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Activity Theory in Didactics of Mathematics

What is taken as shared

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Activity Theory in Didactics of Mathematics
What is taken as shared

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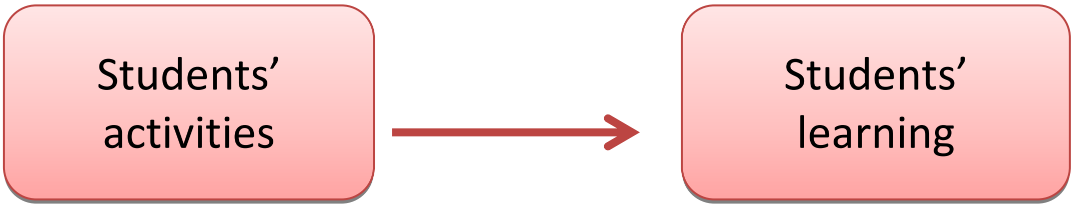
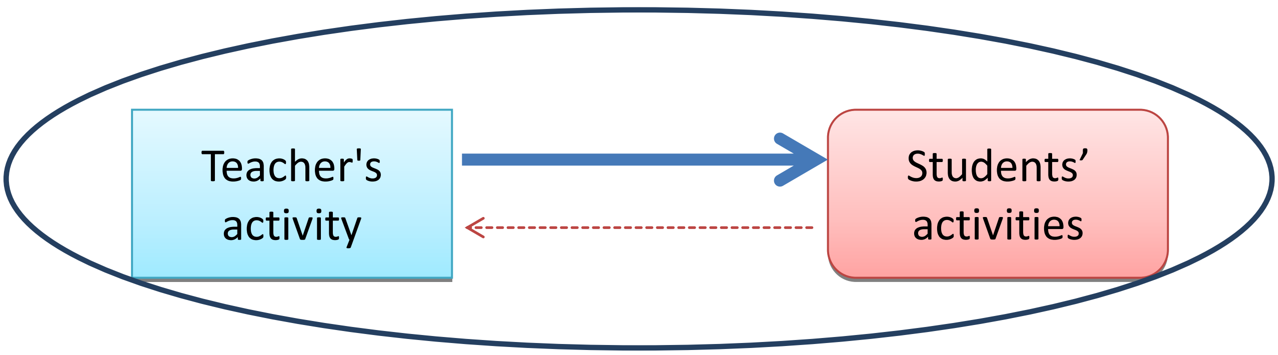
These few pages briefly present the way in which Activity Theory has been adopted for several years now by French researchers in didactics of mathematics and has been adapted to study the learning of school mathematics in relation with the teaching that students receive, as well as teachers' practices. Common general features of the methodology that each one afterwards worked on and continues to develop according to his own research objects, make it possible to specify some principles deriving from well-known learning Theories. Some results are outlined, as well as difficulties and perspectives.

Our research first focus is students' learning, –including the teaching that the teacher is deploying in mathematics classroom (from primary school to the university).

To address this issue, we have chosen to study students' mathematical activities in the class: What the students do (or not), say (or not), write (or not)… As what they think is not directly observable, we work on these activities' observable traces. This theoretical consideration is in line with the Activity Theory approach studying human subjects' activity, in situation, based on the distinction between task and activity. More precisely, it includes the "double regulation" schema of Leplat (Leplat, 1997). These global shared orientations guide our research.



These activities are provoked (to a large part) by the teacher's activities in the working environment of the class.



