Simon Stevin (1548-1620)

Arithmetic in the Low Countries up to 1600 : trade, tradition, terminology

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Abstract

During the Middle Ages merchants went along the houses with their basket with goods. They tried to barter and their job was very different from the occupations of their later colleagues. In the sixteenth century merchants stayed in their offices and sent their travelling salesmen out. Business travels became longer, merchants went overseas to different countries, they had to pay salaries, custom rights, costs of transport, assurances of goods, etc. They needed to change money in many different ways, because each city had its own money system. They visited exchange banks where moneychangers took care of their affairs. Bankers and bookkeepers were needed. Many merchants earned a lot of money and they needed carpenters, bricklayers, gold- and silversmiths and others to spend this money by building houses and filling these with luxury goods.

The job of the merchant and his calculations became more and more complicated and he felt the need of a new arithmetical method, a written arithmetic that could help him to write down his big numbers and his extensive calculations. During the fifteenth and sixteenth century the new scriptural arithmetic with Hindu-Arabic numbers came into use in the Netherlands. Many arithmetic books were written in the vernacular, but in spite of that it lasted several centuries before the new arithmetic had superseded the old one completely.

What are the contents of the Dutch arithmetic books of the fifteenth and sixteenth centuries? Why does it take such a long time before the new arithmetic was the one and only method? At which schools was the new arithmetic taught? These are the main problems dealt with in this article.

1 Schools you could count on

During the fifteenth and sixteenth centuries most children visited primary school during a few years and then left school because they had to help their parents and work for a living. Some children went to Latin school. There all education was in Latin. The pupils were taught the subjects of the trivium: grammatica, rhetorica, dialectica. They also learned Latin prayers and church-singing because they had all kinds of tasks in the Latin service in church. Arithmetic was not a subject on the timetable of this school.

The sons of the merchants, who were predestined to follow in the steps of their fathers, were not interested in Latin and church-singing. They needed a school where they could learn practical and applied arithmetic. Such a school did not exist, but merchants, bankers and other financial and administrative practitioners started their own private schools where their sons could learn arithmetic, book-keeping and French. French was the most important business language in that time. These schools were called 'French schools' although the other subjects were taught in vernacular. It is clear that these schools were good nurseries for future merchants, bankers and money changers.

The arithmetic books used at the French schools were written in vernacular and in the prologue the authors mention the kind of pupils they are aiming at: merchants, bankers, bookkeepers, money changers, but also technical practitioners like carpenters, smiths, mintmasters, etc. The books contain many financial and technical arithmetical problems that had to be solved by these people in their daily practice. The arithmetic with the Hindu-Arabic numbers is very suitable to solve these problems and the authors give long explanations about this new arithmetic. The new arithmetic is the most important subject in the arithmetic books, but besides that many authors also include a chapter on the old arithmetical method, the traditional calculating with coins. Using this old method you don't need to write and that was a big advantage, even in the sixteenth century, when the ability of writing was still quite rare. In primary school the children first learned to read and then to write. Most children left school before they got writing lessons. Around 1600 about 60% of all men and 40% of all women were able to sign their marriage certificate. That means that during the sixteenth century many people could not write. If you can't write but yet want to do arithmetic, you can choose the traditional way of calculating, arithmetic with coins, for which you only need a few lines and some coins.

2 The old and the new arithmetic

In the sixteenth century the counterboard has horizontal lines. You can place coins on or between the lines and in this way you can express numbers and calculate. This board is a variation of the traditional abacus of the old Greeks and Romans. They used a board with vertical columns. During the Middle Ages this abacus underwent some changes and was turned a quarter.

At the end of the Middle Ages some people even use another variation of the abacus. They calculate with coins without lines. In figure 1 you see a picture from the fifteenth century French arithmetic book 'Le livre de getz'. Three merchants calculate with coins without using lines. This method is also found in a Dutch arithmetic book by Christianus van Varenbraken from 1532.

