

HOW TO LEARN JAPANESE MATHEMATICS “WASAN” IN THE EDO PERIOD (1603-1868)

Noriko TANAKA

Naragakuen University, 3-15-1, Nakatomigaoka, Nara-city, Nara, Japan
norikotanaka91@gmail.com

ABSTRACT

There were several schools of mathematics in Japan during the Edo period. Each school had its way of writing formulas and expressing mathematics differently. There are few written records of how students learned mathematics during the Edo period, but one small school, the Shisei-Sanka school, has left behind a rare book describing how students learned.

I will introduce the mathematics studied there and the problems published in the *sangaku* (votive mathematical tablet) to show how the Japanese people were familiar with mathematics then. In addition, each school had a license with several levels of teaching mathematics. One of Licenses is also presented.

1 Introduction

There were several schools of mathematics in Japan during the Edo period (1603-1868). The most famous and popular school was the Seki school of the renowned mathematician Seki Takakazu [関孝和](?-1708). In addition to the Seki school, there were various other schools such as Takuma school, Shisei-Sanka school, Saijyo school, Miyagi school, Omura school etc.

I would like to describe how to express mathematics in Seki school, Takuma school and Shisei-Sanka school. And I also introduce how to learn mathematics, a license and a *sangaku* (votive mathematical tablet).

2 How to express formulas in different schools

In this section, I discuss the representation of mathematics in the three schools.

2.1 Seki school's way of writing formula

The Seki school's way of expressing mathematics is inspired by the motif of *Sangi* [算木](arithmetic sticks), which was introduced to Japan from China.

The sticks were the tools used for calculation (fig. 1). The arithmetic sticks were placed vertically for the first, hundred, and ten thousand places and horizontally for the tenth, thousand, and hundred thousand places. The red blocks represent positive numbers, and the black blocks represent negative numbers. For example, 231 and 5089 are positive numbers, so the red arithmetic sticks are used, and -407 and -6720 are negative numbers, so the black arithmetic sticks are used (fig. 2).



Figure 1. Representation of numbers by arithmetic sticks

231			≡	
5089	≡		⊥	≡
-407				≡
-6720	⊥	≡	=	

Figure 2. Example of representation of numbers by arithmetic sticks

When writing in books, negative numbers were represented by diagonal lines to express them in black ink only. In addition, zero is expressed by ○. For example, II 一○ represents 210. Figure 3 shows the pages of the famous book *Katsuyō Sanpō* [括要算法] (Summary Method) (1712), which describes Bernoulli numbers and binomial coefficients. The number of arithmetic sticks were written.

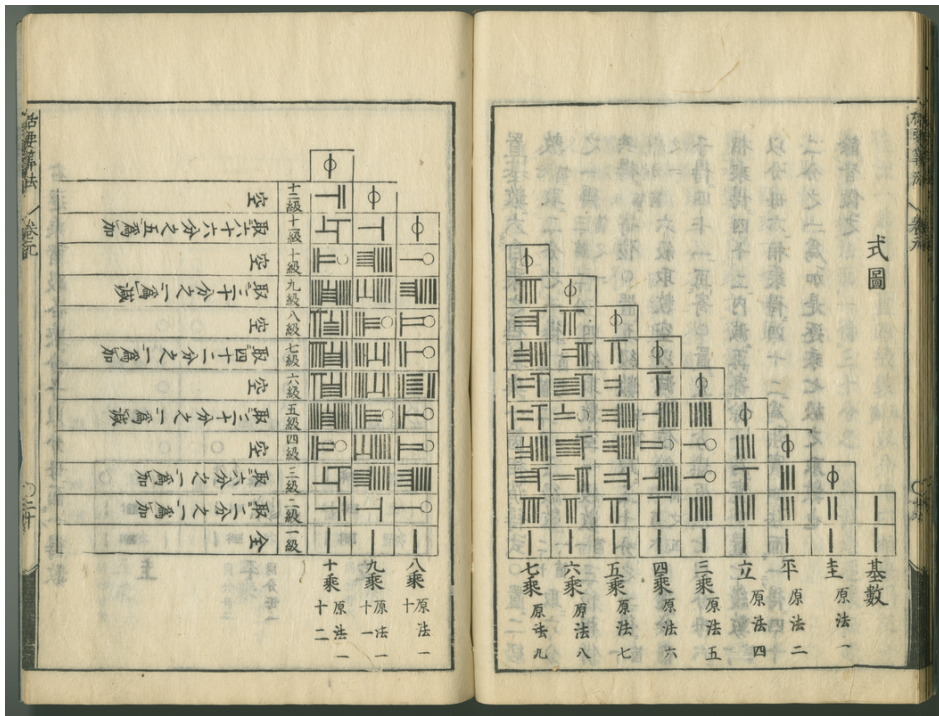


Figure 3. *Katsuyō Sanpō* [括要算法] (1712)

These pages were described Bernoulli numbers and binomial coefficients.