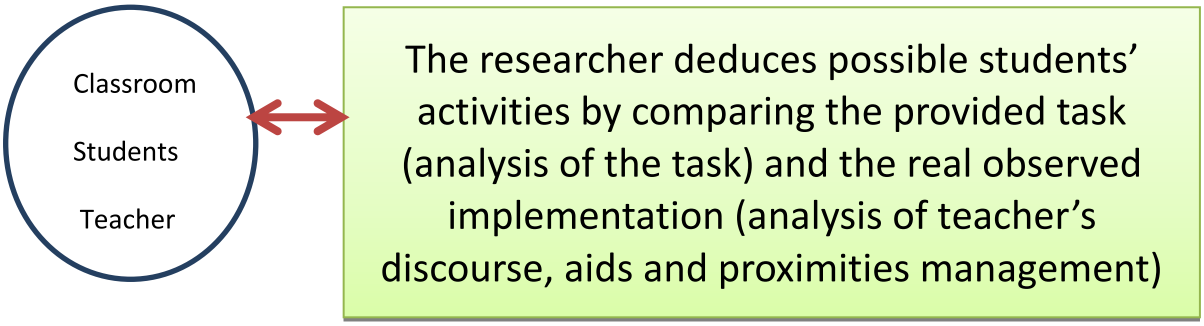
Classroom, working environment

Therefore, our research objects are the connections (schematized by arrows): from students' activities to their learning (even if, actually, it is tied to global hypothesis more than to accurate results) and from the teacher's activities to students' ones (with a little inverse arrow) of course related to the first connection. Our global aim is indeed relevant to give a diagnosis of what occurs in the class or to suggest some new ways of teaching that have to be experimented.

Hence, our approach is both experimental and theoretical, in a dialectical way, involving students' and teachers' observations and data collection, and also data analysis, including possibly new methodological developments.

But how can we access these (partly non accessible) activities and connections (arrows)?

For studying students' activities the general idea is first to analysis provided tasks, which give access to expected students' activities, and to compare them with their possible activities, as deduced of the observed implementation. These activities involve some use of the knowledge to be analyzed. We consider how and when the teacher helps students, sometimes simplifying the task and reducing thus the activity, sometimes developing a discourse near or within the students' ZPD, allowing a larger understanding. During the lecture phases where the students' activities are unobservable, we analyze mainly the teacher's discourse and what we call proximities, which are discourses' elements that could influence the students' understanding according to their own activities in progress.

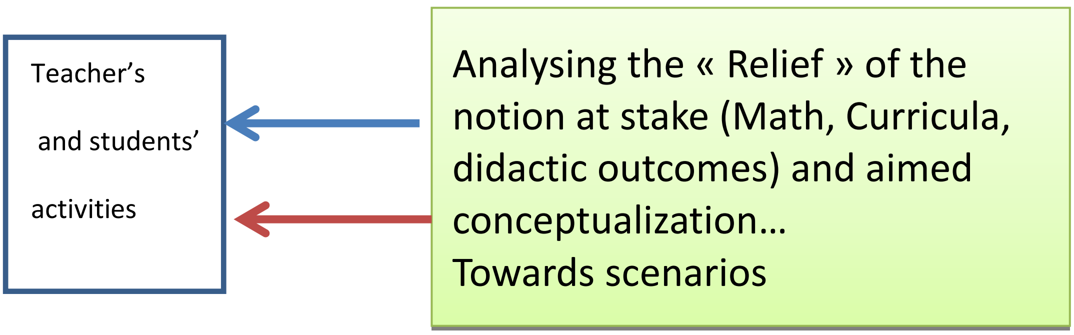


(NB: in the diagrams the blue is used for teachers, the red for students, and the green for researchers)

