Shaping Mind by Shaping Space
- Friedrich Froebel and the Romantic Tradition in Education -

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Abstract

While Friedrich Froebel's "Kindergarten"-idea for early education has been a world-wide success, the "Spielgaben" which he conceived as the corresponding educational medium widely have not been used in that universal and comprehensive sense Froebel had intended. Conceived as a series of building blocks of basical geometrical forms, they were not meant as just another toy, but as concrete symbols of the structures of matter as well as of the human mind, which in the playing child should act as incentives of mental development and of orientation in a complex world.

The specific shape of this idea emerges from "Romantic Natur-philosophy" (RNP) (in which Froebel participated as a philosopher and as a (crystallographer-)scientist) : Based on idealistic "Identitätsphilosophie" (Schelling), RNP developed a concept of nature, that postulated an undividable unity of divine spirit (as the creator of nature), of the structures of the human mind, and of the dynamical structure of nature itself. RNP thus replaced the mechanistic, materialistic selfunderstanding of modern science by laying the grounds for contemporary concepts about matter and mind, both taken as systems of fields of force, of symmetries and of selforganisation.

Thus, Froebel's concept seems to be apt to inspire and to unify contemporary ideas about the role of structures, of selforganisation, and of symmetry in mathematical, physical and chemical instruction.

1 Froebel's "Spielgaben" : the Pedagogy of Romantic Philosophy of Nature

1. 1 Dynamic Explanation of Nature

In 1804, the German Crystallographer Christian Samuel Weiß wrote one of the most charming attacks in history of science. After translating René Just Haüy's "Traité de minéralogie" he vigorously denies that the theories he had just translated are useful for any exact definition or explanation of the different forms of crystals; in an annex to vol.1 he contrasts them with his own "dynamic" ideas. The humorous aspect of such selfassurance consists in the fact that the then 24 years old Weiß, though having grown up in the good "school" of Abraham G. Werner, was a still completely unknown junior scientist, whilst Haüy was the most famous crystallographer of his time.

Haüy's explanations of the regular crystal forms, as they are characterised by specific constant plane angles, can nowadays be found in every textbook on crystallography or solid state physics, and we wonder why Weiß objected to them. Haüy imagines crystals as being composed by tiny regular parallel-epipedical elements whose regular increments small scales let grow the manifold real, geometrically specific forms of crystals on a big scale. Weiß (and other "dynamists") did not absolutely object, as is frequently stated, to the existence of those quasi atomic elements which Haüy assumes in his definitions, nor to the possibility of deriving crystal forms from them. But he denied that this derivation can be accepted as a final explanation, as true knowledge, as knowledge of the truth, of the forms of the elements or of the structures of the crystals : "(The atomist) acknowledges a distinct form as given directly from the existence of atoms. The dynamist denies this"; what really he demands is the explanation also of the structure of those postulated building elements and of the laws of the increments as caused by the innate bonds, forces and energies of the substance of matter, be this now atomistic or not. Yes, he even "opposes the absolute, unconditioned existence of matter at all", which perhaps, under certain circumstances, may be just a "temporary" phenomenon which on its part needs explanation by the eternal, unchanging forces of nature - in my eyes a consideration by no means irrelevant, if we look on contemporary field theories about the structure of atoms, on Quantum-electrodynamics, on laws of symmetry and interaction. Thus, the demands of Weiß can be looked upon as an absolutely modern and legitimate program, although 150 years had to pass for its fulfilment this demands were legitimate also insofar as the atomistic constructions by means of simple addition of building elements do not only take for granted the form of these elements themselves, but could not fulfil even in those days (Werner taught at an institution of applied science, the mining academy in Freiberg!) the postulates of applied mineralogy, an aspect, however, which we cannot discuss in detail here.

1. 2 The philosophical basis : Romantic Natural Philosophy

The deeper motivation of Weiß' critique, however, was of philosophical origin. Atoms in Haüy´s sense could at best render mere phenomenological descriptions of natural phenomena, but no explanations based on causes immanent in Nature. In this point, the critique of Weiß is congruent with that of the other so-called "Naturphilosophen" of those times. They elaborated a fundamental criticism of the mechanical, materialistic understanding of Nature as it was preferred by the Enlightenment movement, an idea finally extended even on the spiritual life of man (de la Mettrie : "l'homme machine"). Contrary to this understanding, the Romantic

Natur-Philosophy (RNP) demanded a definition of Nature as based on internal forces and energies. Nature was imagined as a unit of a whole, interwoven by forces, or, more generally, as a texture of forces unfolding from themselves, guided by their inner dynamics. As it is fully comprehensible in its structures only as a whole, it has the character of an organism. Nature was looked upon as animated, even as endowed with a soul, with a creative urge, which enables Nature to unfold on the basis of the energies inherent to it - if we disregard some quaint linguistic details typical for the time, this concept can well be looked upon as quite a modern program of "selforganisation" of Nature.

Corresponding to a "philosophical" view on Nature, these energies have been endowed by certain general laws of formal kind, postulations of equilibrium, symmetries, polarities - theorems of conservation of the basic qualities imagined as substantial - postulates which, when uncritically applied by weaker philosopher-scientists, in fact often led to untenable structures of mere speculations, which up to now discredited this philosophy as a whole. On the other hand, when critically applied those guiding ideas of empirical research yielded important results which have since proved fundamental to modern science; like Oerstedt's and Faraday's insight into the connection of electricity and magnetism, or like the theorem of the mechanical heat equivalent which Julius Robert Mayer as the first conceived as a general theorem of energy-conservation, yielded on the basis of RNP arguments. But only nowadays the whole value and depth of RNP has become evident, insofar as ideas and general considerations about symmetries of energies, their correlation with theorems of conservation and interactions play a constitutive part in actual physics and express the most general interfering principles of Nature (KLEIN 1992).

In general, and as a focus of our further considerations, RNP considers Nature not as something fixed and finished, a product like a ready but dead clockwork wound up, perhaps by God the creator, and now running down according to mechanical laws ("natura naturata"); rather it is understood as something permanently changing and lively-dynamic, as something spontaneously intertwining according to inner laws, modifying in manifold ways, creating itself by internal activity and change ("natura naturans").

1. 3 Philosophy of Identity

RNP has not only to do with Nature as something exterior to Man, but Man himself is part of Nature. Though finding himself opposite of Nature, he is also interwoven in it in manyfold ways caused by his dependency on Nature as an intelligent and active creature. The idea that Man and Nature are correlated to each other in an isomorphic, complementary sense can be traced back to the philosophy of Friedrich Schelling.

