1. [Aristippus image]

Caption : Euclid's *Elements* (ed. David Gregory), Oxford 1703, frontispiece

Can mathematics education learn from its history ?

FAUVEL John
The Open University (UK)

Abstract

In 1570, at the University of Oxford, a young lecturer called Henry Savile opened his lecture course on mathematical astronomy by telling the story of Aristippus the Socratic philosopher. According to the Roman writer Vitruvius, Aristippus was shipwrecked on the coast of Rhodes, and struggling with his comrades out of the storm-tossed waves came to a beach on which there were geometric drawings in the sand. "Be of good cheer" he said to his comrades, "for I see the marks of human-kind." Humanity is defined and characterised by its being a mathematizing species; wherever there is mathematics there is civilisation. The image from antiquity became a symbol of the University of Oxford's mathematical aspirations, forming a background to common beliefs about the significance and role of mathematics in the ensuing centuries.

This use of the past as a guide and inspiration, testifying to mathematics as a symbol and characterization of humanity, can be seen as forming a thesis which we can hold up in opposition to a second image, also produced in Oxford, three centuries later.

In a book called *Picture logic: an attempt to popularise the science of reasoning* (1874), Alfred Swinburne drew a picture showing all the living world passing into a monster logical sausage-machine, emerging in uniform parcels under the watchful gaze of Professor Logic, who cheerily reflected:

> "It's with our machine here as it is with the ordinary sausage machine, never mind what the material is-so long as the shape is right. Large or small, young or old, flesh or stone, you must all pass through, for Professor Logic isn't particular about the matter, all he concerns himself with is the form." [p. 37]

This symbolic image can be taken to stand for the numerous interventions down the ages suggesting that mathematics is involved in something inhuman, mechanical; that mathematical modeling misses the point and unweaves the rainbow; that mathematics teaching is an anti-humanistic shuttering, a closing down, of human aspirations. And as we will see, the critique made of mathematics and mathematics education as being mechanistic can be extended to the educational policies within which the education is experienced.

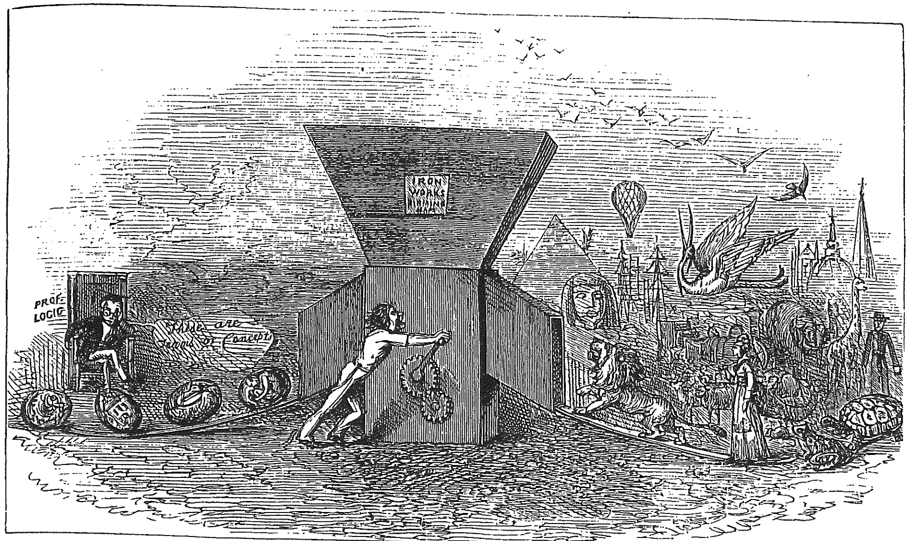
Somewhere in the historical dialectic between the conceptions underlying these two images there may lie a resolution which with strength and energy can be used in constructing the best possible framework for mathematics learners of the future. I'll come back to these symbols later and draw more out of them for this purpose, after making some observations on internationalism in our community.

