined beliefs in relation to classroom actions into objective and reasonable beliefs. Belief systems are change resistant; change can occur only when students engage in personal explorations and are involved in powerful experiences in mathematical thinking and conceptual understanding that motivate a new perspective on students' views towards learning. This subsequently, leads to modified classroom practices, though a change in beliefs does not necessarily translate into changes in practice.

2.1 Efficacy Beliefs about Mathematics teaching

The construct of teacher efficacy grew mostly out of the work of Bandura who identified teachers efficacy as a type of self-efficacy, a cognitive process in which people construct beliefs about their capacity to perform at a given level of attainment (TSCHANNEN-MORAN, HOY & HOY 1998). Self-efficacy is a future oriented construct, which might be perceived as "beliefs in one's capabilities to organize and execute the courses of action required to produce given

attainments" (BANDURA 1997, p. 3). Feelings of competence depend on one's experience in connection to related action; teachers' efficacy beliefs about the teaching of mathematics are mostly shaped on the basis of their own content and pedagogical content knowledge. BANDURA (1997) postulated four sources of self-efficacy information: mastery experiences, physiological and emotional arousal, vicarious experience and social persuasion. These four sources contribute to both the analysis of the teaching task and the self-perception of teaching competence.

Early research in the field identified two dimensions of teacher efficacy, the personal teacher efficacy (PTE) that refers to one's feelings of his/her own capability, and the general teaching efficacy (GTE) that refers to the possibility of teachers, in general, to affect students' learning. Another dimension refers to internal versus external control of reinforcement (TSCHANNEN-MORAN et al., 1998). The former refers to the possibility of teachers controlling students' learning (Internal Control), while the latter accepts the predominance of environmental factors in learning (External Control). A teacher's sense of efficacy is a motivational factor influencing the amount of effort one will expend and the persistence he or she will show in the face of obstacles.

Numerous research studies (PAJARES 1996; TSCHANNEN-MORAN et al., 1998) have confirmed the relationship between efficacy beliefs and significant educational factors, such as professional commitment, enthusiasm, instructional experimentation, implementation of progressive and innovative methods, the level of organization, and certainly, students' affective growth and achievement. There is also some evidence relating efficacy to mathematics learning (PAJARES 1995), but the efficacy beliefs with respect to mathematics teaching is a very little researched area.

3 The preservice mathematics program

In designing the program we assumed that the mathematical background of primary school teachers could rely on an overall grasp of the nature and significance of mathematics. In the light of the foregoing discussion, we hypothesized that history of mathematics would function on one hand as a challenge and motivation. On the other hand, we hypothesized that history of mathematics would facilitate students construction mathematical meanings, develop competence and change their mathematical views. Since the major part of primary school mathematics was originated in the early historic period, the learners' Greek origin was another positive factor. On the grounds of the *similarity principle*, the evolutionary process in mathematical thinking, the solution of big problems that intrigued and inspired top mathematical minds, the study of some of the successes and understanding some failures of famous mathematicians could function as an epistemological workshop. It was expected that such tasks would motivate and attract prospective teachers, improve their conceptions about mathematics and its pedagogy, free students of misconceptions and negative attitudes, and establish a balanced relationship with the subject.

3.1 The content of the program

The program consisted of two "content courses" and one "methods course". The content courses began with prehellenic mathematics, continued with Greek mathematics, moved to Islamic and Hindu contribution and to some mediaeval and enlightenment mathematics. The course concluded with six units of contemporary mathematics (Calculus, Liberation of Geometry, Liberation of Algebra, Set Theory, Logic, and Boolean Algebra). In this section we briefly describe the content of the program, and present two examples in some details.

The work on prehellenic mathematics (numeration systems, arithmetic operations, simple problems, and geometry) aimed a) at drawing attention to the genesis of empirical mathematics, which served the needs of ancient societies, and b) at letting students realize the variety of approaches in meeting everyday social needs. Greek mathematics occupies the major part of the first course. Students' activities are organized around selected works and problems from Thales, Pythagoras, Hippocrates, Euclid, Archimedes, Eratosthenes, Apollonious, Ptolemy, Diophantus, and Pappus. At the opening, we focus on the contrast between empirical mathematics and the deductive mathematics developed by the Greek tendency to ask general questions and raise major philosophical issues. Thales provides the starting point to study similarity, Pythagoras offers many challenging examples ranging from proportions and figurative numbers, to the discovery of irrational numbers, and certainly to the great Pythagorean theorem. We study a variety of proofs and extensions of this theorem, as a means to overcome the myth that "to each problem there is always one best solution". The three classical problems of antiquity help students get the meaning of the term "solution", under certain constraints. Efforts to solve the "unsolvable" problems, which occupied the geniuses for centuries, are life experiences about the nature of mathematics.