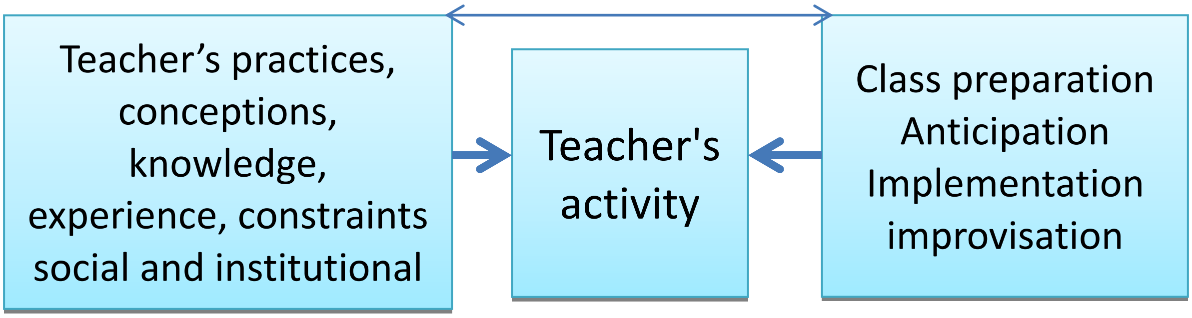
To access the precise aimed Mathematics, to study these activities, we firstly draw up what we call "The Relief" (or landscapes) of the mathematics notion to be taught, combining therefore a threefold studies of this notion: epistemological, curricular combined with the already known difficulties of students regarding this notion. These studies lead us to differentiate some categories of notions. For instance, there are some notions far away from the actual knowledge or skills of the students, because they involve a new specific Formalism, Generalizing and Unifying the previous knowledge. These notions, called FUG, are supposed to be difficult to introduce as there is not preliminary and affordable exercise to prepare the generalization.

Let us notice here that the already known French theories called TSD and TAD (and TACD) offer the same kind of mathematics analysis (involving our shared research principles about the primacy of the study of the contents to be taught).

The set of provided tasks on a notion to be taught is called scenario – including the lectures and the assessments. It is linked to the aimed level of conceptualization, developed into a set of tasks and some knowledge. We distinguish knowledge that is expected to be available, that is to say that students are able to use it even if it is not explicitly mentioned from knowledge that need to be mobilized. Some concepts are involved, in a formal way, as objects. For their use, as tools, wee specify the degree of formalism to be used as the expected degree of rigor and proofs and the way the new knowledge is involved with the actual knowledge (cf. Vergnaud 's *champs conceptuels*).



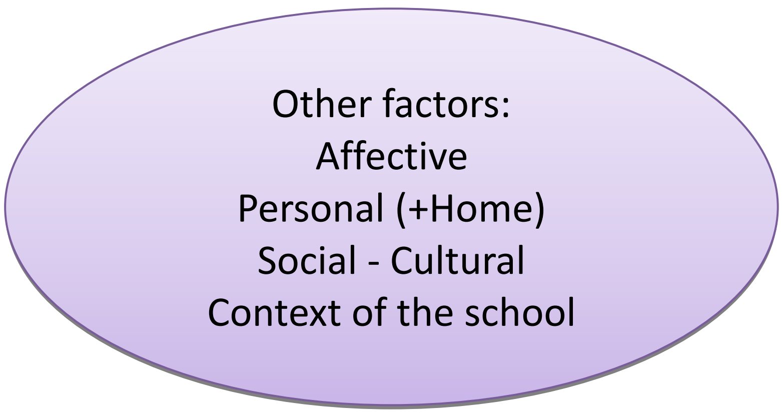
But if the teacher's activity includes what is done before the class (including some anticipation) and during the class (including some improvisation), these elements are not sufficient to understand the teacher's choices. They involve also the professional experience, the knowledge and personal conceptions of the craft and of the mathematics to be taught. We have to take into account also the way the teacher logs into institutional and social constraints (such as curricula, school's social environment and so on). This conception of the complexity of the teacher's practices characterizes the didactical and ergonomic "Double approach" (Robert & Rogalski, 2002). This approach considers 5 components of the practices to interlink: two of them related to the choices of contents and implementation, two other related to the way the teacher takes into account social and institutional constraints and a last personal one, related to knowledge, experience and representations.



We add three levels to study the organization of the practices, which are related to each other. The global level involves the projects, the class designs, and so on, the local one involves what happens in the class (implementation, improvisation..), and the third one, micro level, is devoted to the automatisms and routines. It particularly helps us to study the young teachers' practices, as their global and micro levels are still almost "empty" (to say it quickly).

But even if we are convinced of their importance, we do not systematically study other parameters, tied for instance to affective factors, self-confidence, social and cultural origin. However a lot of research is devoted to the study of disadvantaged classes or schools at the elementary level.