1 Reflections on internationalism: states, people and the New World Order

I'm drawing today particularly on the British experience because that's what I know best, but I would like to think that any ideas and insights we may be able to glean from that situation are transferable elsewhere and make sense in other national contexts too. One of the few reassuring aspects of the "New World Order" is the sense that, through meetings such as the European Summer University and the growth of international travel, people in different countries are able to develop a new sense of kinship and shared experiences in one world despite the selfish self-importance of politicians. We can learn from each others' experiences of the modern state how to improve individual lives as much as possible even while politicians try to impose and magnify divisions.

After all, one of the most moving and important insights into the Great War, which men were fighting not very far from here 85 years ago, came from that incident at Christmas 1914, when the opposing troops came out of their trenches to exchange cards and cigarettes and food rations, and sang together *Silent Night-Stille Nacht, heilige Nacht*-until the officers on both sides discovered what was going on and put a stop to it immediately, pointing out that if this kind of thing spread there would be no point in having a war, and that would not be at all good for armaments manufacturers.

One thing we can learn from international gatherings, such as the European Summer University, is how we foot-soldiers in the educational trenches can learn to control our officer class, the politicians, better. This involves learning from those in other countries how they have achieved certain improvements in educational provision: we in Britain want to find out from you how *the Danes* have managed to get the importance of history of mathematics recognized and written in to the National Curriculum; how *the French* have managed to build up and sustain the splendid IREM structure which enables teachers throughout France to participate in research activities promoting the better utilization of history of mathematics in the classroom; how *the Dutch* have got to the point of talking about the possibility of teachers being seconded to study for PhDs with funding cover supplied to their schools; how *the Italians* have managed to sustain the importance of geometry and its history in their schools when it has nearly vanished in some other countries; how *the Portuguese* managed to support the sending of some 250 of their mathematics teachers to the previous European Summer University, the wonderful History and Pedagogy meeting in Braga three years ago; how *the Swiss* manage to trust their teachers enough to keep them free from the layers of constraints and bureaucracy that keep teachers elsewhere in a state of stress with no pedagogic advantage; and above all, after four happy days in Louvain-la-Neuve, we want to learn how *the Belgians* have persuaded their town planners to design a university city free from cars and on a human scale, embodying so impressively the humanistic structural values of the Middle Ages and High Renaissance. In short, each country has aspects to its educational provision, and its social-political context, from which other countries can learn-both positively and negatively: what to cherish, what to avoid.



THE LOGICAL SAUSAGE MACHINE.

2. [Logical sausage machine]
Caption : Alfred Swinburne, *Picture logic*, 1874, p.35

2 History and mathematics education over 400 years

What have we in the UK to offer you? There may be an object lesson here, in contrasting two apparently contradictory aspects of British history. On the one hand, we have one of the longest traditions of taking the history of mathematics seriously, both intrinsically and in connection with education. On the other hand, the UK doesn't seem to have benefited from all this history in terms of what is actually happening in the majority of our schools at the moment: there is no mention of history in the National Curriculum for mathematics, and pressures on teachers make it hard for them to develop historical interests. So there is some dissonance here which needs to be understood.

From the very first printed mathematics textbook in English, history of mathematics has been invoked as an aid to the pedagogic strategy-in a variety of ways, just like now. Robert Record in his algebra text *The whetstone of wit* (1557), for example, tried to create interest in the activity of taking a cube root, by including a memorable anecdote about the Greek heritage of the problem. In Record's account (derived from Eratosthenes), the Delian problem, that of doubling the cube, was proposed by the gods as a means of deliverance from a plague. But the first mathematics teacher in England, and maybe anywhere, to take the history of mathematics seriously as an integral part of mathematics research, teaching and exposition was John Wallis at the University of Oxford in the seventeenth century. He was working within an ideology already tilting that way, set in train by Sir Henry Savile when he founded two chairs in mathematical sciences, in geometry and astronomy, together with detailed instructions for how the subject was to be taught. The core of the teaching of both subjects, as laid down by Savile, was *explanation* *of ancient texts*. Thus the professor of geometry was to give public expositions of the whole of Euclid's *Elements*, Apollonius' *Conics*, and all the works of Archimedes; in other words, the method for mathematics teaching was historically based, and primary source based. (This may be to do with the Reformation: after all, that is just what protestants do with the Bible). At all events, Henry Savile is a prime example of a great teacher who developed a style of teaching mathematical sciences based on the use of history as intrinsic to the subject and vital for communicating the traditions of the past to the next generation.