In using this method you start by placing coins on a vertical line. These are the so called 'liggers' in Dutch, or 'layers' in English. The first layer indicates the ones. The second layer

indicates the tens, the next one the hundreds, etc. The fields between the layers also have a value, increasing from 5 to 50, 500, etc. Figure 1 shows what the number 3767 looks like in coins.

If you can express numbers with coins you can make calculations with coins and it is indeed quite easy to add up and subtract. Multiplying and dividing is a bit more complicated, but for people who can't write calculating with coins is a good alternative.

During a very long time both arithmetical methods, the old and the new one, stayed in use. In figure 2 you see them together being practised at the same table. On the left you see the modern scriptural method with the Hindu-Arabic numbers and in the middle you see the traditional way of calculating with coins. This picture is from the title page of the arithmetic book by Adam Ries from 1533. Ries explains that the traditional calculating with coins

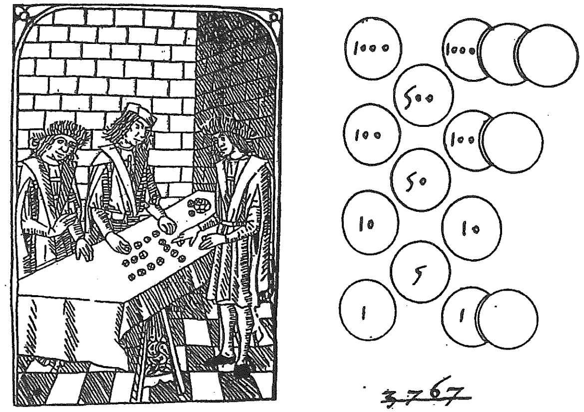

Figure 1: On the left: Three merchants making a calculation with coins without lines, from the French 'Livre de getz', 15th century. On the right: The number 3767 expressed in coins, from the Dutch arithmetic book by Christianus van Varenbraken, 1532.

Figure 2: The old and the new arithmetic at the same table. On the left the modern scriptural method with Hindu-Arabic numbers, in the middle the traditional calculating with coins. Title page from the arithmetic book by Adam Ries, 1533.

is a good preparation for the new method with the pen. He describes both methods. Other authors, also Dutch ones, do the same. It seems that many people in those days could work with both methods. The mathematician Petrus Ramus used the new arithmetical method in his 'Arithmeticae libri tres', but in private, he said, he prefered the traditional way with coins. It sounds more or less like the way we use our pocket calculator nowadays. Sometimes we cipher on paper and sometimes we choose a calculator. It depends upon the problem and the circumstances.

Finally the modern way of calculating with the pen was prefered to the old manner. But this happened only after a very long period. Even in 1698 coins were struck in the Southern Netherlands. And in 1707 Leonhard Sturm explained in his 'Kurtzer Begriff der Gesammten Mathesis' how to calculate with coins.

Why did it take such a long time before the new method was accepted everywhere? It is clear that it has many advantages. For instance you can easily check your calculation afterwards. In arithmetic with coins, the numbers you start with disappear from your counterboard during your calculation. Of course you can check by using the check of nines, but it is impossible to read again the process afterwards. In the new arithmetic you can.

The new method has more advantages. In using Hindu-Arabic numbers it is easy to write big numbers and to extract roots or calculate with fractions. And finally, you only need one instrument for making your calculation and noting the result. So why the delay?

First because many people could not write, secondly the use of zero in the new method. If you are calculating with coins the zero is not necessary. Zero means an empty space on your countingboard and that is no problem at all: nothing means nothing. In the Roman numbers it is the same. You don't need a zero.

But in the new system you do. You have to write a sign, although you mean nothing. And at the same time this magical sign can change the value of a number when it is added to it. 4 doesn't mean the same as 40! People found this difficult, so that authors gave long explanations about the function of the zero.

Some people were even opposed to the new number system. In Florence the guild of money changers forbade their members to use the new numbers in their cashbooks for fear of fraud. This hap- pened as late as 1299. In 1202 Leonardo of Pisa had explained the new method in his 'Liber abaci', when it was completely unknown in the Netherlands.