2.2 Takuma school's way of writing formula

Takuma school was a school founded by Takuma Yoshikiyo [宅間能清] in Osaka. Unlike Seki school's way of writing formula, Takuma school does not use numerical expressions of arithmetic sticks. Negative numbers are written as *fu* (フ) in Japanese katakana [カタカナ], and numbers are one(一), two(二), three(三) in Japanese kanji [漢字].

For example, Figure 4 shows

1 上矢(upper arrow) - 1 小円(the diameter of small circle).

ラ is (-1), in other words, it is said to be 1 negative.

Figure 5 is *Myōyoku Shū* [妙矩集] (Exquisite

formulae) No.6 of Takuma School's Collection [起術解路法]. The following is a description in modern notation.

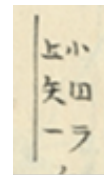


Figure 4. *Myōyoku Shū* [妙矩集] No.6 of Takuma School's Collection [起術解路法].

As shown in the figure (fig. 5), there are two circles C and C' , and C' is inscribed in C . Line L passes through the center of circle C , and 2 lines L and M intersect in circle C . L and M are perpendicular. Also, z is the diameter of circle C' . When C' is tangent to M , answer the following questions for the figure's lengths a , b , x , y , and z .

When 上矢= x , 下矢= y , 小円(the diameter of C')= z , 丑= a , 寅= b , 子= c , the lower right side of Figure 4 is written $1(x-z)y=1b^2$, and the lower left side is written $1yz=1ac$.

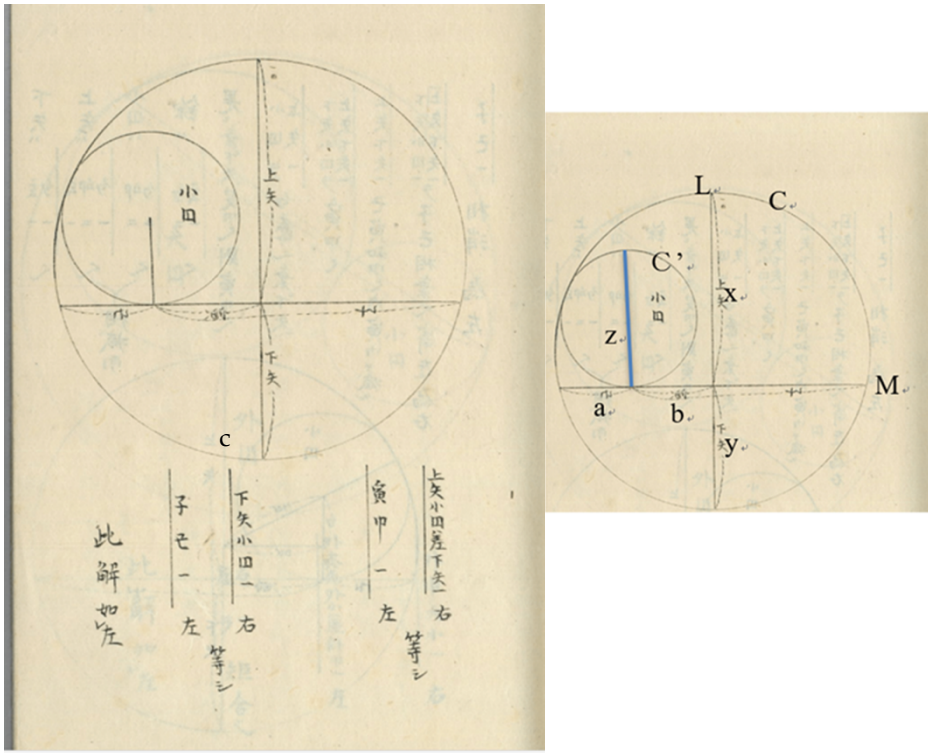


Figure 5. *Myouku Shu* [妙矩集] (Exquisite formulae) No.6 of Takuma School's Collection [起術解路法].

