

A DIALOGUE ON THE EDUCATOR'S WAY TO RELATE TO MATHEMATICAL HISTORICAL TEXTS

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ABSTRACT

There is not only one way to read a mathematical historical text. For instance, there is the mathematician who looks at the past from a modern synchronic plan, or the historian who looks at the past from both a synchronic and a diachronic plan. But there is also the educator's way to read a mathematical historical text, which we consider to be different. Indeed, in dialogue with the past, interlocutors assume a certain attitude of *responsibility* that, in educational context, takes a special meaning. Particularly, the educators must manage a certain complexity with learners in relation to the past which implies realizing conditions so that the relation of learners with the author takes place and accompanying them with inclusive gestures in their interpretation enterprise. This special role singles out considerably the educator's way of relating to the text. On the scene, we have voices from the past, from the classroom, and from a larger sphere of communication - such as those active in the actual sociopolitical environment - that are manifest and that, we think, should be considered in order to understand better the role and potential of the history of mathematics in mathematics education. In this communication, with reference to HPM literature, and focusing on the notion of *otherness* from Levinas and Bakhtin, we will try to push further the investigation of the educator's way to relate to historical mathematical texts. Our paper will maintain the form of a dialogue as a report and expansion of our email correspondence.

1 Introduction

This paper includes the messages of our email exchange, with minimal modifications. For convenience of the reader, these regard erasing of some redundancies or repetitions, typos, and reformatting of the text.

We made these choices as we want to maintain the features of a dialogue, in search to clarify and to develop some ideas that were brought up in the article *History of mathematics in the context of mathematics teachers' education: a dialogical/ethical perspective* (Guillemette and Radford, 2022) where the authors, focusing on the use of original sources in mathematics education with

emphasis on preservice teachers' education and the practice of exploring historical texts, attempt to develop a specific theoretical positioning to think more finely the way preservice teachers engage with the past. They develop a dialogical/ethical perspective that emphasizes on the emergent dialogue in the encounter with the past. Assuming that dialogues always occur in specific forms of activity, with their own object-oriented specificities, the authors highlight how preservice teachers seem to read historical texts in a manner that is neither the mathematician's (looking at the past from a modern synchronic plan) nor the historian's (both synchronously by considering a given historical system of mathematical concepts and diachronically by considering concepts' evolution in time and space). This different manner is characterized by acts that bring with the "potential estrangement" of the text, offering "vicarious aspects around the ways of *being* in mathematics and ways of *doing* mathematics" (id., p. 1501).

In the present contribution, as the reader will see, the focus is principally on ethics and on the notion of *otherness* developed by Levinas and Bakhtin. We try to understand better, in these terms, the educators' way of engaging with the past. Searching for emerging ideas and a theoretical emulation, we decide that our paper will principally report our email exchange prior to the 2024 HPM congress.

2 *Transcription of the email exchange*

Our exchange reported here begun just after Adriano's reading of (Guillemette and Radford, 2022). He considered the opportunity to contribute to giving methodological value to the use of original historical documents (texts) with students. He shares with various authors (for example, with Jahnke et al. (2002)) the idea that these have a great value in mathematics education. However, the educator's choice to use the originals lends itself to critical considerations. One concerns the rigor and reliability of the interpretation (a joint work with mathematicians and historians of mathematics seems necessary). Another concerns the approach to the original document by the students (what can be their involvement and on what can it be based?) and the characteristics of the proposal by the teacher (how does it fit into the educational path? for what objectives?). David proposed to share our reflections with participants in HPM Conference through an oral presentation. Here it is the email exchange:

A. I make reference to a resume of your article (Guillemette and Radford, 2022) but with a part regarding Levinas¹ contributions, a not trivial conjunction, maybe. The perspective to express the point of view of a secondary school teacher intrigues me. Moreover, how can we tackle the objection of “whig history²”?

D. The objection of “whig history” is interesting... There is also a risk of “using” the history of mathematics for other goal than history itself. We could go back to Fried’s (2001) article *Can mathematics education and history of mathematics coexist?*. Whig history is part of the historian voice and perspective, the idea is that the educator does not talk from the same topos, as we understand it in Guillemette and Radford 2022. If we situated ourselves in teachers’ education context, the voice of the historian *could* maybe be invoked, and orient the appreciative orientation of the group (preservice teachers and teachers’ educator)... there is maybe something to develop there... But, I don’t know where we could put Levinas’s perspective in it actually... What do you think?

A. When I write “We discuss...” I want to make reference to Levinas. I imply that an educator has a different "community" to make reference to, that of students (not of researchers, within which specific categories of research questions are shared). I would like to use the expression “today’s di-

¹ For Levinas, the I-Other, face-to-face, relation is the origin of ethics, as responsibility for the Other. This relation is “asymmetrical” (Levinas, 1979, p. 215). This Asymmetry is the ethical core of relation I-Other, where there is neither reciprocity nor measurement. It means that the other and I are never in interchangeable positions, and my role is never transferable, because the other’s life is infinitely more important than mine (“Thou shalt not kill”). Neither moral euphoria, since this act is not voluntary, nor any expectation of reciprocity involved here, as the Face of the other, is the upmost indisputable authority. With reference to mathematics education, the learner is under educator’s authority but also the educator is under learner’s authority. The educators make a work proposal to the learners, but then the ethical relation of responsibility requires a response to the learners, and the educators orient their choices, consciously but also unconsciously.