Though Euclid is visited on various instances, the emphasis is on understanding the first model of axiomatic organization of scientific knowledge. Proposition I,1 draws attention to the level of rigor and opens the way to a respectful but intense critique of the masters. Primitive Pythagorean triples is an interesting activity and the infinitude of the primes is discussed as a model "existence theorem". We Start from Euclid's proposition (IX,21) "Let that A, B, and C be prime numbers, I say that there are more prime numbers" and continue to the modern formulation "Let $P \equiv \{p_1, p_2, p_3, ..., p_n\}$ be a set of n prime numbers; show that there exists a prime number p_k that does not belong to $P, p_k \notin P$". For the work on the fifth postulate, see Example 1.

Archimedes's helix is studied as a means to solve the "quadrature" problem and the "trisection" problem, and his method to estimate the value of π is the departure point for a journey along the efforts to improve the accuracy of π. Eratosthenes's estimation of the circumference of the earth is mentioned as a fine application of mathematical ideas. In Ptolemy's cyclic quadrilateral, the sum of the products of opposite sides being equal to the product of the diagonals is used to compute chords and generate the equivalent of trigonometric identities. Diophantus proposition II,8 "to divide a square number into two rational numbers" is studied and related to the recently solved Fermat's last theorem.

The program includes instances of Moslem and Hindu mathematics, it continues with Leonardo Pissano, the solution of the cubic and quadric equation (Nicolo Fontana and Scipione del Ferro) and the discovery of probability (Pascal and Fermat). Open problem activities are organized on Pascal's Triangle. The unit on Calculus is an introduction to fundamental concepts and specific applications. Zeno's paradoxes provide the introduction to the concept of the "limit", Fermat's computation of the area under a curve and Barrow's method for finding the derivative are discussed, and finally we come to Leibniz-Newton's discovery of the calculus.

The "Liberation of Geometry" (see Example 2) is followed by a unit on the "Liberation of Algebra" to let students "free their minds" from the boundaries set by the Euclidean thinking. After setting the axioms, we refer to Hamilton's quaternions and turn to Cayley's matrices, as a useful example in which the commutative property as well as lot of other rules, often taken for granted,

fail to be satisfied. The unit on Set Theory includes a study of the properties of set operations (union, intersection, and complement), ordered pairs, triples and n-tables, cross-multiplication, the definition and application of binary relations, equivalence relations, and finally the definition of a function. A similar focus also continues in the unit on Mathematical Logic and on Boolean Algebra. The objective in this closing part is threefold. First, to give future teachers a taste of a formal mathematical system, second to let them realize that set algebra and propositional algebra could be united under one abstract system, and third to illustrate the connection of mathematics to electrical circuits and computers.

Example 1. The Unit on Prehellenic Mathematics concludes with a fairly complete treatment of Number Systems. It includes elements of the Babylonian, the Egyptian, the Mayan, the Chinese, the Greek, the Latin, the Decimal and the general place-value system. The structure of each system and its basic properties and operations are discussed and compared. The development of the unit proceeds to connecting the contemporary needs with the properties of the decimal system. Students gradually come to see the influence of culture in creating mathematics and the multiplicity of possible solutions to the same need. The legend about Sessa, the chess inventor, and the King's unkept promise provide a favored example with rich ideas and applications. Questions that were found to motivate the students are included. Why the King could not keep his promise to give the monk one seed wheat for the first square, two for the second, the double for the next and so on until the last square of the chess-board? What is the number of seeds requested $(1 + 2 + 2^2 + \cdots + 2^{63})$? How can this sum be found, how the method is simplified using binary numbers, how can be approximetely estimated, and how can the volume required to store this quantity be found? Extensions of the problem include: What if the request of the monk was to trible the number for each successive square? Which number system is then found? Can we find the sum using an analogous method to the one used above? Change the following numbers over to decimal notation $a_2 = 222 \cdots 2_3$ (64 times), $a_3 = 333 \cdots 3$ (64 times), and generally of the number of the form $nnn \cdots n_{n+1}$ (64 times).