Taking these factors into account would for example involve for the students 'activities some levels of organization, as the global posture according to the school, including the expectations and the relation to knowledge, the local attitude in class, including the participation to collective activities, and the micro level including some automatisms, for hearing for instance.



Methodology and theoretical adapted elements (cf. second schema at the end)

First: the Activity Theory inspires global hypotheses orienting the research, on what is to be studied; the activities of the students and those of the teachers are considered in their complexity.

This leads us to study, as a whole: the tasks and the lesson in progress.

We have developed different kinds of analyses. We first characterize what the student is confronted with in his activity: this is the task analysis – inspired by Vergnaud and completed by different possible uses of the knowledge. Then we have to compare the students' expected activities as detected by the task analysis and what occurs in the class during the tasks' implementation to deduce the possible student activities.

The implementation's analyses involve the way and the time the students are at work, the teacher's comments and helps, the teacher's proximities… It leads us to take account of the possible students' activities: according to the students' working forms and discourses, the teacher's comments, the teacher's helps….

These analyses involve specific indicators and tools, according to the mathematics and the scholar environment. Many of them have been used for years by researchers in didactics. Some of them are in development, for instance to study more accurately the class language, or the technology environments.

These indicators and tools that we use for the analysis of tasks and implementations, often at a local level, are consistent with the theories (complementary insofar as this does not introduce contradiction, but rather articulations): as if we converted the theories to an operational level in order to adapt them to our field of study...

According to Piaget's theories, we adopt the hypothesis of the importance of desequilibrium-equilibration processes. In our context, we suggest that the changes of mathematical frames or registers may be associated with such phenomena. The mathematical usual frames in which students work are the numerical frame, the geometrical frame, the graphical frame, the algebraic one, and then the functional frame. The registers are associated with written representations. The goal for the researcher is to find tasks inducing them. Regarding the importance given to the assimilation, accommodation and abstraction phases, they are detected from the tasks – according to the expected adaptation of the knowledge. Simple tasks do not require accommodation; students need only to apply a general knowledge in a particular exercise, often substituting variables by the values at stake. Other tasks, more complex, may include more adaptations, as recognizing the knowledge to be used, or organizing the resolution with some steps for the calculus, or choosing some specific way of reasoning, or introducing an intermediary notation or an interpretation for previous results inducing changes of frames or registers. These complex tasks entail different activities, contributing to the targeted acquisitions; it may concern the recognition of the knowledge at stake, and / or the organization of task solving, and / or the treatments to be carried out. The search for tasks in which students can work alone, with the possibility of self-control, before the formal and general introduction of a definition or property that decontextualizes them is also a translation (which is particularly developed within the TSD). The generalization from an exercise is usually at the teacher's

charge. Our studies refer also to the quality of the generalizations, even if not directly inspired by the different Piaget' forms of abstraction.
It is a Vygotskian inspiration that pilots the analysis of the teacher's wordings and comments... It relies on the importance of the teacher's and students' mediation in the class. We attempt to operationalize the ZDP notion, detecting all the real or expected occasions of relying on the actual knowledge or skills of the students to make them go further (cf. Bruner). In particular, for the math taught in the classroom, it is possible to try to specify what may be covered by the notion of ZPD and in what extend the teacher can use it, which mediations (see Vygotsky and Bruner), including the teacher's silences (see Piaget for autonomous work) are possible. The proximities in act characterize the attempts of alignment that the teacher operates between what has been done in class and what he/she wants to introduce. We therefore study the way the teacher organizes the movements between the general knowledge and its contextualized uses: we call ascending proximities those comments which explicit the transition from a particular case to a general theorem/property; descending proximities are the other way round; horizontal proximities consist in repeating in another way the same idea or in illustrating it.

The *a maxima* activities express the activities that only certain students do, and that are not shared by all - leaving thus some students, at least partly, out of the field worked by the others. The *a minima* activities are done by all, but for this purpose the corresponding tasks are often reduced by the teacher and therefore the knowledge involved by them is also reduced.