Usually, Schelling is looked upon as one of the three German "Idealistic" philosophers - Fichte, Schelling, Hegel, in this sequence, with Hegel as its culmination and perfection. This view, however, is not what we have in mind here. An idealistic philosophy generally means - misinterpreting and exaggerating Kant's basic idea about the limitation of our cognition by means of Man´s cognitive structures - that our knowledge of Nature can be completely (re)constructed merely by using our knowledge of the structure of our cognition (thus at least Hegel's opinion).

Schelling does not go so far but balances out the relation between Nature and Man more cautiously. His chief question (which is taken up again in science mpre recently in "Evolutionary Epistemology") is this : When taking seriously Kant's critique about the impossibility of direct cognition of Nature - is it possible to derive, from these very cognitive structures, the qualities Nature must have so that it could procreate a living being like Man, with these empirically stated cognitive structures?

Thus in a methodically direct and philosophically comprehensive sense, which even surpasses the mere cognitive function, a unity of Man and Nature can be constituted. The nature of Man, also that of living beings at all, their spontaneity, their permanent agitation, mobility, and capacity for metamorphosies are being declared as structural moments of Nature in general, a kind of universal creative urge attributed to Man as well as to Nature.

This duality now will be completed by the third partner, God himself as the creator of both, in the so-called "Pan*en*theism" of Karl Christian Friedrich Krause (with whom Froebel was in friendly relations). Quite in the sense of RNP, for him "the whole organic realm of Nature on earth" "proves" to be "a unique indivisible organism, a huge all-embracing body, which celebrates itself in the abundance of all plants and species of animals." (KRAUSE 1819, p. 15). But at the same time he looks upon Nature and Man as in their connection with God (l.c. 18) : "Man is not only spirit or merely body, nor both merely imagined as co-existing beneath each other. He is something new, consisting of body and spirit, but being formed by (the power of) God and destined to develop the life of Nature and the life of Reason as a unity, and to represent this unity by a harmonical interchange of all united and spiritual vital energies in common acts." In Krause's opinion God dismisses (releases) the world and Man, both according to His own image, and He persists in both His creations as the creative energy - therefore called pan*en*theism, i.e. the idea of God as certainly existing on His own, yet nevertheless at the same time immanently hidden in Nature as well as in Man.

The formal structures of God (as the creator), of the world and a human mind are identical insofar, as they all are of mathematical kind. Hence, "in practicing mathematics, Man participates in Divine spirit." (Novalis) Consequently, e.g. in Humboldt's plans about the German "Humanistisches Gymnasium", mathematics, together and equivalent with Greek language, becomes the core of the curriculum. The same holds for Froebel's ideas on education of Man, which we will study now.

1. 4 Froebel : "Menschenerziehung" (Education of Man)

Froebel had studied in Jena, so most likely (though not proven) had attended the lectures of Schelling. He was friendly with Krause, as already mentioned, and as a (natural) scientist and crystallographer he at times assisted Christian Samuel Weiß. Thus he participated in all three mentioned directions of romantic thought, and as we will see, he was prepared in the best way and in an highly adequate field of science to his own pedagogical mission, with a sound scientific basis that preserved him, different from many contemporaries, from bad-grounded speculations.

According to what we said before, Froebel's basic idea should be evident, after all. If Nature is considered as a productive, selfgenerating and continuously metamorphosing "natura naturans", instead of a mechanical, statical "natura naturata"; and if Man is also being conceived as a spiritual-organic being that generates itself according to its own internal energies; then the child, insofar as it develops and differentiates from a germ in accordance with internal laws, can be regarded as the most beautiful and most copious symbol of this vital energy in Nature. For Man

in general Froebel states :

> Man - mankind in Man - must not be understood as something ready, as something isolated nor finished, not as something stable or static, but as a living entity, continuously developing, eternally vivid, unfolding and generating itself step by step, actively aiming for its final and eternal goal (Froebel 1826, ME 44f.)

as something in development however which deploys itself only and simply by internal blind and undirected dynamic, but, as it is, and should become a reasonable creature, also guided and unfolding itself by the instruction of the surrounding Nature.

The child now, running through this process of development on an elementary level, also depends upon elementary experience of Nature which, on this elementary level comprises and expresses symbolically Nature as a whole in itself. Here now Froebel's crystallographic basis comes into play insofar as the crystalline forms and shapes appear as such elementary symbols of Nature as a whole which for the child represent nature in an elementary perceptible and conductive manner, initiating and regulating the child's own development, reaching far beyond simple, prima vista chaotic nature, and equivalently important as social relations with parents, the family, and the whole socio-cultural environment (ME 74f).

As a free and spontaneous relation fed by an inner dynamic this connexion with Nature must not result from externally motivated, didactic instruction, but has to be spontaneous in itself, and this becomes obvious for the child when it appears to him as a play :

> In fact all this happens especially by play, by the cultivation of children´s play, which in the beginning has the character of unconscious living-with-nature. Play in this phase is not a silly trifle; it is of high value and has an earnest meaning. Mother, nourish and care for it! Father, protect and guard it ! (ME 75)

Froebel continues emphatically :

> Playing, play is the highest level of the development of the child, of the development of mankind in this time, for it is freely active expression of internal life, a necessary expression, a basic need of innermost life. Play is the purest, the most spiritual produce of Man in this phase, at the same time it is ideal and after-image of the whole human spirit, of the innermost, secret life of Nature in Man and in all things.

The intercourse with Nature also fulfills the requirements of turning to God, i.e. of religion :

> What religion says and expresses, nature shows and exhibits; what is taught by the reflection of God, is confirmed by Nature; reflection of internal processes is verified by external nature; what religion demands, is fulfilled by Nature. For Nature, and everything that exists is a manifestation, revelation of God. Everything is of divine nature, divine essence. (ME 175)

1.5 Pedagogics of the Spielgaben

For the young child, the crystals and crystalline forms are by far too complicated for a playful approach (the same is true for the adult inquiring reason - that is why scientific research

