The role of physics in introducing vectors to secondary school students

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Abstract

Despite the originally geometrical point of view adopted by mathematicians of the 19th century concerning the nature of the concept of a vector, history suggests that vectors were established as a language of mathematics and science (symbolism, terminology, and computational techniques) mainly through physics.

Physics offer intuitively suggestive situations for introducing vector notions and operations in school. Previous research by H. DEMETRIADOU & A. GAGATSIS has identified concrete and persisting difficulties among Greek high-school students (aged 15-18) concerning certain epistemological aspects of the concept of a vector. Some of them concern the confusion between notions like "vector and line segment", "sense and orientation or path", the meaning of symbols like " + " or " = ", as well as the use of vector operations.

It was also found that there is a serious difficulty concerning the differentiation between vectors as "line segments with a definite magnitude and direction (path and sense) attached to a point" (called "tied vectors" in the Greek curriculum), the prototype being the physical concept of force, and vectors as "line segments with a definite magnitude and direction (path and sense)" (called "free vectors" in the Greek curriculum), the prototype being the geometrical concept of a parallel translation.

These are related to apparently different types of addition: the parallelogram and the triangle laws respectively. From a more advanced, mathematical- epistemological point of view, these laws correspond to the distinction between "tangent vectors to a manifold" and "parallel translated tangent vectors on a manifold", the latter concept requiring more structure (i.e. the concept of a connection on a manifold) than just the ordinary differential structure of a manifold. This partly explains our experimentally confirmed result that often, secondary students do not recognize the equivalence of these laws.

We conclude that purely mathematical teaching situations are not the most appropriate means to introduce vector notions and operations. Inspired by the historical development of the subject and based on previous work concerning the students' difficulties concerning vector notions and operations, we suggest the use of real and thought experiments related to displacements, velocities analyzed as successive displacements, and forces, not only for introducing, but also for clarifying vector notions. Some results from a teaching experiment are also presented in this connection.

1 Introduction

Despite the geometrical point of view adopted by mathematicians of the 19th century, concerning the nature of a vector, history suggests that vectors were established as a language of mathematics and science mainly through physics (whether one considers symbolism, terminology, or computational techniques).

In fact, it was the interplay between pure mathematical and physical situations that influenced the emergence of vector concepts and operations. This historical influence of physics is ignored by the secondary curriculum when vectors are introduced in school. In our opinion, the vector concept is too complicated to be introduced in its abstract mathematical form in secondary education. Physics, on the contrary, offers intuitively suggestive situations not only for introducing but also for clarifying vector notions and operations in school.

Vector notions are considered as a rather marginal subject in secondary school mathematics in Greece. Young students are left for many years to form their own ideas about vector concepts on the basis of physics lessons and every day life experience, until the age of 17 or 18, in the last year of high–school. For the first time, at that age, they come in contact with vectors as a mathematical concept having both a geometrical and an algebraic character, and they are asked to use it as a tool for solving geometrical problems. On the other hand, there is a peculiar situation concerning the teaching of geometry in Greek schools. For 5 years (age 13-17) classical Euclidean geometry is systematically taught and only in the last year (age 18) are vector and analytic geometry presented.

Previous research by H. DEMETRIADOU & A. GAGATSIS ([10]) has verified that students are more successful in solving geometrical problems by euclidean methods than by vector ones, towards which they have a rather negative attitude. Similar errors are also made by younger Greek students ([6], [7], [8], [9]). Most of these errors are due to preconceptions and false ideas about vectors. Students' difficulties and strong preconceptions concerning vector quantities and operations, motion, and the distinction between a vector and a scalar have also been verified by other researchers, mainly in physics education (WARREN 1971; TROWBRIDGE & MCDERMOTT 1980, 81; WATTS & ZYLBERSZTAJN 1981; WATTS 1983; MCCLOSKEY 1983; AGUIRRE AND ERICKSON, 1984; AGUIRRE, 1988; AGUIRRE & RANKIN 1989; ECKSTEIN & SHEMESH 1989; GILBERT et al 1982; ARONS 1992; KNIGHT R.D. 1995).