John Wallis was the third Savilian professor, as they were called, of geometry, and took up and developed this pedagogic challenge by writing a great *Treatise of algebra* (1685). This is at once the first major algebra text in English of this size and the first history of algebra, or indeed of any mathematical subject, in English. For Wallis, algebra *is* its history: to learn algebra is become familiar with the progress of the subject up to now, and to do research in algebra is to stand at the frontiers, with the knowledge of the past around you, and move forwards, while dipping in to the past, rediscovering and reinventing the sometimes confusing or hidden messages from long ago.

In this way, there was already by 1700 influential interest in England in the mathematical past, and examples of its use in a variety of ways in teaching the subject. But of course, these studies were at university level and we are talking about a very small number of people. There was no national school system, and such schools as there were taught mathematical skills in practical contexts such as navigation, book-keeping, and so on.

Let the centuries roll on. The nineteenth century was that in which the framework of today's national education system was laid down. In the aftermath of our 'Industrial Revolution', various components began to emerge and take shape, for a range of reasons, including liberal philanthropic responses to the human misery of industrialization as well as its need for an educated workforce. These components included such things as development of examinations; training of teachers; compulsory education, first primary then secondary, in Acts of 1870 and 1902; development of scholarships and State funding mechanisms; and eventually, only recently, a National Curriculum.

Surprisingly, perhaps, the nineteenth century was one in which mathematics education was more historically imbued than at any other time. This was for two main reasons: one was the extraordinary place which was occupied by one 2000-year-old textbook, Euclid's *Elements*. The other, which we may think was not altogether compatible, was a growing educational ideology, that mathematical ideas in the individual develop along the same lines as their historical development-from which it would seem to follow that mathematics teachers, or at least curriculum designers, needed to know the history of mathematics to order materials for presenting to pupils.

First, Euclidean geometry. Other countries taught Euclid, of course, but not with quite such single-mindedness, nor with quite the English obsession with the text of Euclid's *Elements* itself. The French were relaxed about teaching geometry from Clairaut or from Legendre, but in England only Euclid would do. This was widely commented upon. The great British algebraist James Joseph Sylvester recalled in 1869 his mathematical education in the 1820s in these terms:

> The early study of geometry made me a hater of geometry,. . . I know there are some who rank Euclid as second in sacredness to the Bible alone, and as one of the advanced outposts of the British Constitution.

To account for the dominant and sacred status of Euclid's *Elements* it is sufficient to notice that

it served other functions too. In particular, the other skills it was thought to teach were just what were needed for the intellectual part of governing an Empire (Sylvester's remark about an advanced outpost of the British Constitution was shrewdly positioned.) The intellectual case for Euclid's *Elements* was well stated by William Whewell, the Master of Trinity College Cambridge:

> There is no study by which the Reason can be so exactly and rigorously exercised. In learning Geometry the student is rendered familiar with the most perfect examples of strict inference; he is compelled habitually to fix his attention on those conditions on which the cogency of the demonstration depends. He is accustomed to a chain of deduction in which each link hangs from the preceding, yet without any insecurity in the whole; to an ascent, beginning from the solid ground, in which each step, as soon as it is made, is a foundation for the further ascent, no less solid than the first self-evident truths. Hence he learns continuity of attention, coherency of thought, and confidence on the power of human Reason to arrive at the Truth.

Of course, in the hands of unimaginative or badly trained teachers all this was for nothing, and it is not surprising that the controversies around Euclid grew louder as the century wore on. Indeed the UK's main organization for mathematics teachers, the Mathematical Association, was founded out of a society set up to replace Euclid's own text by other geometry textbooks. One of the leading campaigners on this issue at the turn of the century, the engineer John Perry, wrote

> Like all the men who arrogate to themselves the right to preach on this subject, I was in my youth a keen geometrician, loving Euclid and abstract reasoning. But I have taught mathematics to the average boy at a public school, and this has enabled me to get a new view. I have seen faces bright outside my room covered as with a thin film of dullness as they entered; I have seen men [. . .] lose in half an hour (as men did in the first day of slavery in old times) half their feeling of manhood; and I have known that, as an orthodox teacher of mathematics, I was really doing my best to destroy young souls.