("solid state physics") is necessary also for them!) The crystals themselves have more elementary geometric roots, which even more basically symbolize the cosmic structures. For a playfully learning access Froebel conceives a series of donations for play ("Spielgaben") which in their inner logical structure correspond to the structure of the cosmos as well as to the conformities (inherent laws) of the child's unfolding mind. We will attend now to these structures for a while on the basis of the lithographs which the editor added to Froebel´s works on the pedagogics of the Kindergarten (FROEBEL 1862, *cit.* PK)
The "Spielgaben" at the beginning are destined for children of two or three years old. They constitute an ascendant series of elementary geometric basic forms, which in their tactile qualities as also in their conceptual schemes thoroughly shape the child's play, thus rendering comprehensive possibilities of development. Now, for Froebel comprehensive playing doesn't mean just a stupid and mute handling with objects, but also - Man being a creature of language! - to set it into a linguistic intercourse - to give to this action of handling objects a linguistic counterpart : to accompany action by singing, rhythms, little verses and sayings, which arise by themselves, but which also mean describing, conceiving exact linguistic expressions for facts and clear ideas of the structures of the world.
Prior to language, the whole body gets into interference with the material. A ball on a string, for example, swinging to and fro, the hand stimulating the ball to swing in resonance and counterresonance, then also the arm automatically will be involved, and with the arm also the whole body; and murmuring and singing "to" and "fro", and the eyes following the ball on the string, the body will participate nearly dancing; and quite in the sense of Piaget's principle of "internalisation", the structures within and behind action will be internalized pre-rationally, as structure of space, or as structure of swing.

Generally, the first Spielgabe is "the ball" - geometrically : the *sphere* (WAGEMANN, 1957). It is the overarching symbol of the cosmic order : as the most perfect, since most symmetrical regular form, equally extending to all its parts and directions, giving the same aspect from all sides, consistent, perfect, of ideal beauty (Froebel´s "Spherical Law"). For the baby the ball is made of stuff. Moved by a thread we can see again (Fig.1 = PK 1) this simple to and fro, which further on now differentiates the shapes of the ball's movement, revealing the structures of space as well as the physics of movement under constraint : lifting it over an obstacle, again accompanied by murmuring "hither and thither", "hither and thither". The ball moves "up" and "down"; it leaps, "tip, tap, tap", over the floor; jumps up and leaps down again; "rolls away" and "comes back" again, forced by the string. Thus, by swinging to and fro, moving up and down, or "left" and "right", the directions of space and their relations to the body, to its axes and directions, the coordination of sensual experiences, of optical or motional orientation will playfully be internalised. In connection with this, by the impulse to give to these movements of the ball a verbal expression, a term, a description, the linguistic competence will be exercised, quite in the sense as e.g. - there, however, merely verbally, and for the children looking on : merely in a receptive way - the verbal games of Erny and Bert in the Sesamstraße to be carried out in a compensatory way for the socially or linguistically deprived.

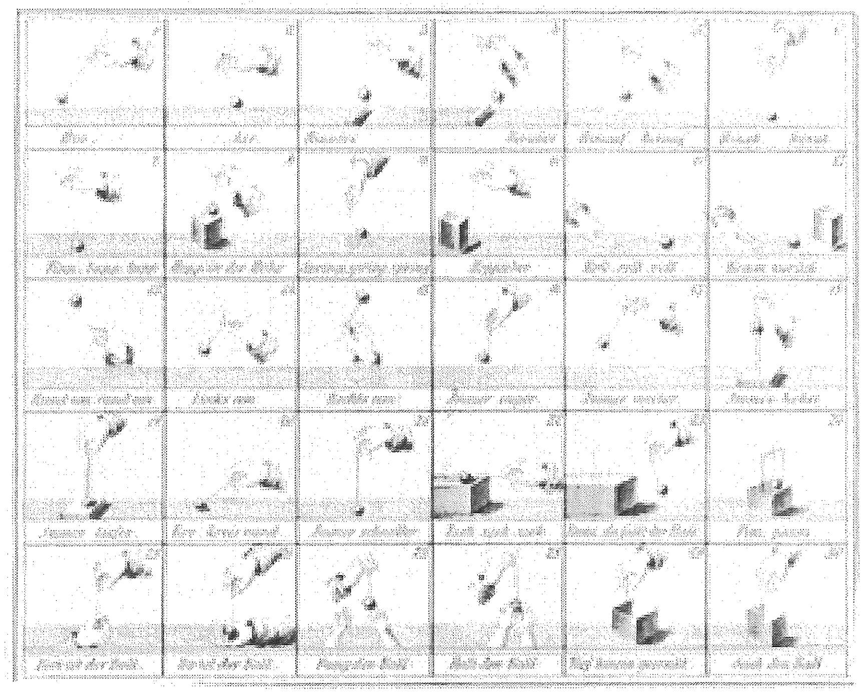
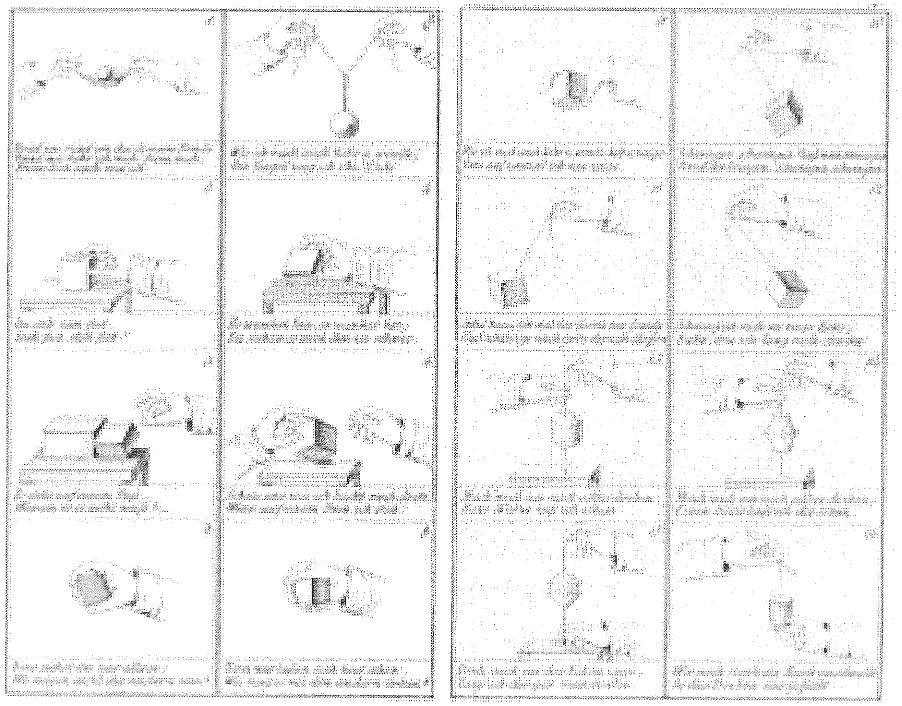
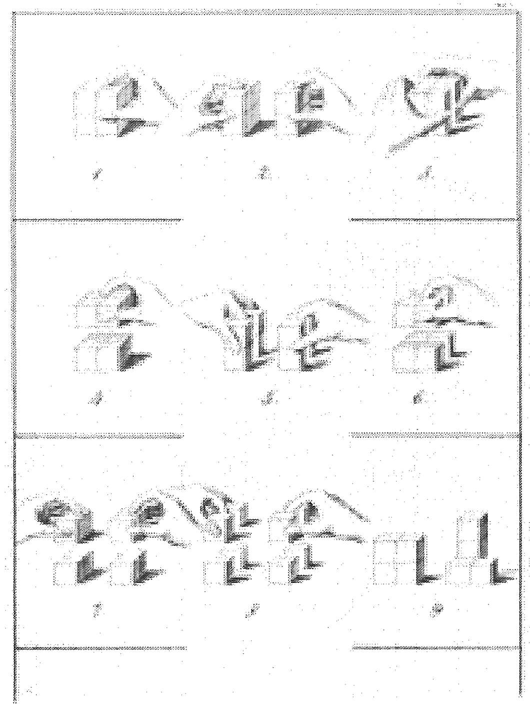