Example 2. The efforts to correct Euclid provide a most challenging opportunity for the students. We begin with Euclid's main propositions on parallel lines I,27, I,28, and I,29, and proceed with some work on equivalent propositions (e. g. Playfair). Next we go to Geminus, the stoic philosopher of the last century B.C., who was one of the first to make a serious attempt to prove the fifth postulate. Examining the principles of the logical building of mathematics, Geminus concluded that "it is as futile to prove the indemonstrable as it is incorrect to assume what really requires proof" (HEATH, Vol. II, p. 227). Geminus was convinced that the fourth and the fifth postulates could be proved. He defines parallelism in terms of equidistant lines, asserting that convergent lines are not necessarily parallel (could be asymptotes). The steps of his ingenious work are followed and the arguments for its breaking down are discussed. We next go to Ptolemy's "proof", which was falsified by Proclus, and Proclus's own "proof" for which he felt so proud. We refer to Wallis and spend more time on Sacheri's work. Finally, we proceed with Bolyai and Lobachevsky to settle the issue. A selection of basic theorems of hyperbolic geometry completes the unit. Experience has shown that the students initially reject any idea of non-Eucledian geometries. The truth of the fifth postulate is taken for granted. Gradually, however, students' resistance to accept some facts of the Hyperbolic Geometry (e.g., that triangles are defective, there exist neither rectangles nor similar triangles) becomes skepticism, and eventually the students are fascinated to follow assumptions and construct proofs to the end.

4 The program evaluation

The program was evaluated with respect to students' attitudes toward mathematics and the mathematics teaching efficacy beliefs of the first graduate's of the program.

4.1 Prospective teachers attitudes

A longitudinal approach was adopted; at the beginning, the subjects were the first year primary students enrolled in the Department of Education in 1992. The students' attitudes were measured, using the same instrument, before they started the first mathematics course (Phase $1 - P1, N = 162$), on the completion of the first course (Phase $2 - P2, N = 137$), and at the end of the whole program in 1995 (Phase $3 - P3, N = 128$). The Department of Education normally selects from among the top 25% quartile. Nonetheless, about one third of the students come from streams with only core mathematics, while about two thirds of successful candidates do not take mathematics at the entrance examinations (it is an optional subject).

Three related scales were used: The Dutton scale[1], which included eighteen items reflecting attitudes, the "justification scales" reflecting students' reasons for liking and for disliking mathematics, and a one-to-eleven point linear "self-rating" scale, reflecting the respondents feelings about mathematics. Two statistical techniques were used to detect patterns in attitude and check for significant change that might have occurred during the implementation of the program. First, the χ^2-test was applied, separately for each item of the Dutton scale, for the Justification scales, and for grouped responses of the self-rating scale (the responses were grouped into five intervals from extremely negative attitudes, to real love for mathematics). Second, the *Median Polishing Analysis* was applied for the three Phases.

4.2 Mathematics teaching efficacy beliefs

The primary teacher population of Cyprus comprises of four groups: graduates of the Pedagogical Academy (PA), graduates of the PA who have obtained a university degree (PAU), graduates of the University of Cyprus (UC), and graduates of Greek Universities (GU). A questionnaire was mailed to selected schools, and 157 were returned (about 65% of the total). The subjects were 91 (58.7%) PA graduates, 15 (9.7%) PAU, 28 (17.8%) UC graduates, and 21 (13.4%) GU graduates. About three months later, we interviewed 18 teachers (ten were UC graduates, six were PA graduates, and two were GU graduates) focusing on issues raised in specific items of the scale. The interviews were tape-recorded and qualitatively analyzed.

The instrument was a five-point Likert-type scale comprised of 28 items. PTE was measured in six different dimensions: Internal interpretation of control, external interpretation, mathematics teaching anxiety, mathematics teaching enjoyment, managing the school climate, and effectiveness of the preservice mathematics program. Four indicators, all of the external interpretation of the student's learning control, measured the general teaching efficacy dimension (GTE) (see Table 1). The negative statements were reversed and the data were analyzed for the whole sample and for each group, on the whole scale, on the sub-scales and on each component, separately. The ANOVA was used to explore possible differences between sample groups and age groups.