As the result of feelings and rhetoric such as this, the primacy of Euclid was successfully challenged and overthrown, followed barely half a century later by the virtual disappearance of geometry as a school subject, though that's another story. The main point is that, for a long period of time in England, learning and reproducing the text of Euclid was more important than developing geometrical understanding, which is the kind of thing to give history (and mathematics) a bad name.

Another strong educational ideology of that century was the view that a young person's mathematical development mirrors the history of mathematics itself. The didactic implication is that mathematical topics should be taught in an order, and perhaps in a way, which is supposed to reflect the historical evolution of mathematical discovery. On the whole this idea was, and is, a force for good as it focuses concern on the development of the individual, thus happily fitting in with other notions of child-centred learning which were becoming consolidated at this time. Throughout its long history, the idea has provided further encouragement for the history of mathematics in an educational context. Indeed, this precise argument and connection was made by the great French mathematician Henri Poincaré, at the turn of the century, who was very interested in mathematics education and wrote:

> One cannot reflect on the best method of imbuing virgin brains with new notions without, at the same time, reflecting on the manner in which these notions have been acquired by our ancestors, and consequently on their true origin-that is, in reality, on their true nature. . . .

> Zoologists declare that the embryonic development of an animal repeats in a very short period of time the whole history of its ancestors of the geological ages. It seems to be the same with the development of minds. The educator must make the child pass through all that his fathers have passed through, more rapidly, but without missing a stage. On this account, the history of any science must be our first guide.

Even at the time, the idea did not go unchallenged. John Perry, who we saw earlier destroying young souls through teaching Euclidean geometry, was equally eloquent in his criticism of a mindless application of the growing recapitulationist orthodoxy:

> Because the embryo passes through all the stages of development of its ancestors, a boy of the nineteenth century must be taught according to all the systems ever in use and in the same order of time. [. . .] Think of compelling all emigrants to pass to America through Cuba, because Cuba was discovered first. Think of making boys learn Latin and Greek before they can write English, because Latin and Greek were the only languages in which there was a literature known to Englishmen 450 years ago. [. . .] It is not merely in arithmetic and geometry, but in the higher parts of mathematics that this waste goes on. Newton employed geometrical conics in his astronomical studies, and mechanics was developed; and therefore it is that every young engineer must study mechanics through astronomy, and he dare not think of the differential calculus until he has finished geometrical conics.

Of course there is still a great deal to be said for a more sophisticated version of recapitulationism, and indeed in our ICMI Study on *The Role of the History of Mathematics in the Teaching and Learning of Mathematics* there is a whole chapter on the historical formation of mathematical thinking. So any problem isn't the model as such, but the crassly unintelligent application of it which has been too prevalent in the past. Whatever its failings, the recapitulationist postulate did have the effect of forcing serious consideration of history of mathematics in education.

Through these and in other ways the history of mathematics has been a regular theme of English discourse about mathematics education during this century, and indeed for several centuries before that, one way or another. By the 1990s, there has developed a lot of serious interest among a relatively small group of people, and more general interest in, or lip-service to, the history of mathematics among a much wider group of people; but no hint of history in the National Curriculum, though of course no positive bar either, on using history as a resource for better delivery of the mathematical objectives and teaching outcomes of the Curriculum. We do know, from surveys prepared for our ICMI Study, that many countries make a stronger encouragement to teachers to use history-of which the Dutch-speaking part of Belgium is a good example-or even require it in some places, such as Denmark and Norway. So to some extent the problem is a British one. But in the structures of what is happening perhaps there are wider lessons too.

