

NEWTON'S DE ANALYSI VS FUNDAMENTAL THEOREM OF CALCULUS

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In *De Analysi*, Newton derives three primary achievements of modern calculus: the area under the curve $y(x) = x^{\frac{m}{n}}$ equals $\frac{n}{m+n} x^{\frac{m+n}{n}}$ (Rule I), the power series of arcsine, and the power series of sine.

Two further rules introduced without proof reinforce Rule I. Rule II and III state that the area under finitely or infinitely many curves equals the sum of areas under each curve.

The standard interpretation of *De Analysi* runs through calculus: adopting the Riemann integral, it presents Rule I as the Fundamental Theorem of Calculus

$\left(\int_0^x f(t) dt \right)' = f(x)$. Accordingly, term-by-term integration of series explains Rule III. However, this interpretation does not correspond to the argument's structure regarding the series of arcsine and sine. In calculus, one first expands the series of sine and then gets the expansion of arcsine by the theorem on the inverse function derivative. On the contrary, Newton finds the power series of arcsine first and then the series of sine. The core of this difference is that Newton does not apply the derivative or limit concept.

We present actual techniques applied by Newton, namely Euclidean proportion, indivisibles, 'infinitely close' relation, and formal power series. Regarding the study of arcsine and sine, we present Newton's results in a broader context, ranging from Ptolemy to Euler.

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During the workshop, we will walk participants through Newton's infinitesimals and formal power series techniques.

The workshop rests on the first English translation of *De Analysi* [1]. We will provide its electronic version in advance. One can also browse it through the Internet Archive digital library.

REFERENCES

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- Euler, Leonhard (1748), *Introductio in analysin infinitorum*. Marcum-Michaelem Bousquet, Lausannae.
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