FIGURE 1

Fig.1/13-20 shows further bound movements, which are coupled with physical laws : with centrifugal force, with translatory and rotary movements in space, with damped oscillations, i.e. the helical movement of the ball approaching zero position "by itself", or moving in an opening helix when energy is added by a synchrone rhythmic movement of the hand.
In Fig.1/25, 26 notice the use of the category of "conservation" : catching the ball, it disappears in the closed hand, re-opening it : the ball is still there; that means : "conservation" is established already at this age - no, not only established but already playfully varied. In fact, the baby is very early and naturally familiar with this kind of "conservation" when, for example, it creeps under a sofa in search for a disappeared ball, because it knows for certain that it must be *somewhere* - thus is the manner by which Piaget's concept of conservation is acquainted at a very early age.



As for Weiß the indexing of crystal axes, so for Froebel it is the play with the ball, the correlation of its movements to the axes of the body (to/fro, up/down, right/left) which already elucidated the marking of the threefold space extension, which, irrespective of rational valuing of space as homogenous, will remain a lifelong unconscious background association. The basic geometrical form, which mathematically specializes the threefold directions of space, is the *cube* (ME 2oo). So the cube is accordingly the second Spielgabe, which shows a wealth of characteristic mathematical and physical regularities (conformities with natural laws) (Fig. 2, a,b = PK 2). Nr.3 shows the stability of the cube, considered as basis for every life, Nr.4 the unstable balance when standing on the edge, Nr.5 the effect of friction, within certain limits making possible stability of unstable mechanical configurations. Nr.6 shows the pecularities of axes and freedom of rotation in case of punctual resting, 7, 8 and 9 show the symmetries of different views of the cube, the number of the appearing planes, in relation to the laws of symmetry : of one plane with its fourfold rotational symmetry (7), the two-planes view and its twofold (8), the threefold symmetry when looked along the space diagonal (9) (cf. also ME 200 f.). The faculty of fancied metamorphosing of geometric structures - the experience of their metamorphoses are being prepared by the winding movements of the cube around different axes and of the then appearing veiling structures (PK 2,13-16).



The third Spielgabe is the threefold divided cube, divided into eight small cubes by three planes vertical to each other, in correspondence to the three directions of space. It gives mathematical reference - and exercises the fine movements of the fingers - when they are separated into the mathematically possible arrangements (Fig.3 (=PK3). The divided cube now is specialised enough to experience the three essential methods of composing complex forms out of elements resp. elementary forms. Different aspects of the world can be represented as

- forms of living
- forms of cognition
- forms of beauty.

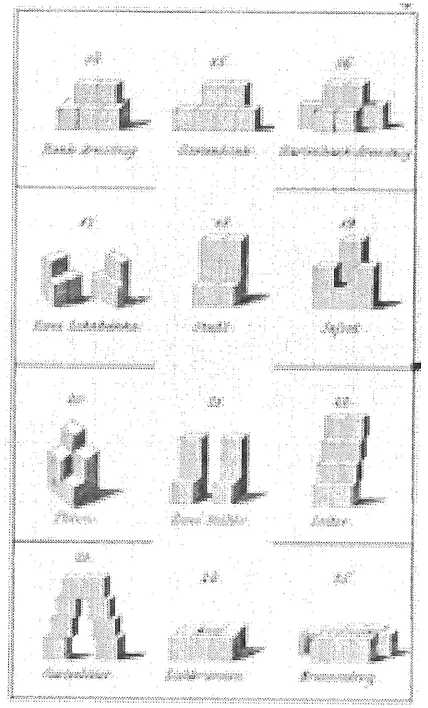


Fig.4 (=PK4) shows some of such *"forms of living"* : objects of everyday's life are constructed by use of the set of small cubes, thus give vivid experience, how the world can be imagined as being constructed from basic geometric bodies. The complex shapes of the world are reduced to elementary forms, in the sense how Kandinsky conceived his elementary course of "analytic drawing" at the Bauhaus. Complex things comprehend elementary parts, and can be constructed from these elements. Abstraction, feeling for space, and imaginative thinking will be exercised by supplying mentally those elements that are hidden behind others (Fig.4; 15, 16, 19, 20) - nowadays a standard part of maturity tests at schools.

