

ERRORS DEALING WITH THE NEGATIVE IN SOLVING QUADRATIC EQUATIONS. AN EPISODE IN THE HISTORY OF ALGEBRA AND ITS TEACHING

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The codex Dresden C80 belonged to Johannes Widmann who used it when he lectured on algebra at the University of Leipzig in 1486. Among the documents it contains there is a *Latin Algebra*, Wappler (1887), in which the rule for solving one of the types of second degree equations ($ax^2 + c = bx$, in current terminology) includes an error. The rule is equivalent to

$$\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

what is correct. The error appears in a prompt at the end of the rule: “if you can’t subtract $[c/a]$, you are allowed to add” (Wappler, 1887, p. 14).

This authorization to add instead of subtract also appears in Bombelli’s *L’Algebra*, 1572. After having solved the equation $x^2 + 20 = 8x$ by giving its two imaginary solutions, Bombelli says that “there is another sophistic method, that since 20 cannot be subtracted from 16, it is added”, and he obtains a third solution of the equation, 10, and adds “and this 10 is minus” (Bombelli, 1572, pp. 262-263), without any explanation.

This is not the only error in the rules to solve this type of equation. In a manuscript before 1504, *Die “Algebra” des Initius Algebras* (Curtze, 1902), the rule is stated correctly, but when applied to an example, the order of subtraction is wrongly reversed.

$$\sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}} \pm \frac{b}{2a}$$

And in Adam Ries’ manuscript *Die Coss* (1527), edited by Rüdiger, Rainer & Folkerts (2023), the rule itself is stated with the subtraction erroneously reversed.

Both errors appear in Marco Aurel’s *Arithmetica Algebraica*, published in Spanish in 1552. He uses the erroneous rules in an example and a problem and explains how to obtain the correct result from the erroneous one or how to interpret the wrong result as correct. Its influence on other books written in

Spanish, and especially the success of one of them, Pérez de Moya's *Arithmetica Practica y Especulativa*, published in 1562 and reprinted a lot of times until 1798, made these errors last long in Spain.

In this communication we study the statements of these erroneous rules, their eventual use in examples and problems, the justifications for the way of obtaining the correct result from the incorrect one or interpreting it as correct, and their dissemination.

REFERENCES

- Bombelli, Rafael (1572). *L'Algebra, parte maggiore dell'Arimetica, divisa in tre libri*. Bologna: Giovanni Rossi.
- Curtze, Maximilian (1902). Die "Algebra" des Initius Algebras ad Ylem Geometram magistrum suum. *Abhandlungen zur Geschichte der mathematischen Wissenschaften XIII*, 435-611.
- Rüdiger, Bernd; Rainer, Gebhardt & Folkerts Menso (Eds.). (2023). *Adam Ries, Coß I*. 2 vols. Annaberg-Buchholz: Adam-Ries-Bund.
- Wappler, Hermann Emil (1887). Zur Geschichte der deutschen Algebra im 15. Jahrhundert (pp. 1-32). In *Gymnasiumzu Zwickau. Jahresbericht über das Schuljahr von Ostern 1886 bis Ostern 1887*. Zwickau: R. Zückler.