2.3 Shisei-Sanka school's way of writing formula

Shisei-Sanka [至誠賛化] school was a school founded by Hurukawa Uji-kiyo [古川氏清](1758-1820). The meaning of the phrase "Shisei-Sanka [至誠賛化]" is to agree that heaven, earth, and nature create and nurture all things if one is sincere in one's heart.

Fig. 6 is one of the problem of *Kugou-Ruihen* [矩合類編] (a collection of equations found in geometric figures).

(Length of one side of the square) = a ,
 (length of the diagonal) = b ,
 then $\sqrt{2}a - b = 0$

The expression of the formula is almost the same as the expression of Seki school's formula.

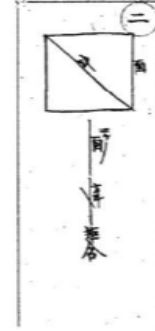


Figure 6. 矩合類編 of Shisei-Sanka School's Collection.

3 How did students learn mathematics during the Edo period?

Few written records exist of how students learned mathematics during the Edo period, but one small school, the Shisei-Sanka [至誠賛化] school, has left behind a rare book *Kiou shu* [淇澳集]. The following is “how to learn mathematics” as written in the *Kiou shu*.

Rule

1. When you hold up a question on a tablet wooden board to a particular student, ask that student by name. When you ask the whole group a question, you should write: 'Ask this to everyone'. Students should have approximately one to two months to solve and answer the question, depending on the complexity of the answer the question requires.
2. If no one can answer a question presented to all students, then all students have lost. Therefore, try your best to solve the math problem and answer first. However, if you cannot find the answer, ask the teacher for the solution.
3. The student should not rely on the help of others, either in the formulation of the problem or in the formulation of the answer. The best way to solve a problem is to use *Tenzan* [天竄]¹, and the second best way to solve a problem is to use *Tengen jyutu* [天元術]². However, since the degree of mathematical training varies from person to person, we do not insist on this limitation.

¹ *Tenzan* [天竄] is a symbolic manipulation technique for solving equations with more than one unknown.

Solve the problem with ingenuity by following the above rules. It is strictly forbidden to discuss our school with those of other schools. Therefore, we do not allow the questions of different schools to be included in the wooden boards of this school. (Jan.1808)

This instruction shows that mathematics was kept secret by each school. Each school had its way of writing formulas and expressing mathematics differently.

The flower arrangement and tea ceremony in Japan today have their schools, and each school has its style of flower arrangement and tea ceremony.

In mathematics in Japan during the Edo period, different schools had different ways of writing mathematical formulas, so it was probably not a good idea to learn from other schools.

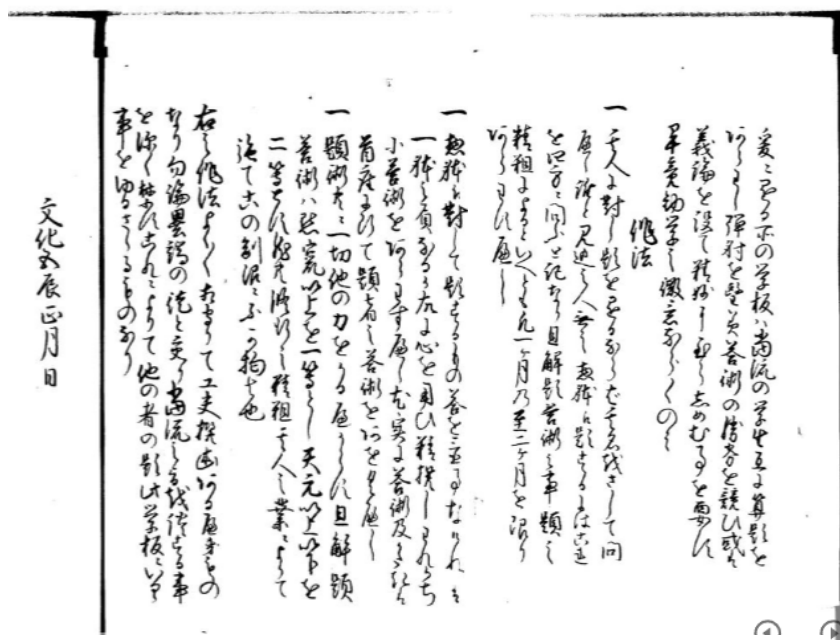


Figure 7. Kiou shu [淇澳集]

The part where the rules for learning are written.

² Tengen jyutu [天元術] is a method of representing and solving equations of higher order using an arithmetic sticks.

4 License

A license was required to teach Japanese mathematics in the Edo period.