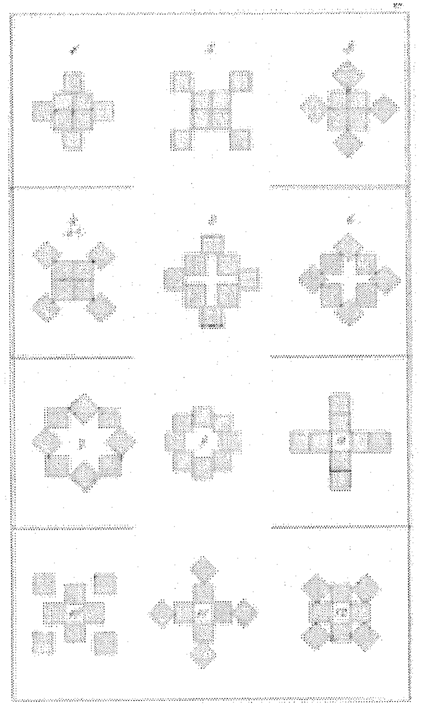
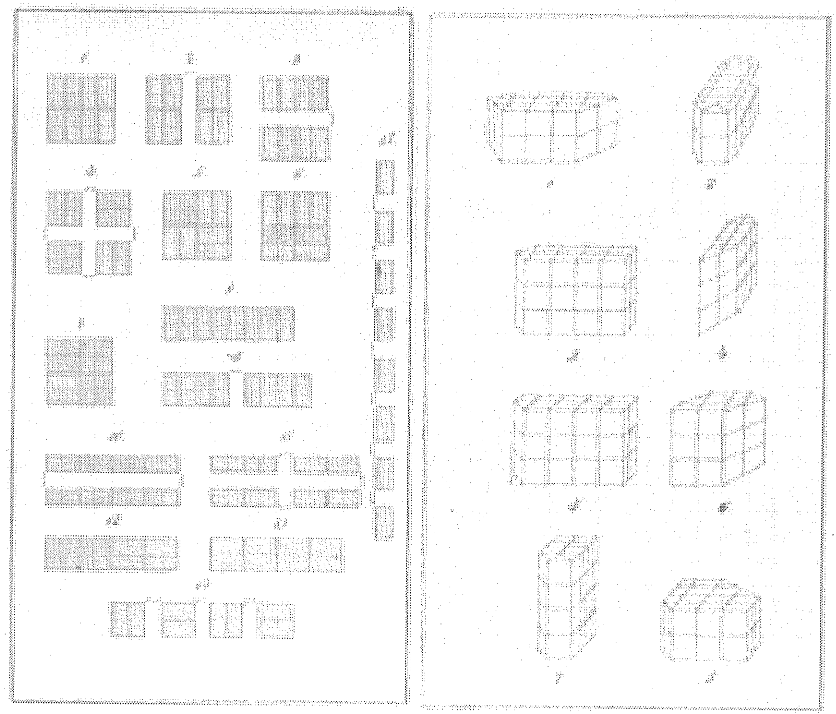


Fig.5 (=PK5) shows *forms of beauty*, regular ornaments without concrete signification, which can be composed from the small cubes and which we perceive as beautiful. Here again the complex comprehends the elementary, now in such a manner that the symmetries of the ornaments result more of less determined from the symmetrical qualities of the elements. Beauty appears as the playful combination of freedom and constraint. Displacements of the structure elements cause fractions of symmetry and metamorphoses on the whole scale.

Finally, *"forms of cognition"* show the regular interchange between elementary and complex arrangements that will be examined according to those laws, that come out as mathematical theorems (e.g. Fig.6 (=PK 8) for the fourth Spielgabe, the "quadered" cube). It becomes evident, that decomposing and synthesising of the world are but different aspects of a world of abstract formal rules "behind" the appearing objects, which result a system of laws (called "mathematics") that governs their concrete manifestation. In a manner familiar for contemporary mathematical didactics this formal analysis can be further developed to divisions and calculations of planes, to the illustration of number systems and of the elementary rules of addition, subtraction and multiplication. That the world is penetrated and ordered by formal rules which we call mathematics will be experienced by play and trained by schooling.

By the last Spielgabe of the pedagogy of the Kindergarten, the diagonally divided cube, the division of planes will be extended into space (Fig.7 (=PK.11)). Columns, complex polygons, discrete divisions of space and cellular structures lead to the metamorphoses of polygons and their zone-rules. Froebel's crystallographic starting point is regained by play.



To sum up, it should become evident how Froebel, in decomposing and synthesising elementary and complex forms, intends to enable children to understand the structure of the world as a beautiful entity ordered according to rational laws, and how this experience is apt to stimulate the mental development of children in harmony with the world.

2 "Games with Rules", or : Symmetry as a Ferment of Mental Development

2.1 Symmetry and Self-Organization : the Modern Paradigm

The scientific ideas of RNP of course did not remain unchanged during the last two centuries of research. On the contrary, the "original" RNP suffered serious criticism and neglect, whilst science itself developed independent from philosophical considerations (at least supposed to do so!). Even the already achieved (or better : postulated) unity of (atomistic) "matter" and (fields of) "force" separated again through the 19th century. They were unified again, however, on the base of renewed and modernised ideas of RNP, that had remained as influential substreams of formal ideas about natural laws, which were recombined since about 1900 by the new paradigm of modern science : to understand atoms as centers of fields of force, that constitute matter

by processes of selforganisation, which are directed and controlled by the symmetries of their fields.

Thus, the concept of *symmetry* became the overarching idea to tie together the various fields of science about nature - and those of science about our mind as well, since the mind must be understood as both : as the tool for getting knowledge about nature, and as a part of nature itself. "Only" the idea of God, as the creator of both, has been expelled from science. (The numerous monographs on symmetry and its application in science and art can easily be found in book catalogues or the internet; the "International Society for the Interdisciplinary Study of Symmetry" (ISIS-Symmetry, founded in 1989) dedicates its work and its periodical, "Symmetry - Culture and Science", to this nearly limitless scientific and artistic field).

I want to sketch now some ideas for implementation of symmetry in education, which I have developed and tested on various school levels. Parts of them were introduced into the Hamburg "General Science Syllabus" for Comprehensive Schools. The full range of the possible interdisciplinary role of symmetry for the formation of mind and for study of science cannot be exhibited here for reasons of briefness. I want to confine myself to some remarks on *mathematics* in *primary*, and to *science* on *secondary level.*

The intention to implement symmetry in school must not be considered as an addition of a mass of new, just different topics of instruction, meant for a merely receptive learning and memorising by heart. Instead, it shows its full wealth only when taken as a general horizon of mental development, a vivid and multifunctional ferment of mental formation ("Bildung"), in about the same manner as Froebel had understood his concept of education. Thus, many of the following consideration are valid for a modern re-interpretation of Froebel-pedagogy as well. With respect to science education, two further views should act as general background ideas :

- The idea that our mind, as the medium of understanding the world, is the product of its interaction with the world. Thus mind receives its shape by the experience of the world during its development, general (in evolution), or individual (in education). Reversely, the mind also shapes the conception of the world, by applying its structures to the world (- the basic idea of Kant´s "Critique of Pure Reason").
- The consciousness that the empirically stated order in nature (e.g. symmetry), and the control of natural processes to verify this order by natural laws ("self-organisation") are just two different aspects of the same thing.