Based on students' difficulties concerning vector concepts, and implicitly influenced by the historical development of the subject, we attempted a teaching experiment based on real and thought activities and situations related to displacements, velocities and forces, for both introducing, and clarifying vector notions.

2 The research

A pilot teaching procedure was performed the previous year. It concerned a class of 30 students in the second year of high-school (aged 14), before they had received any physics lessons in which vectors are introduced. The research indicated difficulties related either to the students' preconceptions, or to the nature of the concept of vector, some of which remained after teaching. These difficulties concern the confusion between notions like "vector and line segment", "direction and orientation as it is used in every day experience", the meaning of "+" and "=" signs, and the use of vector operations.

The main experiment took place during the school year 1998-99. It concerned two groups of

students in the third year of the same high- school (aged 15): the experimental group consisted of two classes (C_1, C_2) of 28 and 30 students respectively, the second being the pilot group. The researcher, who was also their mathematics teacher, taught in this group. Two other classes (C_3 , C_4) of 29 and 31 students were used as the control group. In these classes the traditional geometric and algebraic aspect of vectors, as presented in the school- book, was taught. In this connection it should be noticed that teaching this chapter is optional and in most schools it is usually omitted. The teaching in both groups started after the study of vector quantities in physics. Firstly, a questionnaire (Q_1) was given to both groups to revere their conceptions about vector notions and operations. Two other questionnaires were used, one after the end of the lessons (Q_2), and another (Q_3) after two months, to check conceptions that had been improved or difficulties and misconceptions that still remain strong.

Lessons in the experimental group concerned the introduction of vectors, their characteristic elements, symbols, and geometrical representation, equal and opposite vectors, addition, subtraction and multiplication by a number. The teaching was focused on students, in the sense that they had the opportunity to construct most of the notions, the symbolism and the operations. The lessons were taped and calendars were kept in every experimental class. The teaching was based on physical activities and situations where vector quantities were used. Displacements for the introduction of vector notions, velocities for operations between collinear vectors, and both forces and velocities analyzed as successive displacements, for the study of operations between non- collinear vectors.

3 The experiment: teaching procedures and difficulties encountered

3.1 Vector and line segment

From the very beginning we tried to clarify the distinction between vectors and line segments. The difference was focused on the element of motion, and was given by simple examples of displacements. Comparing different displacements, students realized the significance of origin and terminus points and consequently the significance of magnitude, and direction i.e. path and sense (see Appendix I). They also came in contact with the idea of opposite vectors. After these notions had been elaborated on an intuitive ground, we tried to give more formal descriptions or definitions of them. By successive questions posed by the teacher and discussions in the class, the students finally arrived at a definition. Thus a vector was defined as a new entity of both mathematical and physical character, related to a line segment, and to the concept of motion, and therefore its origin and terminus points are thus determined. A path was defined to be the line on which the vector is lying and every line parallel to it. The notion of sense was described through the movement along the line of path.

3.2 Vector and vector quantities

In our previous research we have identified difficulties concerning the relation between "vector" and "vector quantities". Students see only one aspect of this relation: vectors as a tool for the representation of vector quantities. We tried to present the other aspect as well, i.e. that a vector is an abstract notion whose concrete representations are vector quantities. Students were asked to formulate the concept of a vector precisely. The most convenient way was to use vector quantities as examples of this notion. They mentioned velocity, force, weight, acceleration as examples of vectors, since they are characterized by magnitude, path and sense.