3 Analysis of the problem

To put this into perspective, and give us a framework for analysis, I want to use the recent work of the young Hong Kong researcher Lit Chi Kai. For his MPhil thesis at the Chinese University of Hong Kong, Chi Kai has investigated aspects of introducing history of mathematics in classrooms, including teacher reactions. He garnered the views of 360 mathematics teachers in 41 Hong Kong secondary schools, and found that although most think highly and positively towards history of mathematics, most, too, do not make use of it in their teaching. He then went on to ask them why not, why there is a dissonance between their beliefs and their actions, and he found four reasons in particular

- lack of knowledge
- lack of supporting materials
- students and parents too concerned about examination scores
- curriculum too packed.

This overall perspective, the four main reasons given by Hong Kong teachers for their hesitation in making use of historical materials, ties in with our experience in the UK, in the sense that it is very difficult to find anyone-any teacher, any member of the mathematical community-who will not say that they think history of mathematics is a good thing. And yet the use of history remains a minority pursuit for those relatively few pupils lucky enough to have teachers with the confidence or enthusiasm to over-ride these difficulties.

So we need to address these reasons. They fall visibly into two parts. The first two, the lack of knowledge and the lack of supporting materials, is being rectified actively by teacher trainers in collaboration with organizations such as the Summer University, the HPM committee of ICMI, and national bodies in many countries. So in time these need not be major stumbling blocks.

The second two, pressures of examinations and curriculum, are to a great extent outside direct control of teachers and take us to the effects of what are primarily political decisions. This is true even though the proponents of using history might want to claim that deeper understanding would benefit students' examination scores and would make a crowded curriculum easier, not harder, to assimilate. But there remains a problem of perception.

I should observe too that the first two reasons are also the result of political judgements. The increased difficulty, in the UK, in supporting teachers with issues such as historical knowledge and resources arises directly out of political pressures for local management of school budgets. Ten or so years ago the Thatcher government forced local education authorities to devolve budgets down to schools, which has had the entirely predictable consequence that there is less money for things which only larger budgets have the flexibility and vision to accommodate. Teachers who used to be able to apply for central funding, for such things as attending meetings to be informed and trained in using history, now must persuade their headmaster or school manager to divert funding away from repairing the classroom roof. That is just one instance of the rippling consequences of decisions taken for other reasons.

This is not to say that politicians are unaware of the importance of history. In 1999 the UK's Education Secretary, David Blunkett, made an apparent exception to the lack of history of mathematics in the National Curriculum when he advised schools that the wartime exploits of the Bletchley Park codebreakers should be on the history curriculum for all pupils over the age of eight. It was at Bletchley Park, a Victorian mansion in Buckinghamshire, that from 1939 to 1945 a team of mathematicians and other codebreakers, among them Alan Turing, worked with great success to decode German Enigma signals and others. Among the decoding apparatus constructed by workers at Bletchley Park was the world's first large electronic programmable computer, called Colossus. The Education Secretary was impressed with the way in which studying this history could motivate young people. "This is a good opportunity", he said, "to turn them on to something that brings history alive and which makes them think positively about how they might contribute in their own lives." [Carvel 1999, p. 4.] It should be noted, though, that this use of history is for its morale-raising value, not for any contribution it might make to mathematics education.

Overall, we can see that the state of mathematics education, and the role history is allowed to play in that, are deeply political issues. To understand where we have got to and how, and what to do, involves looking again at the political and social influences and constraints on educational policy and practice. I now pull out again the two introductory images, and explore the idea that mathematics education has developed within a tension between humanization and mechanization over the past centuries. I will just give a few pointers to ways in which these show themselves.

4 Mechanization versus humanization

The specific image of the logical sausage machine is interesting because it captured the spirit of the age, and we find Henri Poincaré, once again, using similar imagery in his analysis of current trends within mathematics. Poincaré was rather suspicious of the formalist approach to mathematics, of the school of David Hilbert, and satirized it in these terms:

> Thus it will be readily understood that, in order to demonstrate a theorem, it is not necessary or even useful to know what it means. We might replace geometry by the reasoning piano imagined by Stanley Jevons; or, if we prefer, we might imagine a machine where we put in axioms at one end and take out theorems at the other, like that legendary machine in Chicago where pigs go in alive and come out transformed into hams and sausages. ['Mathematics and logic', in *Science and method*, p. 147].