Fig.8 is a license for Takuma School of Mathematics (private collection, 1852). A person who is granted a license can teach mathematics. Depending on the type of license, the content of mathematics that could be taught differed. Some licenses allow teaching only basic mathematics, while others enable teaching complicated mathematics. Note that the license is written vertically and reads from right to left. In addition, the license is kept wrapped up.

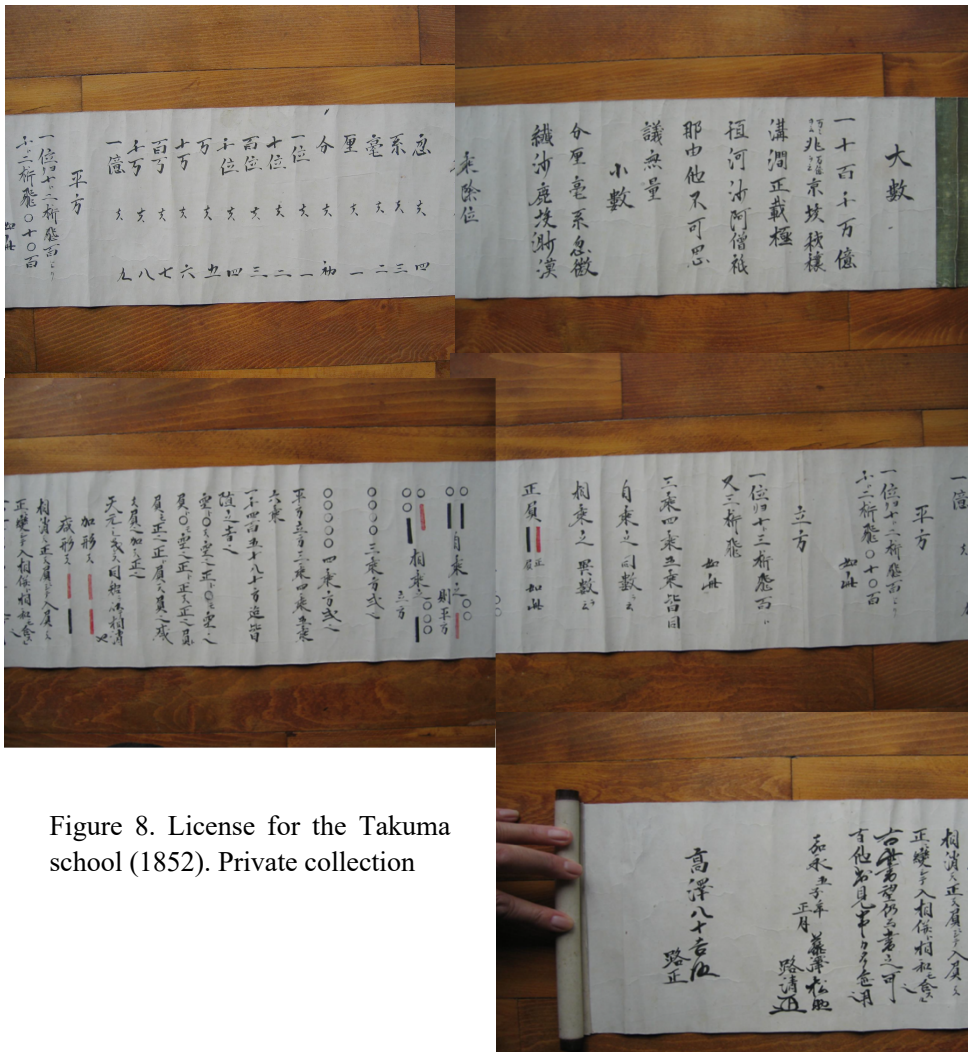


Figure 8. License for the Takuma school (1852). Private collection

5 *Sangaku*

When students solved an excellent mathematical problem, they thanked the gods or Buddha, wrote the problem and the formula for solving it on a wooden panel called *sangaku* (votive mathematical tablets), and dedicated it to a shrine or a temple. In the Edo period, shrines and temples were the most crowded places, not only for festivals, celebrations, and funerals, so by displaying *sangaku* there, students could show many people the problems that could be solved. The names of the students and their schools were written on the dedicated *sangaku*, which would have promoted the schools.

The geometrical figures on the *sangaku* were painted in colorful colors. The *sangaku* are still preserved in shrines and temples in Japan today.