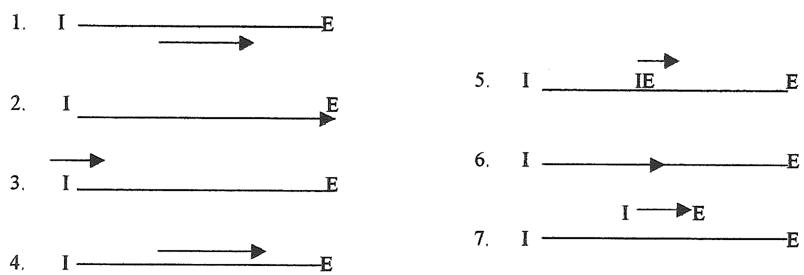
3.3 Symbols and geometric representation

After the distinction between vectors and line segments was discussed, the need for an appropriate symbolism and geometrical representation of vectors arose naturally. Students were asked to make their suggestions, which were written on the blackboard: $IE, x, \vec{IE}, I \rightarrow E$. For every suggestion, comments were made and finally it was accepted or rejected by the majority of the class. The first two ones were very close to the notation used for line segments or straight lines and were rejected since they do not show the sense of motion. During the second lesson, the last symbol was also rejected, since it covered a lot of space.

After vectors representation had been discussed, a new symbol $\overleftarrow{BO}$ appeared in both experimental classes to show the displacement opposite to $\vec{C}$ [1]

This symbol was discussed a lot, and appeared sporadically through lessons. Although it was finally rejected, two children, one at every class kept it until the end and in the final questionnaire it caused confusion to both of them.

Concerning the geometric representation of vectors, the following suggestions were made:



The students finally kept the 2nd and the 4th representation. When the teacher drew some vectors using model 4, with paths cutting each other, most of the students realized that it was complicated. It was finally left out after the second lesson.

3.4 Misconceptions about Sense and path

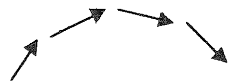
Several difficulties and misconceptions concern the notions of sense and path, mainly related to preconceptions about orientation used in every day experience. The following misconceptions seem to be closer to the notion of sense. However, in most cases they also have a negative

[1]It may be interesting to notice that this notation invented by the students is currently used in research mathematical domains such as non-commutative geometry, stochastic calculus, etc!

influence in understanding the notion of path. The most characteristic error is that nonparallel vectors are considered to have the same path.

3.4.1 Sense related to circular motion

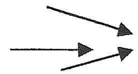
Vectors which are either successive, or with paths which could be similarly oriented as tangents to the same circle were considered as having the same sense.



Students were strongly influenced by the use of the term "sense" in physics lessons for the orientation of circular motions. A characteristic case is that of two students, one at every experiment class, who insisted that identically oriented arcs on a circle are vectors of the same sense. It is remarkable, that it took quite a long time before some other students realized that those were not vectors at all. There was a long discussion about the different points of view concerning vectors and circular motion.

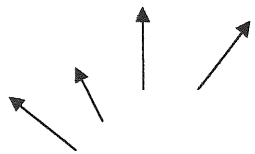
3.4.2 The concept of sense related to destination

Vectors whose terminus points are very close were considered as having the same sense.



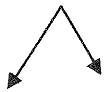
3.4.3 The concept of sense related to points of compass

Some students consider vectors to be of the same sense when "they go up", "down", "left", "right", "southwest", etc. Or they consider two vectors of horizontal and perpendicular paths respectively, as opposite. E.g. These vectors are considered as having the same sense, since "they go up".



3.4.4 The concept of sense related to the same origin point

This could be regarded as closely related to the previous conception. These vectors are considered of the same sense, since "they go to the same place" or "they go down"