Considering the realisation in school now, the basic methodical postulate would be : keep away from mere verbal instruction, but aim for a participation of the pupils´ whole person. Esp. in lower classes, this may best be achieved, in a relaxed manner which is very suitable to the topic, by true manipulating work with appropriate materials and techniques. Preferable esp. for scientific modelling would be types of material that do not command a *special* interpretation (e.g. in the manner of "look, this is a model of a NaCl-crystal"), but rather by the use of types of building elements which show only certain symmetrical qualities, but further are free for concrete scientific or artistic interpretation. Froebel proposed a manifold of materials to make children play with structures, yet favoured his Spielgaben as optimal. To my knowledge, the most versatile modern system of material in this respect is "Geomix" from Ratec/Frankfurt (it was used for the illustrations in the 2nd part of this paper).

The advantage of these symmetrical qualities would be that constructing and modelling activities achieve the character of games that follow certain rules, yet have an earnest intellectual background. It would even be a serious obstacle of understanding and of mental development to declare a certain activity as a topic of "art" or "mathematics" or "chemistry" (not even as a part of the background-theory in the mind of the teacher!). A vivid variation of the different meaning-levels seems the best method to improve both, formal development of mind, and topical learning.

With these basic guide-lines in mind, we will find symmetry an outstanding medium of erudition, since it fulfils nearly all formal requirements which modern education would demand as principles of fertile instruction :

- It aims at interdisciplinary approach since it deals first with formal conditions of understanding applicable to all possible objects of experience;
- It relates objects of learning to each other, thus rendering possible *shaped, understanding* learning;
- All formal laws raise from and remain closely related to *sensual experience*;
- A deep feeling of comfort is raised by having symmetrical orders open to our senses; this affects our sense of *beauty*;
- Learning with symmetries may be based on action, and action will continue to give a basis of understanding for complicated problems; the promoting unity of action, of sensual and intellectual activity in understanding will be experienced;
- These activities may be abstracted and *formalised* towards mathematics, simple enough, yet basic, thus *evolving* the mathematical interpretation of the world;
- On the other side, they are applicable for detailed concretisation, yet with very general principles as promoters and guideline;
- These problems also cover all school levels, but its typical subjects are not too far from traditional school topics, so that implementation would not raise serious difficulties in syllabus : It is more a new spirit than new contents that are aimed for.

Of course, there will be, and shall be, some topics which school hitherto has not dealt with, or topics, that will seem to be introduced "too early", so that teachers might be in fear of additional stress and of extension of an already overcrowded curricula. This would be an unnecessary fear, since, as you remember, the intention is not to introduce problems artificially, but that the problems *introduce themselves*, so that, on the contrary, it would be an artificial obstacle to keep them from affecting pupils' minds : the intrinsic logic and inner dynamics of the symmetries of the used materials and methods should (nearly) always generate the problems being treated,

thus creating a sort of *self-organisation of problems*, so that the result, though it demands hard motoric and intellectual work, should be joy and relief.

I will now give a survey of topics I treated in schools on the various levels I had access to. The illustrations are but a small selection of the activities being done. On the other hand, this necessary restraint could be an advantage for the reader : Instead of being limited, your imagination should be inspired – so feel free in imagining!

2.2 Preschool Level

The fields of activities mentioned here respectfully remember Froebel´s pedagogy. Activities should only *prepare* exercises in symmetry by laying emphasis on *topological* aspects of action and imagination in space, such as :

- Orientation in space (up – down, left – right, before – behind, inclined, diagonal, through, around. . .)
- Spatial relations between different objects.
- Connect different objects ("trains", garlands), let them close (chains, necklets, fences), cross each other (bridges, warps), encircle areas, erect boundaries. . .
- Study qualities of simple geometrical bodies : balls (look alike from all directions), bricks (different types : cubes, blocks, pyramids, bars; different views : surfaces, edges, corners); stable and unstable positions.
- Generation of lines by stringing points, of planes by shifting bars (or whirling bars around your fingers, or around axes, or. . .), covering space by shifting planes, asf.

Do not drill the children to do certain exercises, but let the challenge of material do the job; vary problems; embed them into "meaningful" games, into narratives, artistic activities; or, at different occasions, abstract them and make them conscious. Very important : make the children speak about what they do, this give them implicit selfreference; but do not intend a canonic, nor an even "professional" terminology, on the contrary : intend a versatile common language, rich with associations, yet precise.

All these are well-known axioms of preschool teaching; just make them rich, and be sure they will effect implicitly what you intend.

2.3 Primary Level

The "New Math" reform of primary mathematics, in spite of its condemnation and pretended failure, in fact resulted a considerable enrichment of classical geometry and of calculating towards activities with symmetries and structures. Apart from regional differences, one nevertheless might criticise in general :

- that they do not go far enough. Especially the symmetries of space usually are lacking – for bad reason : aversion of teachers against true actions ("no time", "no materials") and the difficulty of illustrating them in textbooks. The latter however is the reason for their importance : symmetry operations in space would need a fourth dimension - the simulation of line-symmetry in (dim n) by a true operation, turning around an axis on (dim n + 1), is no longer possible – so action in space basically must happen in imagination.

- A further criticism is that symmetries are fixed on the mathematical use, but neglect the colourful enrichment they could gain by looking on biology, on arts, on music and dancing; in reverse direction, artistic activities, or the beauties and functions of biological objects, lack a deeper understanding by structural insight, which would lead immediately into mathematics. (In general, the split into the "two cultures" caused a completely unwarranted fear of mathematics teachers to lose "strictness" by artistic fantasy, of teachers in arts, to hinder "creativity" by disciplined reflection on the process of creation – numerous books on ornament construction in art nouveau, many of which have been reprinted recently, show the contrary.)

- Finally, the inner-mathematical use of symmetries in school is too poor, too : the study or the construction of ornaments is shown immediately to lead to classical geometrical problems, whilst the autonomous value of symmetry problems, e.g. first steps into group theory, are estimated low – caused by the flop of New Math., but without reason, since group theory allows us to denote the formal (= mathematical) structures of empirical space.

To turn to primary instruction now, symmetry would appear especially as a matter of "mathematics", but with respect to the media by which it should be introduced, mainly as a matter of "art".

In fact, at this level an approach of a certain systematic character should introduce the various types of symmetry operations. First would be finite symmetries : line symmetry and rotational symmetries (Fig.8); their combinations constitute the full range of rotational structures and fulfil the axioms of cyclic groups. When studying them (in the sense of : playing with them), notice how originally non-symmetrical problems come into consideration, too : rotational subgroups and submultiples; line symmetries of rotations, odd and even numbers, order of rotational symmetry, fractions and angles.