² I think about the negative connotation (for us) of “Whig history” (that focuses on the successes that led to present-day mathematics, while ignoring failures, to go too directly to look for the present in the past, as would say Butterfield) considering that educators are usually oriented to today’s mathematics.

achrony”³ to mean that in teaching we have not to “say immediately everything” of a historical document to the students instead, what we have to say depends on particular educational needs, so that we may postpone some considerations. So, Levinas helps us to highlight the interactional aspects. Of course, historians, and mathematicians, are inevitable references.

D. Yes, OK, but I wonder how Levinas can help us to describe the interactional (interesting!) aspects as you said... It seems easier with Bakhtin⁴, as we did with Luis. Maybe we could get back to *trace* and *face* concepts, but how do they get special meaning when describing the educator perspective?

A. Levinas can help to “sustain or justify” our point of view of educators, considering the ethical relations of the educator with the students. Face and everything in which there is trace of the Other produce the establishing of those relations, but also of that with the author of the document. I believe that Levinas can enter where you and Luis speak of “Engagement and answerability” (I would like to add *responsibility*, for the document). So, I refer to what I say in Demattè (2022). I suppose that the relations with the

³ By using “diachrony” I underline the development over time of interpretation by students of a historical text. For example, about interpretation of a passage from Euler’s *Introductio in Analysin Infinitorum* on logarithm (Demattè, 2022): I introduced Euler as an eminent character of 18th century (in a multidisciplinary project); the class faced exercises and problems taken from Euler’s originals concerning the concepts of exponential and logarithm (from both *Introductio in analysin infinitorum* and *Vollständige Anleitung zur Algebra*); individually, each student read the passage and wrote a report; these reports were briefly discussed; more exercises and problems inspired by the passage were solved... I do not pretend that all this exhausts the interpretation of the passage (for instance, I did not deepen the term “imaginaris” - which has been translated as “imaginary” and as “complex” by different translators - with students, a couple of examples excluded).

⁴ In his phenomenology of the practical act, Mikhaïl Bakhtin (1895-1975) developed a dialogical perspective on human being, and human sciences. In his perspective, being-in-the-world is dialogical through and through, as we are co-participants in a world where other subjects exist and that our very identity is constituted of these relationships. The Other has, for Bakhtin, a “surplus of vision” on us, and vice versa, making possible a “reversal of perspectives” which is capital here in the human intersubjective experience and the formation of a proper ethical consciousness.

students and with the author of the document characterize the educators' role and are different from those of historians and mathematicians.

D. I think that it could be interesting to develop about the *responsibility* in the triad (teacher/students/mathematician from the past), as we insist more on engagement with Luis. Levinas would surely help us here indeed. I'm interested in your idea of bridging the educator roles and its relation to the educator way to relate with the past... It makes me think that we assume that the educator way to relate with the past is different from the mathematician and the historian way - different... but maybe not entirely, historian and mathematician also have their "third party" (*tiers*) interlocutor. Plus, and I don't know exactly how to fit it, but the responsibility is also shared with to the students, as you clearly pointed it in your article in DdM (*ibid.*)... What do you think?

A. The educator roles and its relation to the educator's way to relate with the past, I think, depends on the different people that the educator refers to. I agree: the educator way to relate with the past is different from the mathematician and the historian way. Do you think that historians and mathematicians can have as their "third party" (*tiers*) the educators? Where? I try to answer: in the HPM group!

Achieving the sharing of responsibility with the students through an educational initiative... It risks to be, I think, an oxymoron - being the responsibility based on an ethical relation (in order to preserve it, that relation must be lived without making it an object of reflection...). As educators, I think we can only present us as "models" of responsibility with the mathematician of the past, in front of the student (I consider an imposition as an institutional act that would preserve "form but not substance", that is the exteriority required by the Third Party instead aiming at the I-Other relation; but I don't exclude that this imposition could leave space for an I-Other relation)

D. Of course, the educator lives a kind of tension in relation to the truthiness of the interpretation of the text, but it is not the same as the historian one (his entanglement in the hermeneutical circle – the tiers here is the History itself as this never-ending, never-achieving, always constructing narrative) and the mathematician one (the search for the expansion of the mathematical realms– the tiers here is Mathematics itself as a science). For

the educator, I think it is much more a kind of prudence (*prudentia* - the cardinal virtue with all its ethical deepness) that is different and that has to be explored more finely (the tiers here is for me, mathematics education as a social and political project). Lacan would say a different big Other (*grand Autre*)⁵. You are right, the educator role brings different way to relate the text and its interpretation. I would say that the educator role is oriented generally on *giving sense*. But this *giving sense* is taking a special sense (!) when classroom engage with the history of mathematics. Indeed, mathematics educators seek, within the exploration of the history of mathematics, to restore to mathematical activity its most precious ontological force, its recreation against the backdrop of culture and history... For me, *responsibility* of both teacher and pupils takes on special meaning here also... This said, as I said, in my paper in ESU 9 proceeding, criticizing Fried's notion of "moderator in dialogue attitude", that it is not enough to simply have this position in the presence of learners – to enact this attitude *in front of them*. One could say that, as educators, we should situate ourselves as moderators between the past and present *in preparation* for the encounter with the past in our classroom. But again, this dialogue between the old and new must engage the entire classroom if we want learners' horizons to expand. We would like the whole class, teachers and students, to dialogue with the past...