3.5 The equality relation - The "=" sign

As verified in our previous work, students consider vectors of the same magnitude to be "equal", and they use the notation: $\vec{a} = \vec{b}$ for such vectors. This misconception could partly be due to the manipulation of vectors as line segments, and partly to the presentation of equality relation in the school- books. Indeed, the use of the term "equality", as well as the use of the "=" sign for vectors, seem to be rather confusing in school mathematics. Mathematically speaking, equality of vectors is actually an equivalence relation with respect to magnitude, sense and path, which is completely different from the concept of "equality" of line segments; the latter is an equivalence relation with respect to the length of segments. However, the same word "equal" and the same symbol "=" are used to denote both equivalence relations. This is a subtle point for the clarification of which no effort has ever been made in Greek textbooks or curricula. This point was discussed in the classroom, and other examples of different types of equality in the sense of equivalence were also mentioned, like the above mentioned equality between line segments or fractions (see also MARJORAM 1966; DAVIS & SNIDER 1987). Simple situations with pairs of equal displacements, forces on the same rigid body, and velocities were used for the clarification of the subject. These vector quantities were considered equivalent because they lead to the same result. Concerning notation, students suggested the notation $|\vec{a}| = |\vec{b}|$ for vectors of equal magnitude, inspired by the symbol used for the absolute value of a number, in distinction to the notation $\vec{a} = \vec{b}$. However the misconception mentioned above was rather strong and persisted for many children, as the final questionnaire indicated.

During the lessons, the algebraic aspect of vectors was manipulated in parallel with the geometric one, especially in subjects like opposite or zero vector, operations and their properties (distributive and associative laws, etc).

3.6 Vector operations

3.6.1 Addition of collinear vectors

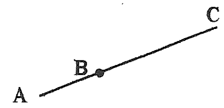
Addition between collinear vectors was presented, first through simple examples from physics, like that of two men pushing a car, and second through more complicated cases of relative motions, like that of a little ball rolling on a moving board, where velocities were analyzed as successive displacements in a unit time interval. In every case the students made the drawings of vectors to scale, and they tried to find out the relation of the magnitude and direction of components and those of the resultant. They also produced the corresponding models of vector operations and vector patterns.

A remarkable misconception about addition is related, on the one hand to the nature of this operation and on the other hand to the manipulation of vectors as line segments by the students: vector addition is considered as addition of numbers, probably because the sign of addition "+" is used with two different meanings (cf. the comments on "=" in 3.5 above). Consequently, the magnitude of the resultant vector is the sum of the magnitudes of the two vectors, even in the case of opposite or non-collinear ones. After measuring the lengths of the sides of some triangles, the students' wrong conception of vector addition led them to contradictions, since they realized the validity of the triangle inequality for the lengths of the sides of each triangle. A long discussion concerned the meaning of the symbols "+" and "=" in relations like: $\vec{F_1} + \vec{F_2} = \vec{F_3}$, which indicate that $\vec{F_3}$ is **equivalent**, i.e. gives **the same result** as the two others **together**. Both terms "sum" and "resultant" were used, indiscriminately.

3.6.2 Subtraction of collinear vectors- Multiplication by a number

The introduction of the subtraction of collinear vectors came naturally when adding displacements of opposite sense.

After the notation $\vec{AC}+\vec{CB} = \vec{AB}$, was used, a student suggested also: "$\vec{AC} - \vec{CB} = \vec{AB}$. Let's put "-" to show that we have a displacement and then we go back again".



Following this, other students tried to improve this notation:

2. $\vec{AC}$ + $\overleftarrow{BC}$ = $\vec{AB}$,
1. $\vec{AC}$ - $\overleftarrow{BC}$ = $\vec{AB}$,
3. $\vec{AC}$ - $\vec{BC}$ = $\vec{AB}$: " By "-" I indicate that $\vec{BC}$ is the opposite vector of $\vec{CB}$ "

These remarks gave the opportunity to see the subtraction of two vectors as "the addition of the opposite vector".

The introduction of multiplication by a number was presented on the basis of the addition of equal forces.

3.6.3 Addition between non-collinear vectors

Serious difficulties were found concerning the differentiation between "tied vectors" (i.e. line segments with a definite magnitude and direction attached to a point, the prototype being the physical concept of force) and "free vectors" (i.e. line segments with a definite magnitude and direction, the prototype being the geometrical concept of a parallel translation). These are related to *apparently* different types of addition: the *parallelogram* and the *triangle* laws respectively. From a more advanced mathematical - epistemological point of view, these laws correspond to the distinction between "tangent vectors to a manifold" and "tangent vectors translated in parallel on a manifold", the latter concept requiring more structure (i.e. the concept of a connection on a manifold) than just the ordinary differential structure of a manifold. The result of the pilot research that often students do not recognize the equivalence of parallelogram and triangle laws respectively may be due to this fact.