[1] DUTTON (1988)

5 Results and discussion

5.1 Attitudes and attitude change

An alarmingly high proportion of students bring extremely negative attitudes to Teacher Education. For instance, 24% of the students endorsed the statement "I detest mathematics and avoid using it at all times" at the entry phase. Similarly, the statements "I have never liked mathematics", "I have always been afraid of mathematics" and "I do not feel sure for myself in mathematics" were endorsed by 28%, 15%, and 47% of the students, respectively.

The χ^2-test showed significant difference ($p \leq .05$) on 14 out of 18 statements, 13 of which indicate positive change during the program implementation. The proportion of those who "detest mathematics" dropped from 24% to 12%, of those who "never liked mathematics" from 28% to 18%, while of those who "enjoy working and thinking about mathematics outside school" raised from 20% to 40%. Finally, the proportion of students who "never get tired of working with mathematics" raised from 19% to 27%.

The same pattern of responses appeared in the Self-rating scale. On entrance, 36.9% of the students located themselves in the range 1-5 indicating negative attitudes about mathematics, 20% expressed neutral views, and only 43.1% of the students' felt positively towards mathematics. In the course of the program implementation the proportion of subjects who detest mathematics dropped from 14.6% to 5.9% ($p \leq .01$), while the proportion of subjects on the positive side of the scale raised from 39.8% to 51.6%.

On entrance, students stated that they liked mathematics because "it develops mental abilities" (47%), "it is practical and useful" (39%), "it is interesting and challenging" (35%), and "it is necessary for modern life" (35%). They disliked mathematics mainly because "they were "afraid of it" (29%), due to "poor teaching" (27%), and "lack of teacher enthusiasm" (25%)". The χ^2-test showed significant change on nine out of the ten statements of the liking part of the scale and two of the disliking part, all in the positive direction. For instance, the proportion of those who like mathematics because "it is necessary for modern life" raised from 35% to 76% ($p \leq .001$), and because "it develops mental abilities" from 47% to 72% ($p \leq .001$), while significantly more students disliked mathematics due to "teachers' lack of enthusiasm", at the end of the program than at the beginning ($p \leq .001$), and fewer students continued to believe that "mathematics is never related to everyday life" ($p \leq .03$).

The *Median Polishing Analysis* was applied to responses of students' on the Dutton scale, which was partitioned into three sub-scales focusing respectively on *anxiety* from mathematics (five items), *usefulness* of mathematics (four items), and *satisfaction* from mathematics (eight items). This method partitions two-way tables into three interpretable parts, the Grand Effect (GE), which indicates the typical response across all the items, the Row Effect (RE) which tests for differences between responses in different rows (phases), and the Column Effect (CE) that reveals relative differences among the items. The results showed a low endorsement of the anxiety part (GE=21%), a rather high acceptance of the utility part (GE=41%) and a moderate agreement on the satisfaction part of the scale (GE=34%). Row effects showed consistent positive change during the three phases in all sub-scales. Specifically, a reduction was observed on the anxiety part $3.0 \Rightarrow 0.0 \Rightarrow -3.0$, meaning a positive development from one phase to the next, an increase on the utility dimension $-4.5 \Rightarrow 7.5 \Rightarrow 9.5$, showing a steady improvement of attitudes, and finally the change in the satisfaction sub-scale was from $-14.5 \Rightarrow 3.5 \Rightarrow 3.5$. Thus, an improvement was observed between the first and the final phase, in all three sub-scales, that might be due to the mathematics program.

Extracts form the interviews tend to affirm the conclusions concerning beliefs and views about mathematics and the role of the historical approach. The first three refer to students' feelings before and the fourth after exposure to the program.

- "My attitudes were extremely negative thanks to my teachers. Mathematics was for me a piece of work based on getting the right answer and I just could not do that".
- "The proper way to learn mathematics was by memorizing facts and procedures",... "any statement or answer in mathematics was either right or wrong".
- "When I entered the University I felt relief; I was happy, thinking that I had finished with mathematics. The moment I learned that the program required 3 more courses in mathematics I felt frustration. I felt that mathematics will hunt me for ever".
- "History of mathematics provided me with a variety of interesting, new, experiences. I realized that mathematics has always been and continues to be a very useful subject... I followed the efforts of people to use mathematics to solve daily problems. The course showed me that mathematics is normally a human activity. I felt more confident when I realized that even great mathematicians did mistakes as I frequently do".

5.2 Mathematics teaching efficacy

Table 1 shows indicative items from each dimension of the scale and the percentage of participants who endorsed one of the two positive alternatives, for UC graduates, PA graduates (together with PAG), and GU graduates, respectively. A variability of endorsement on items and among groups of participants is evident.