From local analyses to global ones

Indeed, The factor" time" seems to be missing in our schemas that describe more local analyses than global ones. It is true that local analyses are more frequent in our research than global ones.

Actually, for teachers, global analyses are possible thanks to the practices' stability[1] (shown by several of our research studies), ensuring therefore the validity of the extension of our local outcomes. It enables us to reconstruct some global logics of the teacher's actions, beyond the practices 'complexity. The comparison of the practices of several teachers leads us to go beyond these descriptions by expanding the range of options and alternatives. The latter allow us to propose a certain variability of practices, at the intersection of observed practices and the didactic qualities of supposed "robust" scenarios, targeting thus teacher professional development.

Expanding the range of teachers' options (in terms of choices of contents and implementation)
& professional development

[1] For experienced in-service teachers, the component tied to the implementation choices seems particularly stable.

We have extended our approach to the teachers' training, broadening the notion of ZPD, for the students' learning into the ZPDP for the practices' development. We conceive the training scenarios basing the collective work on real practices. It leads to settle new questionings, awareness, discussions on the teacher's choices linked to the contents and the implementation. It also leads us to work on the way of transposing some of our research analysis tools to educators and teachers.

For the students, it is more difficult to infer global results from our local analysis. We have only hypothesis on the quality of the scenarios. We suggest that the recurrence of teaching ways is an important factor for students' learning.

Some results

In terms of results, we can stress obtaining a range of analyzed practices, related to different mathematical domains (numerical and geometric in primary school, algebraic and functions in secondary school) and to the use of technology environments.

A number of introductory activities and even of robust scenarios (in particular, on orthogonal symmetry) have been proposed.

Unexpected difficulties of students have also been brought up to date. Some of them are related for example to knowledge adaptations to be used by students when solving exercises. In some case these adaptations are not detected by teachers and are left implicit. We talk about teachers' "naturalizations" when it is as if these adaptations are too familiar to teachers to be located. Often some students ask questions on these implicit adaptations, especially since the class is diverse. But if not, they may be overlooked and this likely blocks some students, even for a long time as it is often repeated. In elementary geometry for instance, some changes in points of view may be unintelligible, and thus have to be explained, for example, between right angles and perpendicular lines. Other implicit elements may occur, for instance about the curve / function link, and proximities might enlighten the expected transfer, making easier the meaning of the algebraic point of view.

Our research also enables to propose a critical view of curricula and institutional instructions. Tensions may exist between diverse expected rigor requirements needed for the different contents that have to be taught during the year. In order to give sense to mathematics, the institutional injunction makes the students to work on complex tasks. To solve these complex tasks, students use diverse procedures. This diversity makes difficult for teachers to highlight the knowledge and the path that have been used. .

A set of published books and articles (see bibliography) present these results.

Some issues and difficulties

In addition to the shift from local to global mentioned above, the difficulties of our research are due in particular to the time they take and the large amount of data to be gathered, and all the variable parameters involved.

Moreover, the "appropriation" that we did of the ZPD must be specified, insofar as this notion is related to individuals whereas we use it in the context of a class.

In general, validating this type of research is not simple, whatever the framework is. Researcher-teacher relationships, which are essential during the experimental phases, are not always transparent, and also require some clarifications.

Recent developments and perspectives

New developments in this theoretical frame are conceiving tools to better target the distance between what students do and / or know and the teacher's actions and mediations (proximity-in-action). But also studying the moments of knowledge exposure through the development of analyzes in terms of discursive proximities, in order to appreciate opportunities for possible or even missed proximities between what is general and stated by the teacher and what the students already know or do. The specific analyzes of activities with technological tools allow access to what is new in terms of working on these instruments, both for the teacher and for the students and to provide the means to take more account on it. Other developments are about the practices related to assessments, to collaborative research and the clarification of roles, to training and support of school teachers in very disadvantaged classes.

Before concluding this overview of our frame, one could ask what does it brings in addition of other frames well known in didactics.