A convenient first step for establishing the equivalence of these laws seems to be connected with the commutativity property of addition. Activities were given for the verification of this property. On the one hand groups of two and three line segments as well as groups of vectors were used. In both cases, the students were asked to make them successive and find out all possible ways that this could be done. It is obvious that, for vectors where not only magnitudes and paths, but also senses were considered, the result was always the same. This verified the commutativity of vector addition.

The triangle law was verified on the basis of successive displacements on different paths, and velocities considered as successive displacements in the unit of time. Certain cases reminded some students of the parallelogram law, which they used in physics. This was the reason for studying and finally verifying the equivalence between these laws. The parallelogram law was also studied experimentally with the aid of an experimental arrangement comprising a system of two pulleys, and three weights balancing each other. By changing the weights, the students were given the opportunity to verify not only the parallelogram law, but also the inequality between the sum of magnitudes of the two components and the magnitude of the resultant. The final result was that although both methods of addition are different, they are equivalent, in the sense that they lead to the same physical result. It was also discussed that in some cases it is more convenient to use the one or the other law. More precisely, the triangle law is more convenient for successive vectors, while the parallelogram law is more convenient for vectors having the same origin.

4 Results

Concerning the analysis of the questionnaires, work is still in progress. However, the first results indicate some interesting points. The first questionnaire (Q_1) identified specific errors related to the understanding of notions like path and sense, difficulties in the distinction between line segments and vectors, and also errors concerning vector operations.

The questionnaires (Q_2) and (Q_3) indicated some improvements of the experimental group in comparison with the control group. On the other hand, it seems that some misconceptions are strong and persist after teaching. In the following we refer to some types of errors met in all classes. Classes C_1 and C_2 constitute the experimental group, while classes C_3 and C_4 constitute the control group. Concerning their performance in school, C_4 is considered to be the best class, with students having a very good level in mathematics and physics. Classes C_2 (the pilot class) and C_3 come 2nd and 3rd respectively, while C_1 has students of a rather low level in both mathematics and physics.

The numbers in the following tables correspond to percentages of errors made by the whole class.

The figures for each test exercise are given in Appendix II.

4.1 Errors related to the concept of path

Exercise 1(Q_3): *Which of the following vectors have the same path?*

Confusion between path and orientation:

C_1	C_2	C_3	C_4
11 %	10 %	43 %	24 %

4.2 Errors related to the concept of sense

Exercise 3d(Q_2): *Vectors $\vec{c}$ and $\vec{b}$ have the same sense.*
Correct Wrong Why?

Sense related to points of compass:

C_1	C_2	C_3	C_4
4 %	13 %	62 %	52 %

The following table gives the maximum percentages of errors concerning sense, met in Q_1 before teaching, compared to errors met in Q_2 after teaching. All classes were on a rather equivalent level concerning their preconceptions about sense. However, after the teaching, the experimental group showes a greater improvement.

Errors (%) related to the concept of sense

	C_1	C_2	C_3	C_4
Q_1	40	57	43	56
Q_2	11	27	62	52

4.3 Vectors regarded as line segments

Exercise 4 (Q_2): *a) $\vec{n} = 3\,\vec{k}$ b) $\vec{m} = \vec{k}$*
Correct Wrong Why?

Answers (%): "Correct" in 4 (Q_2)

	C_1	C_2	C_3	C_4
4a	14	17	38	23
4b	11	10	24	16

Exercise 5 (Q_3): *The triangles ABC are all isosceles (AB = AC). Examine which of the following cases expresses the relation between $\vec{b}$ and $\vec{c}$ for every one of these figures:*
a) $\vec{b} = \vec{c}$ b) $\vec{b} = -\vec{c}$ c) Another answer. Which?