This kind of critique has been an important thread in mathematics education down the ages. Whether you need to understand what you are doing, in demonstrating some technical skill, is a perennial issue that keeps returning. Its *locus classicus* is perhaps the dispute between William Oughtred and Richard Delamain in the 1630s over whether in order to use a slide rule you needed to be taught the principles of logarithms. Ever since then question has been a recurrent one of how to strike the right balance, in education, between deep understanding and technical facility.

The perception that mathematics is somehow damagingly close to the mechanistic aspects of the age has long been held in some quarters. A famous early proponent of this cultural criticism was the English poet William Blake:

> I turn my eyes to the Schools & Universities of Europe
> And there behold the Loom of Locke, whose Woof rages dire,
> Washed by the Water-wheels of Newton: black the cloth
> In heavy wreathes folds over every nation: cruel Works
> Of many Wheels I view, wheel without wheel, with cogs tyrannic
> Moving by compulsion each other, not as those in Eden, which,
> Wheel within Wheel, in freedom revolve in harmony and peace.

One of the most influential treatments was that of the English novelist Charles Dickens, in his novel *Hard Times* (1854). Sometimes considered Dickens' greatest novel, *Hard Times* is *about* precisely the dialectic of my opening images, the rival claims of humanity and mechanization in education. Early in the book, the schoolmaster Mr M'Choakumchild (who would be *Master Kindkeler* or *M. Etouffélève* in Belgium) is introduced as both a victim and a symptom of the mechanization of education:

> He and some one hundred and forty other schoolmasters had been lately turned at the same time, in the same factory, on the same principles, like so many pianoforte legs. . . . Orthography, etymology,

> syntax, and prosody, biography, astronomy, geography, and general cosmography, the sciences of compound proportion, algebra, landsurveying and levelling . . . were all at the ends of his ten chilled fingers. . . . Ah, rather overdone, M'Choakumchild. If he had only learnt a little less, how infinitely better he might have taught much more! [pp. 52-53].

This message has been well absorbed in the University of Leuven, it is gratifying to notice, whose publicity brochure contains an explicit disavowal of mechanistic *kindkeler* principles: speaking of Leuven's commitment to the broad ethical, cultural and social context of education, the brochure goes on to say that "Rather than passing on mere factual knowledge, [KUL] promotes the skills of identifying, formulating and solving problems. It creates the necessary conditions for a stimulating educational experience."

Dickens' criticism is just one part of a long tradition of socio-cultural awareness from long before him, until now-Charlie Chaplin was making a similar point in his great film of the 1930s, *Modern Times*. The same mechanistic ideology criticized by Dickens in education, and Chaplin in industrial management, fuels the limited imaginations of our present politicians. The havoc being created in mathematics education today, and education generally, through managerialism and market solutions-the pressure towards examining and assessing everything so that what is unassessable drops out of the aims of education-results from just the same attitudes of unimaginative self-righteousness that Dickens noted.

We may ask what the forces of humanization have been doing, whether, apart from criticizing the mechanistic trends, there is any countervailing force with a positive constructive ideology. There is, of course, the whole movement in which we are involved, towards the incorporation of history as a humanizing influence, restoring and sustaining for young people and their teachers some of the important human values in education. But we have other examples from history too and I mention just one, the case of Mary Boole (1832-1916). She is not very well known now, and indeed was never very influential outside a small circle, but English mathematics educators have in recent years increasingly found inspiration in her work, particularly at primary and middle school levels. Mary Boole was the widow of George Boole, the great English mathematician after whom Boolean algebra is named. He died young, in the 1860s, and she lived for another 50 years and contributed a number of books towards developing the mathematical imagination of the child. Her books include such titles as *Logic taught by love*, and *Philosophy and fun of algebra*. These give enough idea, for now, of people who have been striving to promote other than mechanistic values and whose influence within educational circles is a good one.

5 Conclusions

The main direction of these remarks is that there are grounds for guarded optimism. My reading of history is basically a cyclic one: those who forget history are doomed to repeat it. Few are more vulnerable to this inexorable law than the politicians who pride themselves on starting afresh, discarding the past and obliterating it from their memories.