The oldest Dutch arithmetic book in which the new arithmetic is described, was written in 1445. There are two other Dutch arithmetic manuscripts from the fifteenth century, but most of the Dutch arithmetic books were written and printed in the sixteenth century. At present we know 36 arithmetic books from these two centuries: 12 manuscripts and 24 printed editions. Eleven of the 36 arithmetic books are either a reprint or a copy of an earlier work. On first view the books do not look like arithmetic books. Dutch authors didn't use arithmetical symbols, but described their calculations in words and complete sentences. The German Johann Widman already used arithmetical symbols in 1489 but he was hardly followed by his Dutch colleagues.

3 What could you learn from a sixteenth-century arithmetic teacher?

The most important aim of the authors of the Dutch arithmetic books was to teach their pupils the new scriptural arithmetic with Hindu-Arabic numbers in order to apply this to all kinds of

technical and financial arithmetical problems.

First the pupils had to learn how to read and write the new Hindu-Arabic numbers. The authors wanted to show that the new number system is very suitable for writing down big numbers systematically. Christianus van Varenbraken gives an example of a number consisting of 18 figures. Try to imagine what this number looks like in Roman numbers and you will realize the advantages of the new system.

If you know how to read and write the new numbers you will have to learn how to do calculations with the pen. All Dutch arithmetic books start explaining how to add, subtract, multiply and divide. Sometimes the authors even treat halving and doubling. The arithmetical operations look rather modern. We use the same methods nowadays, with a few differences when it comes to division.

The different money systems of the cities could make calculating rather complicated. In figure 3 you see a subtraction with two amounts of money: 298 pounds, 19 shillings, 10 pennies and 16 mites are subtracted from 334 pounds, 13 shillings, 9 pennies and 13 mites. You have to know the Ghent system, in which: 1 pound equals 20 shillings, 1 shilling equals 12 pennies and 1 penny equals 24 mites. It is clear that this made calculations very difficult and there were many mistakes, as you see in the final result of this example: 11 pennies ought to be 10 pennies.

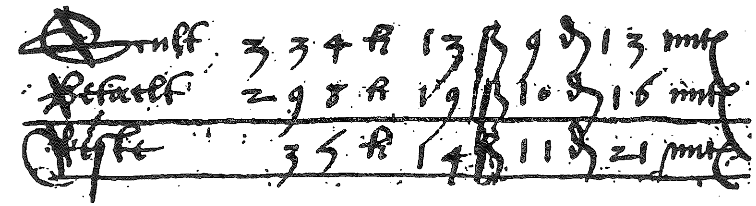

Figure 3: Subtraction of two amounts of money from the arithmetic book by Christianus van Varenbraken, 1532.

When the authors have explained the arithmetical operations they next discuss rules to solve the applied and practical arithmetical problems. The rule of three is the most important one. It is used to find the fourth number in proportion to three given numbers. Because of its importance some authors introduce this rule in a beautifully decorated frame, Figure 4. This picture is from the arithmetic book of Peter van Halle, 1568. The text says: *The rule of three, how you can find the fourth number out of three numbers.*

Figure 4: Introduction of the rule of three in the arithmetic manuscript by Peter van Halle, 1568.

With the rule of three you can solve all kinds of problems in the life of merchants and financial, administrative and technical practitioners : problems about buying, selling or exchanging of goods, about partnership, changing money, calculating of interest, about insurance, profits, losses, making alloys, etc. Look for instance at the problem on selling cloth in figure 5. This problem is from an anonymous arithmetic book from 1578.

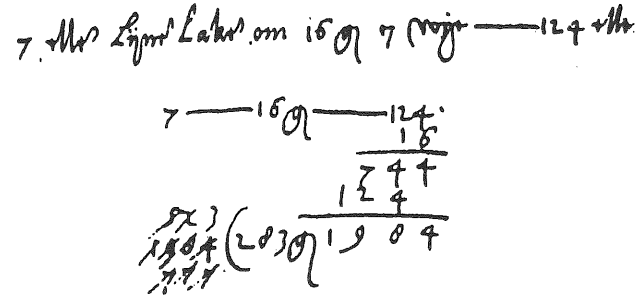

Figure 5: Linen sale from an anonymous arithmetic book, 1578, fol. 32r.