Answer $\vec{b} = \vec{c}$

C_1	C_2	C_3	C_4
11 %	7%	36%	20%

4.4 Subtraction of collinear vectors

Exercise 6 (Q_2): *Complete the second part of every equality by the correct vector:*
a) $\vec{EA} - \vec{BA} = \cdots$ b) $\vec{CD} - \vec{ED} = \cdots$

As it is indicated in the answers of exercise 6 the experimental group made greater progress. We should also mention the high percentage of no answers in the control group.

Answers (%) in 6a (Q_2)

	C_1	C_2	C_3	C_4
Correct	54	67	34	29
No answer	0	10	17	19

Answers (%) in 6b (Q_2)

	C_1	C_2	C_3	C_4
Correct	57	63	31	23
No answer	7	10	21	23

4.5 Parallelogram law-triangle law

The experimental group had a better performance in manipulating the triangle law.

Exercise 6(Q_3): *Replace the question mark by the correct vector by using vectors $\vec{a}$, $\vec{b}$ or $\vec{c}$:* $\vec{a}+\vec{b}=?$

The experimental group made greater progress in this question, where the use of the triangle law is more convenient than the parallelogram law.

Answers (%) in 6 (Q_3)

	C_1	C_2	C_3	C_4
Correct	43	43	7	19
No answer	4	0	10	8

Exercise 5 (Q_2): *Replace the question mark by the correct vector:* $\vec{AB}$ + ? = $\vec{AC}$

Answers (%) in 5 (Q_2)

	C_1	C_2	C_3	C_4
Correct	86	100	69	65
No answer	0	0	10	6

4.6 Relative motion

It seems that children's strong preconceptions about motion and vector quantities are in most cases obstacles for understanding relative motion. Our groups had not been taught about such motions in physics lessons. During the experimental teaching several preconceptions appeared, especially among good students. Many of these conceptions were so strong that they led to long debates among students. Our research suggests that although the concept of relative motion is rather difficult for young children, teaching can modify some of their preconceptions.

4.6.1 Relative motion - Collinear vectors

Exercise 7 (Q_2): *The railroad car is moving rectilinearly, at a constant speed of 30 m/s from west to east. A passenger is moving along the railroad car, at a constant speed of 2 m/s. Using vectors indicate the passenger's displacement after 3 sec in the following cases:*
a) The passenger is moving from A to B. *b) The passenger is moving from B to A.*[2]

[2]The figures used in this exercise have been taken from [?].

Answers (%) in 7a (Q_2)

	C_1	C_2	C_3	C_4
Correct	68	63	0	42
No answer	11	7	38	23

Answers (%) in 7b (Q_2)

	C_1	C_2	C_3	C_4
Correct	57	47	0	39
No answer	11	20	41	29

4.6.2 Relative motion - Non collinear vectors

Exercise 9 (Q_3): A marble ball is moving across the floor of a railroad car, at a speed of 6m/s relative to the floor. When the ball starts moving, the railroad car starts also moving rectilinearly, at a constant speed of 8 m/ s. With what speed does an observer outside the railroad car see the marble ball moving?

Answers (%) in 9 (Q_3)

	C_1	C_2	C_3	C_4
Correct	36	40	14	19
No answer	11	17	17	35

Exercise 10 (Q_3): An airplane is moving horizontally with velocity $\vec{v}$ and packets of cattle food are thrown over a mountain village. There is no air resistance. Draw where a villager sees the packet moving towards, when it leaves the airplane from point A.

Answers (%) in 10 (Q_3)

	C_1	C_2	C_3	C_4
Correct	25	43	7	6
No answer	4	7	7	0

5 Discussion

Vectors are introduced, as a tool for solving geometric problems, only in the last year of Greek high school mathematics. Our previous research has verified specific and persisting difficulties among Greek students (aged 15-18), about some epistemological aspects of the concept of a vector, which have a negative influence on geometry problem solving procedures.

In our opinion, these difficulties are mainly related to the fact that vector notions are presented in their abstract form right from the beginning, without introducing them first in a more intuitive and physical way. Physics is a suitable field for introducing vector notions and operations in a more intuitive way. However, the role of physics in the development and establishment of vector notions suggested by history is ignored in the curriculum of secondary school.