By giving *a place to subjects (students and teachers) in their singularity*, this framework is specifically adapted to study what effectively happens in class, whether practices are ordinary ones or not.

Other complementary frameworks, such as TDS, are particularly concerned with the design of learning situations of which the implementation has to be studied. Some evolution occurred in the research so that now the ordinary classes and the resources production are also studied. But the main aim remains to study the cognitive potential of a given situation, that is the study of what the students may learn according to the contents' choices. This leads to the identification of what could be due to the milieu (present in the situation independently of the teacher), but also the didactic variables that enable the teacher to play on possible student activities[2]. The didactic contract is used to specify the expectations, explicit or not, of the teacher and the students towards each other, and considering it, make possible the understanding of what could distort or reinforce the activities in which the pupils are engaged. This induces a conception of the actors as generic subjects, having a special function (student, teacher) rather than as singular, active subjects as is it the case in our theory.

The TAD, concerns more a global vision of the education system and the profession (for mathematics), emerging from existing constraints and norms. The mathematical reference analyses are based on the identification of the involved praxeologies, ranging from the types of tasks and techniques to the technologies in use and the theories on which they are implemented. Moreover, the phenomena identified are related to different levels of determination, ranging from class to society. This leads to a conception of the actors as subject to a given institution, and, again, not as singular actors.

Finally, it is important to be aware of the fact that theories, whatever they may be, are by no means a sort of enclosures but rather guarantors. It is important to use them to guarantee a certain

[2] Here the word is used in an ordinary meaning, as action.

coherence in the division of the observed reality, but also to identify what could be unexpected, and even to know how to transform what first appears as a "disturbing noise" into a new development (for example, the notion of FUG was conceived, because we could not find a "good" situation to introduce the concepts involved in terms of tool / object dialectic).

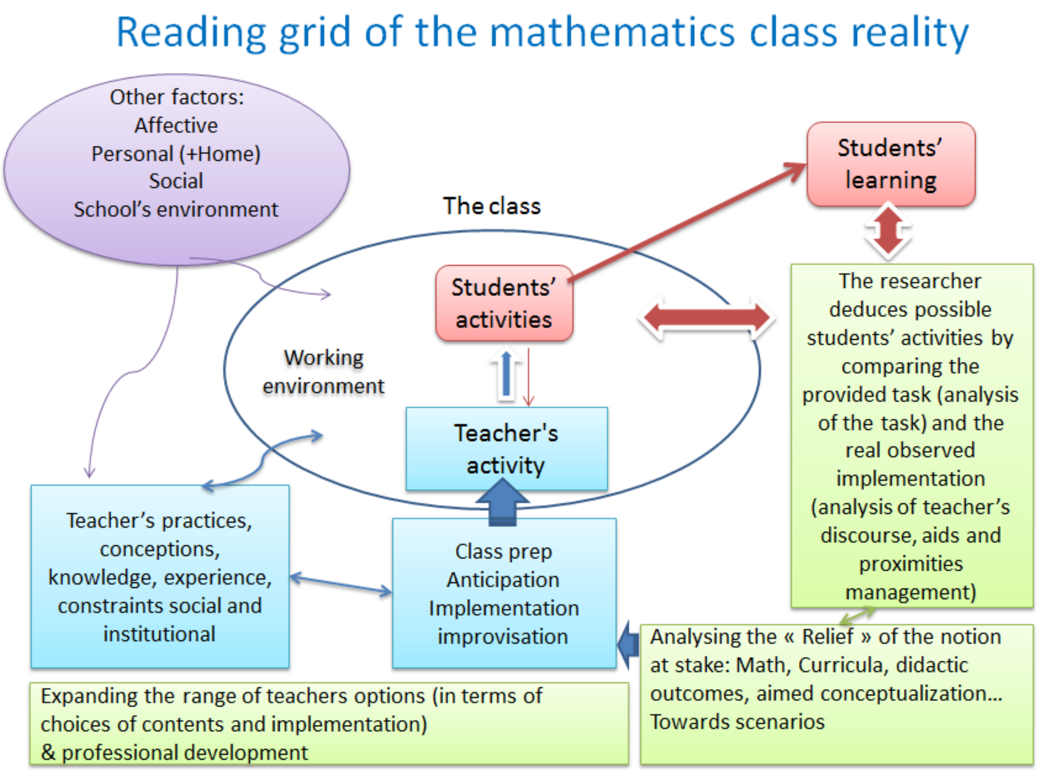
Likewise, if data gathering must be adapted to the theoretical frameworks, this, fortunately, may still produce unexpected phenomena... opportunities that the research has to grasp!