If 7 ells of linen cost 16 pennies, how much do 124 ells cost?
Place the three given numbers on a line. Multiply the last two numbers and divide the product by the first one: $(16 \times 124) : 7 = 283$ pennies. In figure 5 you see that 3 is not ticked off. That is the remainder.

The following problem is a bit more difficult. You can read it in figure 6. It is from the arithmetic book by Peter van Halle, 1568: *If 10 mowers can mow 15 hectares of land in 7 days, how many days do 16 mowers need to mow 20 hectares of land?*
In this problem you have to use the rule of three twice: once in the normal order and once in the inverse direction. The final outcome is 5 5/6 days.

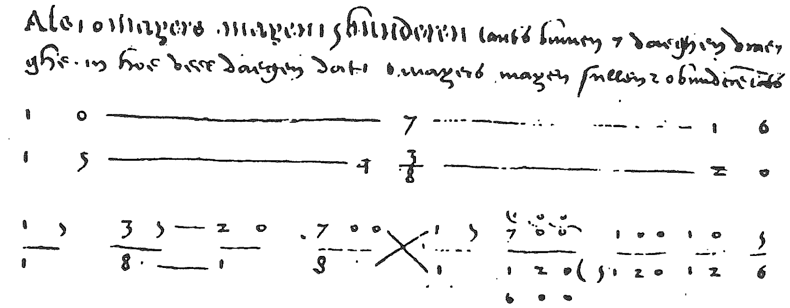

Figure 6: The problem of the ten mowers from the arithmetic book by Peter van Halle, 1568, fol. 95r.

Money changers had to solve problems like the one in figure 7: *A merchant from Florence went to the exchange-bank in London in order to change 120_ ducats at 42_ pennies each into angelots at 66_ pennies each. The question is: How many angelots will he get in London?*
The calculation here is: $(120\_ \times 42\_) : 66\_ = 76$ angeloten and the remainder is 594.

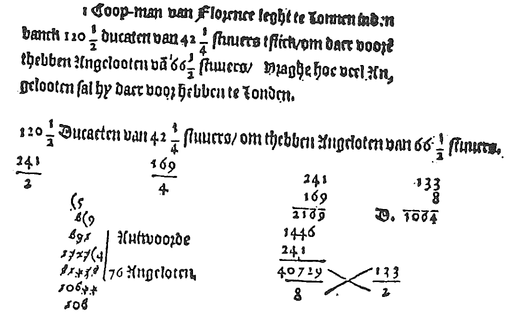

Figure 7: The problem of changing money from the arithmetic book by Adriaen van der Gucht, 1569, fol. 96r.

4 Words used for arithmetic

Mostly the teachers at the French schools were the authors of their own arithmetic books. They did their very best to make the arithmetic as clear as possible to their pupils. They gave long explanations and very many problems to prepare their pupils for their future jobs.

Alas, especially in the early arithmetic books, there is one aspect that made their explanations a bit obscure. Although the authors wrote their books in the vernacular, they used many traditional Latin arithmetical terms and that was difficult for pupils who did not understand Latin. But an arithmetical vocabulary in Dutch did not exist.

In the sixteenth century the authors tried to make the Latin terms clear in different ways:

1. they gave the Latin words Dutch endings;

For instance: addito — *adderen*
summa — *summe*
unitas — *uniteit*
quadratus — *quadraet*

2. they tried to describe the Latin word and added Dutch expressions to it;
For instance: *in sich selve multipliceren*
(multiply in himself = to square) *getal dat gi begeert te delen*
(number that you want to divide = division number) *gebroken getal*
(broken number = fraction)

3. they tried to find Dutch translations of the Latin words. Most of the time they used existing Dutch words;
For instance: differentia — *verschil*
radix — *wortel*
substractio — *aftreckinge*

4. sometimes the authors created completely new Dutch words;
For instance: arithmetica — *rekenconste*
additio — *optellen*

and soon.