The above difficulties, and indirectly the role of physics in the historical development of vector algebra, led us to an alternative teaching approach, which can help students construct basic vector notions and operations on the basis of physical and geometrical situations and activities.

Our experimental teaching gave us the opportunity on the one hand to face and discuss difficulties in depth, and on the other hand to attempt to eliminate them. Although our results presented here are mainly based on a qualitative analysis of teaching, they indicate that our approach is successful. The first results suggest that our teaching helped students in the experimental group to overcome some of their wrong conceptions. However, a more detailed description of the consequences of our teaching approach, based on a quantitative analysis of the questionnaires distributed to the students, is still in progress.

Nevertheless, we believe that the teaching approach of using intuitively physical situations, proposed here, would be helpful for a deep understanding of such notions and their construction. This understanding is necessary before vector concepts and operations are used as a tool in vector geometry.

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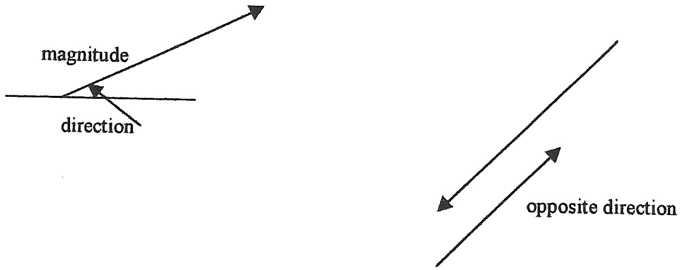
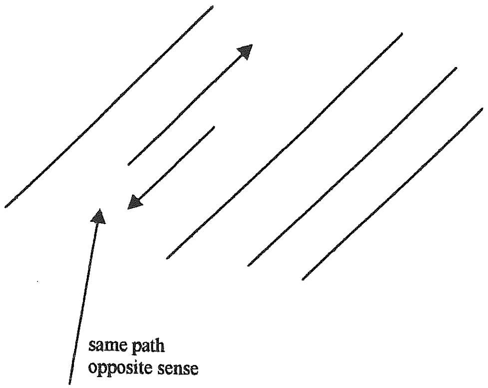
Appendix I

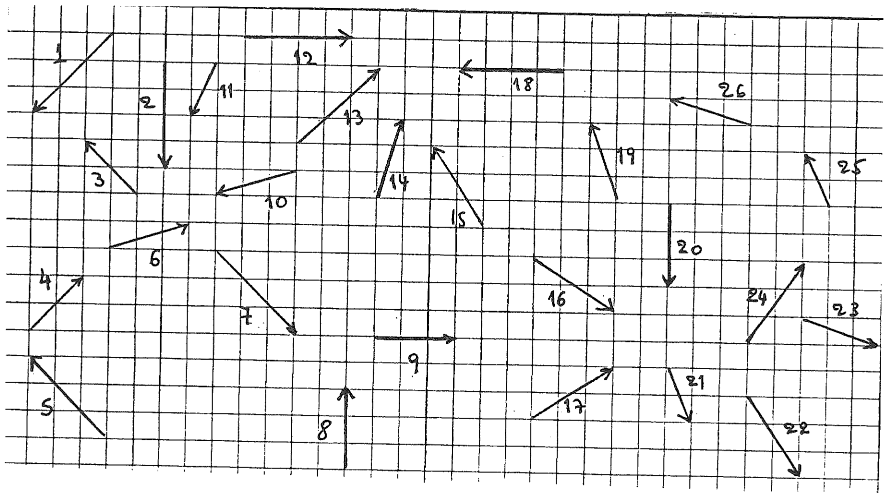
In English one describes a vector by two terms: Its **magnitude** and its **direction**.



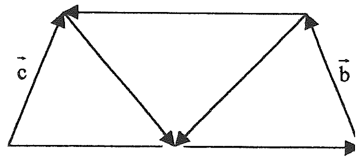
In Greek (and in French) one describes a vector by three terms: its **magnitude** (longueur), its **path** (direction) and its **sense** (sens).