Table 1

Endorsed Proportions to selected Efficacy Items by CU, PA, and GU Subjects

	Personal Teaching Efficacy (PTE)	
Internlal Contrl (P-I)	11. When a child becomes better in mathematics, I believe that it was due to the variety of different ways I found to help him/her	(82%, 76%, 67%)*
External Contr (P-E)	12. I feel that irrespective of my effort, I cannot teach mathematics as successfully as I can teach other subjects	(93%, 81%, 86%)
Teach Anx	21. Sometimes I feel anxious that a student might ask me a question that I do not know how to answer or I cannot explain	(86%, 89%, 90%)
Teaching Enjoy	30. If I were to choose one subject in a colleague's class, I would have opted for mathematics	(57%, 40%, 29%)
School climate	15. When I have difficulties as to how to teach mathematics, I seek advice form experienced colleagues in my school	(50%, 66%, 52%)
Preservice program	9. The preservice mathematics program, offered me the necessary basics to become an efficient mathematics teacher	(50%, 25%, 28%)
	General Teaching Efficacy (GTE)	
External cont. (G-E)	5. As now things really stand, the weak students cannot get the required help to get through in mathematics	(50%, 43%, 19%)

* The percentages represent the positive endorsement by CY, PA, and GU graduates, respectively

Four out of five participants endorsed the positive side in six items, which reflect that teachers felt competent "to help pupils make progress even in topics of mathematics considered as difficult", "to help pupils think mathematically", "to consult experienced colleagues, when facing difficulties", "to answer the pupils' questions", "to correct pupils' assignments", and "unwillingness to give away mathematics in case they had the chance to give away one course". On the other side, the majority of participants ($\geq$ 50%) endorsed the negative side of the scale in five items reflecting "anxiety to cover the subject matter", "efficacy of the preservice program", "capability to help weak students", "possibility of weak pupils to get help", and the ability to "discipline a student who is not used to from home".

The ANOVA showed significant differences between the four sample groups (UC, PA, UPA, and GU graduates) on the general teaching efficacy dimension (GTE), on the preservice mathematics program, and on the total scale according to years of teaching service (0-5, 5-10, and $\geq$ 10, years). The first finding of this study was that the UC graduates, more than the other groups, believe that students are teachable, i.e., that students progress is controlled by internal to the school factors ($\underline{F}$ = 3.150, d.f. = 3, $\underline{p}$ = .027). Specifically, UC graduates were found to have better beliefs on each one of the four items comprising the GTE dimension of the scale. The peak of the difference was found on the item reflecting beliefs that "as the situation really stands, the weak students cannot get the required help to get through in mathematics" ($\underline{X}_{UC}$ = 3.3, $\underline{X}_{rest}$ = 2.9). UC graduates were also found to hold better beliefs that t he other three groups concerning their preservice program of mathematics education ($\underline{F}$ = 8.992, d.f. =3, p = .000). PA graduates (both sub-groups) expressed the most negative evaluations of the preservice program, while GU graduates expressed moderate feelings ($\underline{X}_{UC}$ = 3.29, X_{PA} = 2.75, X_{PAG}=2.27, X_{GU} = 2.90).

The significant difference in efficacy beliefs found according to the length of service on the total scale ($\underline{F}$ = 3.257, d.f. = 3, p= .042) means that teachers beliefs tend to get worst during the first years of service and improve later on (the mean value varie d in the form 3.59 ⇓ 3.47 ⇑ 3.65). This affirms and extends earlier findings (Hoy & Woolfolk, 1993) that efficacy feelings improve with experience. The UC graduates felt uncertain about the best procedure to adopt in teaching certain topics ($\underline{F}$ =3.150, d.f. = 3, p = .021), whereas they did not, as much as the other teachers, endorse the idea that "there are children facing so many difficulties that I am unable to help". The former finding can be interpreted as an excessive responsibility of CU graduates during the first year of their employment.

Analysis of interviews

In interviews we encouraged the subjects to talk about their experiences and express their evaluations with respect to mathematics teaching. The responses were classified into three categories: as positive, neutral, or negative, on the basis of the text. We present excerpts on perceived efficacy a) to influence the learning of non-mot ivated pupils (NM), b) about the preservice program (PSP), and c) to manage the school climate (SCL).