Some of these inventions became part of the Dutch arithmetical vocabulary.

If a sixteenth century arithmetic teacher wanted to write his own book he started by using books of his predecessors. We would call this plagiarism, but in those days it was quite normal to compose a new book with material from existing books. Of the 36 transmitted Dutch arithmetic books at least 31 are based on one or more sources. There is a dense network of relations between the arithmetic books. Take for instance the book of Gielis van den Hoecke from 1537. He may have used the works of Euclid, Ptolemy and Regiomontanus. He translated and adapted fragments from Grammateus, de la Roche and Rudolf. In the same way Van den Hoecke used the works of his predecessors, his work was used by later authors. Some colleagues copied his whole book, others used fragments or made adaptations. Some books seems to have some relation with the book of van den Hoecke although it is difficult to prove that. Probably some linking books between those have been lost in the course of time. The book of van den Hoecke is just one example. Arithmetic books of this time heavily depend upon earlier books.

If an author copied pieces from the works of his predecessors he also copied their terms and if he feared that the copied terms were not clear enough for his pupils, he added some extra terms or descriptions. The consequences of this practice are that the amount of arithmetical terms in the books is growing during the age. Authors made no choices between terms and used all synonyms they knew.

Sometimes several synonyms appear in the same sentence. Gielis van den Hoecke starts his chapter on addition with the title: "Additie, vergaerderijnghe, sommerijnghe". He uses three different words to announce 'to add'. Van der Gucht has used the work of Van den Hoecke, he starts addition with the title: "Additie, vergaerderijnghe, sommerijnghe ofte toedoeninghe". He even added a fourth term.

Van der Gucht used many different sources in composing his book. The consequence is that he

has a striking number of arithmetical synonyms. For instance: he uses 13 different words for 'To add': *adderen, brijnghen, inrekenen, rekenen, sommeren, toebrijnghen, toedoen, verzaemen, tegader adderen, tegader doen, in een somme brijnghen, tegader tellen, tsamen tellen.*

In our eyes the big amount of synonyms in the sixteenth century arithmetic books is remarkable and it seems confusing, but in these days there was no standardisation of terms. Finally, a long time after the sixteenth century, the biggest part of these terms disappeared, but it is striking that among the sixteenth-century arithmetical terms that many of them are still in use nowadays in the Dutch arithmetic.

An important author was Simon Stevin. He was born in Brugge, probably in 1548. He moved to the North in 1581 and became a student in Leyden. He published many works on mathematics, astronomy, physics, law. He was the teacher of Prince Maurits and published his instructions in the 'Wisconstige gedachtenissen'. Stevin was convinced that of all languages Dutch was by far the best for scientific purposes. In his works he introduced many new Dutch words. Some became the only correct terms in the Dutch scientific language. Words like: *wiskunde* for 'mathematics', *evenwijdig* for 'parallel' and *evenredigheid* for 'equality of two ratios', etc. In 1585 Stevin published his "De Thiende". He described a new method to write decimal fractions and how to use them in fundamental arithmetical operations. None of the Dutch terms he used here are his own invention; he made a selection from the existing 16th century terms. We may conclude that Simon Stevin invented many new Dutch scientific terms, but for our Dutch arithmetical terms we must thank his predecessors.

5 The combination of business with pleasure

Let us return now to the contents of the arithmetic books.
Among the many serious practical arithmetical problems for future merchants, bankers, money changers, etc, you find problems that are not realistic at all. These are mostly funny story-problems, probably to amuse the pupils and make the program less serious.

Christianus van Varenbraken (1532) starts this kind of problems always with the title: *Questie uut genouchten*, which means: Problem for pleasure. Martin van den Dijcke (1591) has collected all his recreational problems in a special chapter at the end of his book. He introduces his collection as follows: *Here you will find many different beautiful problems to sharpen and enjoy your mind.* It is clear that these problems have a recreational function.