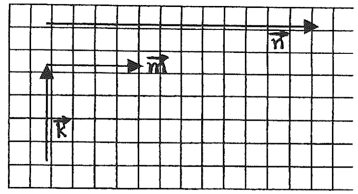
Appendix II



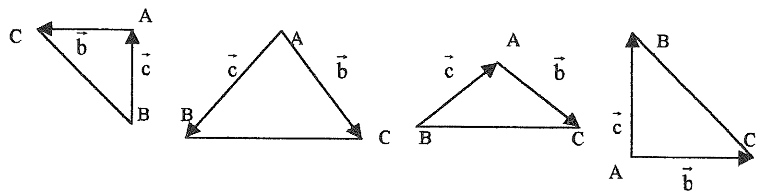
Exercise 1(Q3)



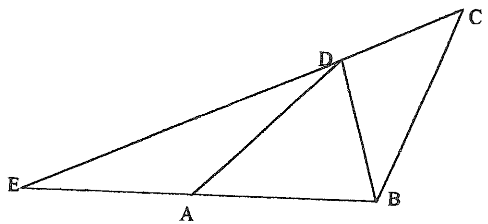
Exercise 3d(Q2)



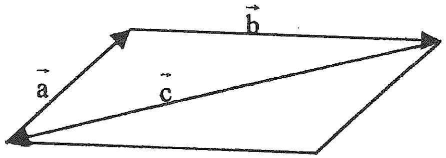
Exercise 4(Q2)



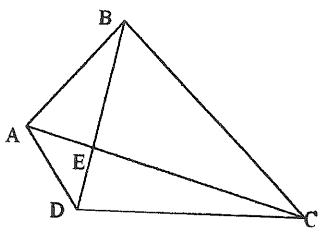
Exercise 5(Q3)



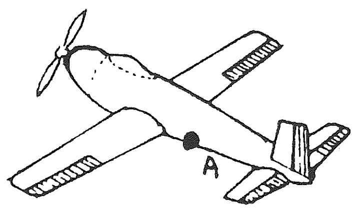
Exercise 6(Q2)



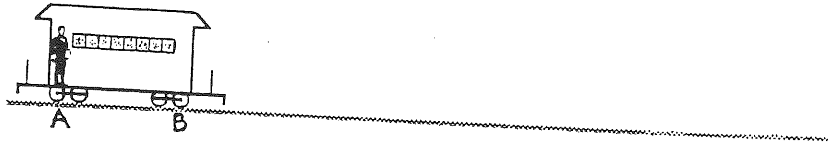
Exercise 6(Q3)

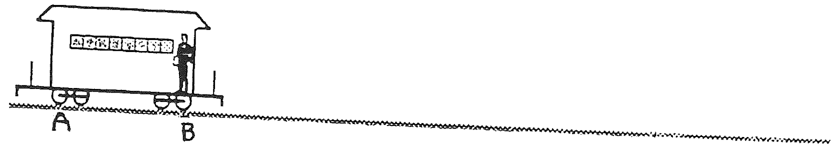


Exercise 5(Q2)

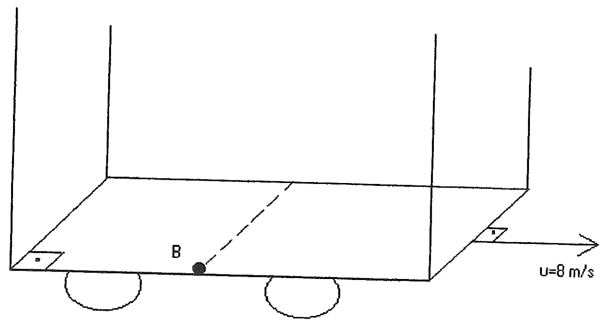


Exercise 10(Q3)





Exercise 7(Q2)



Exercise 9(Q3)