In 1861, for example, nearly 140 years ago, a Royal Commission in the UK reported that the best and cheapest way to extend elementary education was to fund teachers according to the examination successes of their pupils, so-called 'payment by results'. Because this system looked to be very cheap, the government of the day enthusiastically adopted it. In vain did wiser voices urge that it would lead to terrible consequences for the education system: it would lead to a reduced curriculum, where only the minimum that was examinable would be taught; it would encourage the neglect of moral and other dimensions of a rounded education; the status of teachers would be lowered; grants to schools would be reduced; so would local interest in schools;... and on it went. All this happened just as foreseen, and it took 30 years to get the system of 'payment by results' stopped and to repair the damage. Those who notice tendencies in later governments towards payment by results have a responsibility to point out the historical precedents, and hope that a government is not so set on abolishing the very idea of history that it cannot heed the omens.

In his great novel on mechanization in education, *Hard Times*, Charles Dickens foresaw, it is interesting to observe, precisely the consequences of Reagano-Thatcherite policies of the 1980s:

> It was a fundamental principle of the Gradgrind philosophy, that everything was to be paid for. Nobody was ever on any account to give anybody anything, or render anybody help without purchase. Gratitude was to be abolished, and the virtues springing from it were not to be. Every inch of the existence of mankind, from birth to death, was to be a bargain across a counter. And if we didn't get to Heaven that way, it was not a politico-economical place, and we had no business there. *Hard Times* (1854), p.304.

It is the application of such principles to education that has led to some deplorable consequences in current practice, but the lesson of history is still that these things will pass. Mathematics education can indeed learn from its history, but most effectively if those responsible for educational policies will listen.

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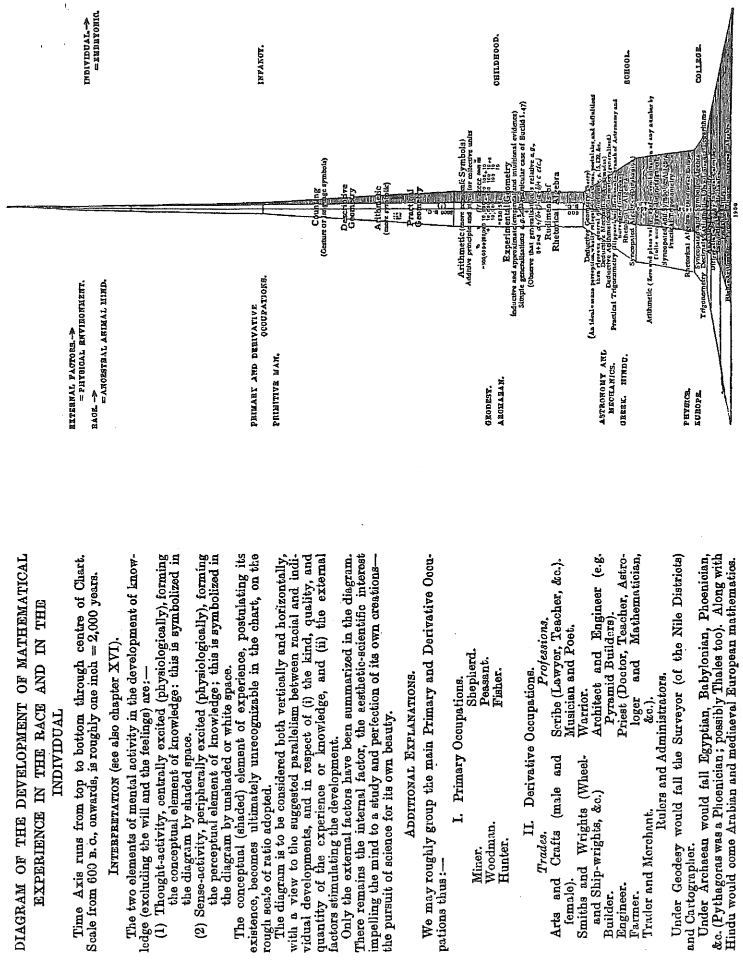

3. [Branford 'Diagram of the development of mathematical experience in the race and in the individual']

Caption Benchara Branford, *A study of mathematical education*, 1908, frontispiece.