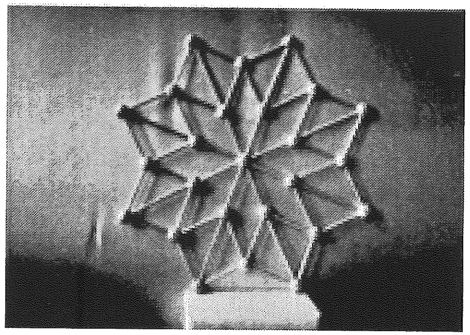


FIGURE 8

Adding shift operations (translatory symmetry) will cause infinitely extended ornaments : strings in one dimension, mosaics in the plane (Figs.9,10), finally lattices in 3-dim. space. It is an enormous step for children of age eight or nine to understand what really it means to say just "and so forth", and to operate with it – this needs careful elaboration. New types of symmetry operation will arise from this addition, fulfilling the group axioms again, now resulting the two types of (Felix) Klein's four element group; so the structure of space, insofar it is constituted by symmetry operations, is active mathematics on Klein's group.

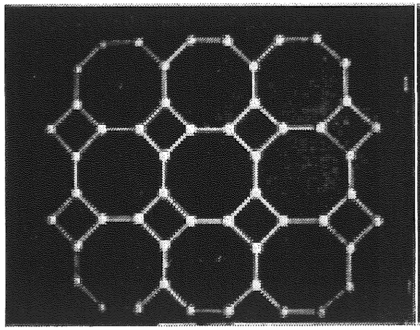


FIGURE 9

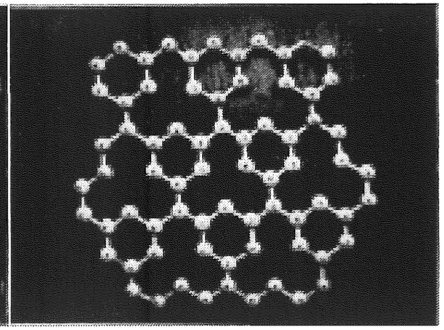


FIGURE 10

Take the simplest and most general one of the polyhedra, a parallel-epiped, and shift it translatory into three independent direction over whole space, thus children will get first experiences with lattices; construct regular aggregates of bricks, of matchboxes, cells; for oblique epipeds take sphere-and-rodlets structures; these immediately render experience with symmetry centres, mirror type symmetry in space and the crystallographic laws of rational indices - all this will happen nearly automatically.
"Automatically" – that´s the key-word for the basic experience when dealing with all these things; for it is by no means intended to "learn" all this by verbal or formal drill; instead, all these subjects mentioned should evolve intrinsically from the structures of materials and methods, most of which, as mentioned, under classical categories would appear as "arts".

The intention with them all is that the resulting symmetries are automatically generated by the techniques used, so that the more or less surprising effect causes the question why that is so, and what are the details. Well known in this respect, dealt with in many publications, is the generation of linear symmetries by paper folding, gluing, blotting; for rotational symmetries : folding through centres, puttering with straw... Regular plane ornaments are constructed with "elementary ornaments" (paste board or wood) which are encircled and added to each other by continuously repeated rules of symmetry operations. Translatory symmetry of all types is most easily achieved by constructions with bricks or spheres regularly pierced and tied together by rodlets (e.g. Geomix-type).

All plane ornaments can also be drawn or painted free-hand on paper. But this would be a conscious *decision* to regularity instead of an instrinsic automatism of elements or techniques.

In some techniques, one finds even both. In Javanese batik, for instance, the restraint to regular design in spite of free "painting" technique serves as a medium of contemplation, whilst modern, semi-industrial techniques of stamping batik patterns by use of a "cap" represent in-strinsic regularity, the laws of which must be considered during production and afterwards are fixed. The same holds for European "blue print" on textiles with wooden stamps, whilst the well known "potato prints" allow both types of design. The various techniques of weaving, by different rules of interweaving warp and welt, render convincing examples how the technical process itself implies different symmetrical textures.

Of course, all these *active* methods should be paralleled by studying and contemplating given ornaments and works of art, by pictures or better with the originals. Combine the study of mere formal textures with possible meaning, for instance, when enjoying a Gothic window for its fine ending, tell the narrative of the pictures in the field, too – imagine why the medieval artist has combined them!

2.4 Secondary Level

This is the appropriate age to experience the dependence of structures in nature from the symmetries of building elements, and this will last as an "open end" situation - its still ours in scientific research.

You may start with "lattices", on basis of linear and plane translatory symmetry, as mentioned in the preceding chapter; this would be in particular adequate if, as usual, an extended phase of primary level work with symmetries does not exist.

You may also start with particles of highest symmetry, spheres, which would restitute the 17th century atomistic approach, with anticipated success today, however, since knowledge increased since to steer this process of learning. In Germany, there has been done a lot of conceptual work in this direction (listed in SCHMIDT, 1987) which one might call a "starting chemistry with structures"-concept ("Strukturorientierter Chemieunterricht").

The principal idea is just to put together a lot of spheres of provisionally same size (made from styropore, wood, or metal), and to look what will happen. There will result densely packed, hexagonal plane layers, which when stacked up will render the densely packed spherical lattices; there are two of them, the plane centered cubic and the hexagonal densest lattices (plus the energetically interesting bodycentered cubic).

The spheres of course are meant to represent particles of spherical symmetry, i.e. particles which commit equal forces into all directions. This is the case for electrically charged particles, ions (with stripped or added electrons) and metals (with their outer electrons delivered to the "electron gas").

The three mentioned lattices indeed are the typical structures of metals, and the study of lattice gaps and configuration groups will render many chemical problems, while plausible arguments on stacking faults, dislocations and particle exchange lead immediately on problems of plastic flow and of alloys, thus to metallurgy and solid state physics.

Ions, to their side, are geometrically characterised by different particle radii, and in fact, when

throwing together spheres with different size (preferred ratios of radii 1:2 [for the NaCl-lattice, Fig. 11] and 3:4 [for the CsCl-lattice]), the main types of salt crystals will immediately result.