$Q1 - NM$ How confident do you feel to help the non-motivated pupils?

Positive view I am sure that I can help all students to make progress. For the 2-3 n on-motivated or slow learners that are normally found in every class, special provisions are needed, such as to simplify activities, allow for more time, and be in close contact with parents.
Neutral view : I can help slow learners make progress, but "it is not possible for all students to reach the same level". In every class "there are two or three special cases (e.g., problematic families) for whom it is very hard to do anything". "They need special attention and I have no time".

$Q2 - NM$: In the case that a child makes progress, to whom should this success be credited?
Positive view : There are many factors that influence students' learning. The final outcome is due to a combination of joint efforts. However, in my view the teacher is the factor to be primarily credited.
Neutral view : I think that a student's progress is due to the teacher, the parents and the student himself. a) "Teacher's influence on students' learning is limited because they are at school only for about 4-5 hours a day". b) "In cases with serious problems, the teacher cannot do much".
Negative view : a) "The crucial factor is the child, quite a lot depend on him/her". b) "I think that everything depends solely on the child".

$Q1 - PSP$: How do you judge the preservice mathematics program you passed through?
Positive view : I believe it offered me all the necessary background to teach mathematics. When I was a student, I was frequently wondering whether several of the issues and ideas discussed were practically applicable, now I am convinced they are useful.
Neutral view : Most useful was the Methods course. History of mathematics helped in the sense that one appreciates the developmental nature of mathematical ideas, but I think that the tutorials could have been more profitable.
Negative view : We only did one course on teaching methods, "which was just an introduction, rather irrelevant to teaching".

$Q2 - PSR$: A frequently raised point refers to the balance between theory and practice, what do you think about that?
Positive view : I now think that the teacher needs a strong theoretical basis. We covered a wide spectrum of topics and this provides the teacher with a basis necessary to choose and create

didactical situations on his own.
Neutral view : I think it could have offered us more. There were quite a few topics, which were not useful for the primary teacher in the content courses, that caused us additional anxiety. I think we should stick to methods of teaching specific topics.
Negative view : "It paid too much attention to mathematics instead of methods of teaching mathematics, it was very poor... Instead of sets and probability, we could have done more didactics of mathematics". "We did a lot of mathematics at the high school, we should do more teaching methods instead".

$Q1 - SCL.$: How much concerned do you feel about the subject matter coverage?
Positive view : a) "I don't worry about that, though I am quite behind schedule. I do not like to rush and miss some pupils. I will finally be able to get through the subject matter". b) "In my view it is not necessary to cover all the topics. The issue is to let pupils learn what we do, so I don't worry". "I do not proceed beyond a certain point, unless I am certain that 98% of the pupils have learnt it well". The motto in our school is quality, nor quantity". "We all protested. The inspector stated that we were behind schedule, but I cannot push the children, they need to understand".
Neutral view : a) "Yes, because there is so much in the books, and I realized that the level of my pupils is not so high". b) "One has to insist on the basics, not to rush, and that worries me a lot". c) "We hurvied to cover the prescribed matter without going in depth".

$Q2 - SCL$: How do you feel when the principal or the inspector attends your class?
Positive view : "Well, it's natural not to feel as easy as when you are on your own; it may cause me some tension but not really anxiety. I have nothing to hide, I want them to get the real picture of the class, to bring possible problems on the surface". "It is a matter of self-confidence".
Neutral view : "There is a certain degree of anxiety. After all, you are under assessment, they are examining your results".
Negative view : "I feel anxious and uneasy... I think it is in my character".
Table 2 summarizes the responses on the three issues of each of the three teacher groups. The general picture seems to affirm the results found from the questionnaires. UC graduates expressed better efficacy beliefs to influence the non-motivated students and about the preservice program, while they felt relatively less efficient to manage the school climate. The latter might be a consequence of antagonism, which was developed in some of the older teachers due to the abolition of the Pedagogical Academy. There are indications that this climate is getting better. PA graduates do not feel as efficient to help the non-motivated students and they consider their preservice program as inefficient. The responses of UG graduates seem to be rather normally distributed, indicating that they hold moderate efficacy beliefs.
Table 2
A summary of classified responses in interviews

		Univ. Cyprus	P. Academy	Greek Univ.
Efficacy to influence the Non-Motivated	Positive	85%	33%	2/6
	Neutral	15%	50%	4/6
	Negative	0%	17%	2/6
Efficacy to manage the School Climate	Positive	54%	78%	4/6
	Neutral	32%	17%	2/6
	Negative	14%	5%	2/6
Efficacy of preservice program	Positive	50%	0%	2/6
	Neutral	50%	50%	4/6
	Negative	0%	50%	2/6

6 Conclusions

The results of this study provide support for three hypotheses. First, prospective teachers bring to teacher education serious misconceptions and negative attitudes towards mathematics. Second, the designed mathematics preservice program was effective to change preservice teachers' attitudes and third, the UC graduates hold better efficacy beliefs in mathematics teaching than the graduates form other institutions.