The following example is taken from this collection:
Hanneken Vinc and Lysken Rinck are getting married. The bridegroom and his family are waiting for the bride. The bride is walking towards her bridegroom, but suddenly she stops because her family has told her that she is breaking the rules. The horse has to go to the hay and not the reverse. There is a fight between the two families but finally they reach a compromise. The bride goes 7/8 part of the distance between her and her bridegroom backwards end then again 9 2/3 steps forward. The bridegroom goes forward 1/5 part of the distance. Finally the distance between the couple is only 3/5 part of one step. So they can kiss each other and the audience laughs loudly. The question is: How many steps were there between bride and groom when the bride stopped walking?

Probably this problem was nice entertainment in the sixteenth century classroom, but it could play the same part in a modern mathematics lesson. Funny problems are timeless.

In the following example from the same source we see that rivalry between cities is of all times.

Once in an inn two men, one from Louvain and one from Mons, were boasting. The man from Louvain said: 'Compared with Louvain, Mons is just a pigeon house.' The man from Mons answered that the length of the Mons citywalls was 5/8 of a mile. But then it seemed that the Louvain walls were 1 4/11 miles long. The conversation ended with a bet: 'I believe it is possible to place Mons more than 4_ times inside the Louvain walls. If not, I will pay for the drinks.' Did the man from Louvain win?

These types of problems were not only solved in the classroom but also probably in pubs or at parties. It is like a modern quiz: a way to challenge each other.

We do not know how old Van den Dijcke's problems are. He copied them from Godevaert Gomparst, a sixteenth-century arithmetician from Antwerp and probably they have not been used in books of an earlier date. But some of these arithmetical 'riddles' are extremely old and can be found in books from different times and cultures, like the 'world famous' problem of the ring.

In a group of people there is a person who has a ring on one of his fingers, around a certain phalanx of that finger. The leader of the game doesn't know where the ring is, but he asks the group to do some calculations and finally when he sees their results he knows where the ring is. The calculations are:
Double the number of the person that has the ring. (Each person has a number)
Add 5 to this.
Multiply the result with 5.
Add the number of the finger
Place the number of the phalanx behind the result and subtract 250.

If for instance the third person has the ring on his second finger around his first phalanx, the final result of the calculations will be: 321!

I believe that this problem can also be a big success in our modern classrooms. You can add extra questions. For instance you can ask your pupils what will happen if the ring is on the tenth finger. You can ask them why the trick works and if it will work with all numbers. And perhaps you can even challenge them to create their own hidden ring trick. In doing this you can turn this exciting historical game into a rich and valuable math problem.

I found this problem in the arithmetic book by Peter van Halle (1568) and it also appeared in the arithmetic book by Adriaen van der Gucht (1569). But the problem is much older. It appeared already in the problem collection of Beda in the 8th century and also in the works of Abu mansur (eleventh century), Fibonacci (thirteenth century), Chuquet and Calandri (fifteenth century) and in many sixteenth century books: De la Roche, Ghaligai, Rudolff, Apian, Tartaglia and Trenchant. And last but not least, the Dutch arithmetician Willem van Assendelft described the ring problem in his book of 1621. It is clear that long after the sixteenth century these problems were as popular as they turned out to be at a summer university in Louvain in 1999. Which shows us that good problems will keep their charm forever.

6 Conclusions

In the title of this article there are three T-words: Trade, Tradition and Terminology. During the fifteenth and especially the sixteenth century the increasing trade caused an increasing need for a written arithmetic method. Merchants and practitioners of financial, administrative and

technical professions learned to apply this new method to practical arithmetical problems.

During a long time both the new arithmetical method and the traditional method of calculating with coins stayed in use and many new books contained old and well-known problems, that used to be solved for fun.

Many of the modern Dutch arithmetical terms have been invented by sixteenth century authors of arithmetic books.

The sixteenth century was a time of tradition and ambition, but people also liked partying. During these parties people may have been challenging each other by trying to solve arithmetical problems. Probably these occasions were a kind of Summer university-banqets-avant-la-lettre. The most important lesson we can learn from our sixteenth century colleagues is: Always try to combine business with pleasure!

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