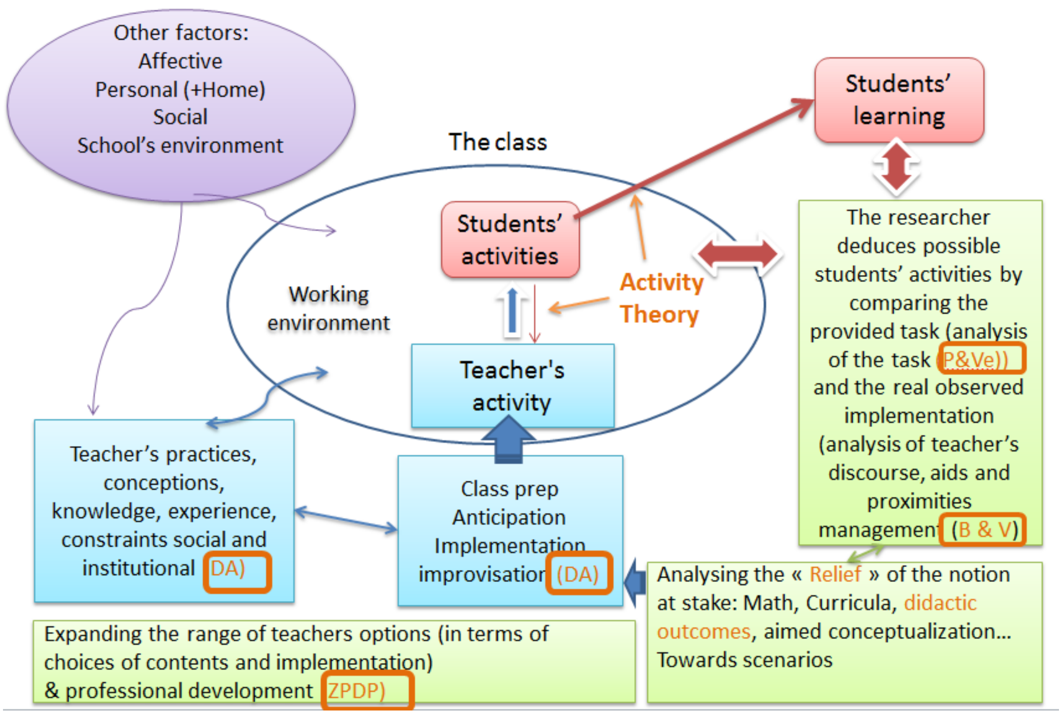
And in other teaching/learning fields? An open-ended question ... Towards an activity theory in didactics of mathematics and sciences?

The approaches presented above have been developed in the context of mathematics. The issue of their relevance to other areas of teaching / learning is raised and has already received positive responses in some research, including teaching practices (Kermen, 2016), of course, taken into account the epistemological differences and specificities of the area to which they were extended.

Moreover, just as the notion of a conceptual field was developed by Vergnaud for mathematics - of the elementary school - and then widely used in research on teaching / learning in science, the concept of "relief" could be relevant for other fields of science. But it is possible that there exist some elements that we do not analyze in mathematics and that should have to be taken into account. For instance, in physics, it is sometimes useful to consider "physics of common sense" that does not only concern the primary school teaching and learning. More generally, the relation with the reality of experiments and theories in physics or chemistry (differing according to subjects) does not have its counterpart in the mathematics on which we have worked.

A global Diagram, synthesizing the above developments, with a version including theoretical elements





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