Negative feelings towards mathematics are mostly due to teachers' shortcomings, which lead to students' failures and negative attitudes. The situation calls for urgent measures, otherwise, it is highly probable that a considerable proportion of teachers will continue to view mathematics as a fixed discipline, teach along the traditional lines, influence students in non desirable directions, and perpetuate the same situation. One of the tasks of Education Departments is to break down the vicious circle so formed.

In the present study, we sought change in belief systems as a secondary goal of the teacher preparation program. History of mathematics was used throughout as the vehicle to develop mathematical understanding, though two more special environmental factors were conducive to that effort. First, the establishment of a new Department offered the chance to design a mathematics program from the beginning, and second, the historical heritage of the student population. At the end of the program, changes in the attitudes of the students were observed as evidenced by three complementary instruments and several statistical analyses. A significant change in students' responses was found in most items of the Dutton scale, on each of the three sub-scales, on the Justification scale (mostly the liking part), and on the self-rating scale. The fact that there has been non-desirable change of attitudes in two items indicates that in some cases the improvement has not reached the level to overcome deeply rooted mathematics anxiety. Most of students' feelings were formed over their entire school life and, in many cases, were influenced by long prejudices of the social environment. It seems that some emotions in the minds of students are resistant to change; longer exposure and more challenging experiences seem to be essential in order to override them.

CU graduates more than other teachers felt that they are capable to help even the unmotivated pupils, that pupils in general are teachable, and that they were relatively satisfied with their preservice mathematics program. It should be noted, however, that the "success" of the implemented program was found to be satisfactory in comparison to the other two groups, who showed a low esteem of their own preservice programs (in absolute terms the acceptance of the program is rather moderate). The overall evaluation of the preservice program is considered, however, as positive because CU graduates were found to outperform their counterparts in almost all scale dimensions as well. This indicates that the program based on the history of mathematics was effective in improving students' attitudes and developing positive mathematics teaching efficacy beliefs.

It is surprising and it deserves further investigation that the fact that no differences were found in attitude change in terms of gender, type of high school, mathematics performance, and the family socio-cultural conditions. It would be very encouraging, if the program is really so powerful as to affect students' beliefs and efficacy feelings, irrespective of individual characteristics.

The present study did not disentangle several factors that might have been operative. One of these factors relates to the mental models that the program created in the students' minds about mathematics. Another factor is the presence of the university instructors themselves and the way they implemented these models in the classroom. A final reservation concerns the permanence of this change and its long lasting effect on actual teaching behavior. This final dimension is one of the directions in which we plan follow up and further evaluation.

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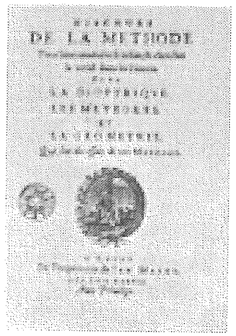
Sur les modes du savoir[2]

RADFORD Luis
Université Laurentienne,
Ontario (Canada)

Abstract

Ce travail se penche sur le problème de la construction du savoir mathématique dans une optique culturelle. En prenant la rationalité mathématique d'une certaine époque comme partie de la rationalité culturelle dans laquelle elle se trouve inscrite, il s'agit d'investiguer comment la rationalité mathématique se forme et donne lieu à un type particulier de savoir à la lumière des pratiques de signification culturellement reconnues, pratiques qui délimitent le pensable et l'impensable, le possible et l'impossible. Le "texte" mathématique est vu ici comme un discours se déployant suivant les possibilités sémiotiques offertes par les relations issues des activités sociales des individus et la conceptualisation du monde et des objets mathématiques que ces activités permettent selon représentations sociales en place. Le cas des mathématiques Babyloniennes sert à illustrer cette approche culturelle épistémologique.



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