In January 2024, we visited Zenkoji Temple in Nagano Prefecture to see *sangaku* [320cm×133cm] (fig.9). This is relatively large in size. This *sangaku* contains five problems, five geometric figures in colorful colors, with the problem, the answer, and the short explanation for the answer written below each figure. The first large kanji characters on the right side of the *sangaku* are “dedication[奉納],” indicating that the *sangaku* was dedicated to the gods. It was considered beautiful and excellent to use a small number of letters to write on *sangaku*.



Figure 9. Seki school's *sangaku* Zenkoji Temple 1833. Photographed by author

6 From the Edo to early Meiji (1868-1912) period

The many versions of the Edo period mathematics book *Jinkōki* [塵劫記] became a huge bestseller. The people of Edo learned mathematics by reading the *Jinkōki*.

The pace of Western mathematics introduction quickened as the Edo period drew to a close. However, it's important to note that Western mathematics introduced during this period was not in its pure form but rather as a tool for specific applications or practices. The more advanced branches of European mathematics, such as calculus and algebra, were yet to be systematically integrated. From the Meiji period onward, Japan promoted modernization on all fronts, following the lead of the West. A mathematics education policy was instituted based on Western mathematics rather than traditional Japanese mathematics.

This meant that many *Wasanka* [和算家] (Japanese mathematicians) could teach elementary Western mathematics if they could learn it. In other words, many people who knew Japanese mathematics were teaching at elementary schools in the Meiji era. Japanese mathematics has quickly abandoned the more than 200-year tradition of Japanese mathematics. Facing such a situation, I wonder if those involved in Japanese mathematics felt no lingering regret for their Japanese mathematics. Some Japanese mathematician expressed their displeasure with the penetration of Western mathematics.

Despite the discontent expressed by some Japanese mathematicians, the trend towards the adoption of Western mathematics persisted. This trend, rooted in the core support of science and technology, underscored the enduring influence of Western knowledge on Japanese education during the Meiji era.

Japanese mathematicians who also studied Western mathematics from the end of the Edo period to the beginning of the Meiji period were among the first to realize the value of Western mathematics. It did not matter whether the mathematics was Japanese or Western as long as it was useful. Those who promoted the introduction of Western mathematics also studied Japanese mathematics. The shift from Japanese to Western mathematics was an inevitable movement that accompanied a general policy shift in science and technology.

7 *Teaching WASAN in the High School Mathematics Classroom*

The history of mathematics in Japan is not clearly positioned in the curriculum. However, almost all mathematical textbooks provide students with access to the content of traditional Japanese mathematics, "WASAN". Teachers do not deal with it much. I would like to describe a class in which Japanese high school students solve problems by looking at the original text.

I dealt with the problem in Figure 5 (*Myouku Shu* [妙矩集] No.6 of Takuma School's Collection [起術解路法]).

The Math lesson is as follows.

- 1 Target group: 20 interested third-year students of a certain Japanese high school
- 2 Timeframe: 1 Lesson, December 2022
- 3 Contents (Lesson Flow)
 - 1) The teacher handed out copies of the book of the Takuma School and explained about mathematics in the Edo period.
 - 2) While explaining how to read mathematics in the Edo period, the teacher wrote problems on the blackboard in the modern writing style.
 - 3) When the teacher finished writing the problem in the modern writing style, the students thought about the problem.
 - 4) The teacher wrote the answer on the blackboard in the modern writing style in the book.
 - 5) Since the students did not understand the answers, the teacher discussed the solution with the students and they all understood the solution together.
- 4 Student's Impressions
 - 1) This was the first time I saw mathematics written vertically.
 - 2) Just by looking at the problem, it was too difficult to solve.
 - 3) It was difficult to understand the solution written in an Edo period book because it did not describe the way of thinking in the middle of the problem.
 - 4) People in the Edo period may have been amazing in their ability to solve plane geometry problems.
 - 5) I would like to see other problems.

8 Summary

In Japan during the Edo period, there were various schools of mathematics, and different schools had different ways of writing mathematics. In this report, we introduced literature describing “how to learn mathematics” at that time, licenses, and *sangaku* dedicated to shrines and temples. In addition, we discussed how Japanese mathematicians viewed Western mathematics from the Edo to early Meiji period.

Few studies have been conducted on Japanese mathematics in the Edo period from an educational perspective, and I wanted to introduce it internationally this time.

In the future, I would like to deepen my consideration of the spread of the schools of Japanese mathematics during the Edo period and why the Seki school, which was the largest, could not unify all the schools in Japan. I would also like to continue my research to see if there are any other documents written about education at that time.

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