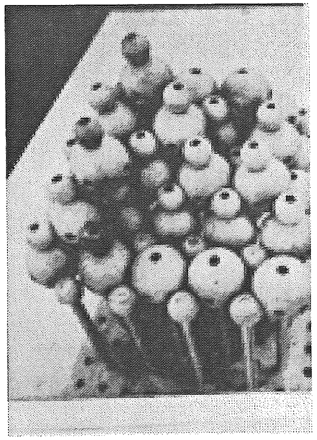


FIGURE 11

Isolated molecules are constituted by atoms which commit directed forces, so their symmetry is low, and also shows a great variety of subtypes. But the very representative of directed (homeo-polar) forces is the *tetrahedral* configuration, since the symmetry of four equivalent forces, which is the preferred symmetry of the four electron pairs of the rare gas shell, shows this shape (KLEIN, 1980). Tetrahedral forces may be represented by all carbon models, or, as a "merely symmetry"-particle, with Geomix by a tetrahedron brick or a tetrahedral pierced sphere. Consequent use of symmetrical orientation of neighbours in tetrahedral configurations ("cross" or "parallel" (Fig. 12)) will automatically lead to wide fields of organic chemistry. Nearly all subjects of basic organic chemistry are comprised, but also subjects which rarely if ever will appear in school - like cyclo-alcanes or polyhedral alcanes (Fig. 13). Even important modern materials like silicates and silicone come into view by the automatisms of modelling by means of symmetry.

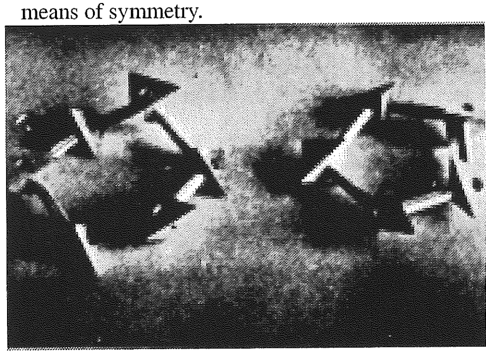


FIGURE 12

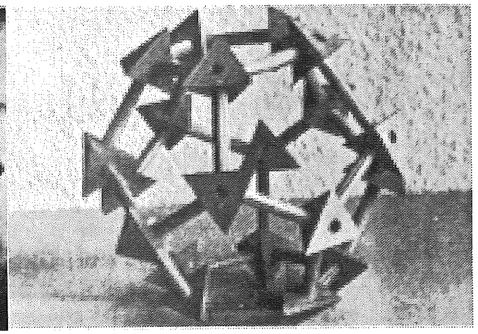


FIGURE 13

The tetrahedral configurations may also be extended infinitely into three dimensions, so that there will result the structures of diamond (Fig. 14), semi-conductors and ice or clusters of water (Fig. 15 : Pauling's "Chlatrat"-model of water), rendering a structural approach to their interesting large scale phenomena.

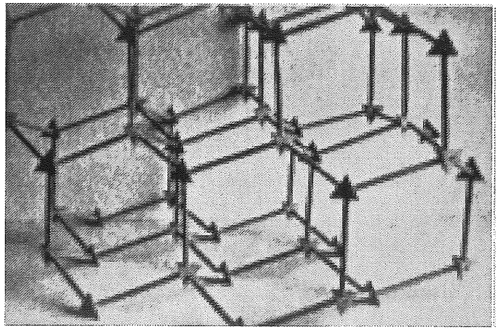


FIGURE 14

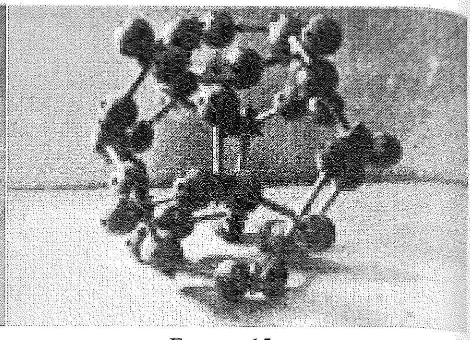


FIGURE 15

Parallel to these structural games, and their interpretation as and application in "classical" themes of science education there should be rich experiences and activities with topics connecting phenomena with structures as KELLER (1980) proposed for crystal growth.

Let this be enough for a gross sketch of the fruitful problems which may be generated and mastered by symmetry considerations in science education. I have confined myself to science here for reasons both of briefness and competence. I hope the interdisciplinary role of symmetry stretching also into the fields of "humanities" has become obvious, though.

This comprehensive role does not mean, however, that it should happen mainly within comprehensive - so called "interdisciplinary" or "integrated" - courses; though frequently favoured, this aim usually can result only a more or less diffuse unit of a whole. In contrast, education on secondary level should make use of the full clearness and differentiation of knowledge and problem-solving that is rendered possible by the discursive nature of mind and which happens most effective "classically", namely in different sciences. "Different" must not mean "isolated". It was my main intention here to stress the understanding that especially during phases of parallel strings in different sciences, symmetry can act as an overarching background organiser of learning in school that ties students' minds together and protects them from the stupidity of learning into separated boxes.

Thus, instead of being just more or less efficient *instruction*, learning would become a medium of true cultural formation. By its intrinsic beauty, symmetry would connect knowledge with morality - and that would be the *humane* task of symmetry in education.

Bibliography

BOLLNOW, Otto Friedrich (1952); Die Pädagogik der deutschen Romantik (= Geschichte der Pädagogik Bd. 4); Stuttgart

FROEBEL, Friedrich Wilhelm August (1826); Die Menschenerziehung,. . . ; quoted as ME from H. Zimmermann (ed.), Leipzig (Reclam) 1926

(FROEBEL) (1862); Die Pädagogik des Kindergartens (= F.F. coll. papers (Wichard Lange (Ed.) vol. II); Berlin (quot. PK)

KELLER, E. (1980); Wachstum und Aufbau der Kristalle; Köln (Aulis Praxis Schriftenreihe Bd. 10)

KLEIN, Peter (1980); Das Tetraeder als Baustein räumlicher Strukturen; in : chimica didactica vol. 6; pp 173-191

KLEIN, Peter (1992); Symmetry Arguments in Romantic Natur-Philosophy; in : Symmetry - Culture and Science; vol. 3; pp 401-420

KRAUSE, Karl Christian Friedrich (1819); Das Urbild der Menschheit; Dresden (2.ed.)

SCHMIDT, S. (1987); Die Struktur von Stoffen als Leitlinie zum Verstehen chemischer Prozesse; Kiel (PH (=Teacher College); Doctoral Thesis)

WAGEMANN, Elmar Bussen (1957); Quadrat - Dreieck - Kugel; die Elementarmathematik und ihre Bedeutung für die Pädagogik bei Pestalozzi, Herbart und Froebel, Weinheim O.J.

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