HOW CAN THE HISTORY OF THE EXISTENCE OF FOURTH PROPORTIONALS FROM EUDOXOS VIA OMAR KHAYYAM AND NASIR AL-DIN AL-TUSI TO ISAAC NEWTON FOSTER A MODERN MATHEMATICAL NUMBER CONCEPT?

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A concept development towards a modern understanding of real numbers is explored using the historical genetic method. First, we consider the concept of the fourth proportional (given magnitudes *a* and *b* of the same kind and another magnitude d, then there is a magnitude *c* of the kind like *d*, such that $a : b = c : d$) at the hand of excerpts from book V, attributed to Eudoxos, in Euclid's Elements. Then we discover and discuss situations in contemporary school textbooks in which the existence of fourth proportionals is assumed and used. We look at these textbooks through the eyes of Eudoxos and Omar Khayyam (1048–1131). In Euclid's book V, we study definition 5 of the equality of two ratios and compare it with the "antiphairetic" definition based on the Euclidean algorithm. With Khayyam, for us (positive real) numbers are nothing but proportions of magnitudes that, surprisingly, can be calculated with if at least the calculation laws are proven, which Euclid and Khayyam partly did. The proof of the is in Euclid's book V and later in Nasir al-Din al-Tusi (1201–1274). In the early modern period, the concept of number as a proportion of magnitudes was anew clarified by Isaac Newton (1642-1726) and later attributed to him (cf. Frege, p. 25). The existence of the fourth proportional can serve as "Grundvorstellung" (mental model) for the arithmetic of real numbers, which we try to develop together with the participants along the historical perspectives such that the educational potential becomes visible.

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