⁵ Lacanian psychoanalysis describes the subject as split between different interpellations, more or less authoritarian voices anchored in the symbolic order, sometimes called *le grand Autre* [the big Other]. In our discussion, we try to describe the ethical relation to the historical text as *responsibility*, a responsibility beyond any ontological condition, a responsibility that is not coming from a decision or a freedom, but in the authority of the Face of the other. This ethical relation of responsibility is never a simple isolated face-to-face. Indeed, for Levinas, "the disquieting imprecision of third parties shrouds the face-to-face encounter with the other signifies an insistence on disturbing, on preventing the two from remaining within the two" (Bensussan, 2006, p. 36, our translation). In the investigation of the educator's way to relate to the past, one of the main objective could then be to determine the sense of this Third Party and its manifestation in educational context. The ethical relation of responsibility when reading an historical text is something always shaped by a Third Party, other voices than the one of the reader and the readed, determining the appreciative orientation in the encounter with the past and ways to relate to it.

A. Here my comments. Prior to the truthiness of the interpretation, I see the meaning the educator gives to the text (depending on the personal experience/competence but, first of all, the aim to involve students, in a sort of “today's diachrony” of the text inserted in the process of students’ learning: a never-ending process, abandoning the myth of a grasp “hic et nunc”/in “exhaustive” form/without “possibility of remediation”... see also Freire (2000). Why not consider the tiers (Third Party) only as human subjects dealing with a common object (see Radford’s theory of objectification) - that is the mathematical content/the text itself or more generally history - for a radical humanization of mathematics? An object belonging to the teacher (as the proponent of school activities), therefore – hopefully – desired by the students (see Levinas quotation in my ESU9 workshop), and progressively constructed in teacher/student interaction? Why this reference to “prudential”? This reference to Lacan reminds the Levinasian Third (but again, why mathematics education instead of teachers and other people involved in mathematics education?). Could you explain with reference to learning your reference to “ontological force, its recreation against the backdrop of culture and history”?

D. Indeed, there is the meaning and the truthiness of this meaning. There is empirical evidence about the fact that teachers do not introduce history of math in classroom because they don’t feel enough competent. Another problem is that they are interested in introducing it, but they don’t know precisely why. There’s a double problem.

It is the case of Fried who problematizes clearly the differences between the two communities considering their relations to the past.

Bakhtin would say that when someone talks (interacts), he always has a third interlocutor (*surdestinataire* ideally just) who is also responsive (ideally). I do not consider the tier as human subject, because the Third is voiceless concretely (yet it is ideally). In fact, for Lacan, the truth is that there is no big Other, we are alone, but this is another story. My idea is that if we want to explore the relation that educators have with the past, maybe we should try to understand from *where* they stand.

This is an attempt to understand the educators’ way to relate to the past, taking in consideration where they stand. My first intuition is to situate the educator as embedded in a political and social project that is mathematics education, which demands constant care, judgement, and adjustment.

3 *Educators and historical texts: synthesis and concluding reflections*

For some mathematics educators, the reading of historical texts is a privileged way for learners to encounter history, in school and in the context of teachers' education (e.g., Arcavi and Isoda, 2007; Demattè, 2021; Demattè, 2022; Guillemette, 2017; Jahnke, 2014). Several studies have emphasized the experience of “dissonance” or “alienation” (e.g., Arcavi and Isoda, 2007; Guillemette, 2017), which students can feel when reading historical texts. Others have suggested that these reading activities could help the prospective teachers to learn something about their own mathematics knowledge and positions by experiencing and “reflecting on the *contrast* between modern concepts and their historical counterparts” (Fried et al., 2016, p. 218).

This said, as pointed above, the different ways to read and to encounter mathematics from the past in the context of mathematics education are still keenly discussed. There is a need to understand better how educators relate to the past, both when referring to teachers' classroom implementations related to the history of mathematics and to teachers' educators. Fried (2018) observes a variety of relationships to the mathematics of the past. He focuses and discusses, among others, mathematics educators' way to engage with the history of mathematics, highlighting their own specificities. We would like to suggest that this educators' way to relate to the past should be investigated, both theoretically and empirically.

A fundamental point concerns the choice of texts and the implementation of activities with the learners, for interpretation. The choice is completely up to the educator, it is part of his pedagogical authority, and is conditioned by his epistemological beliefs regarding the construction of knowledge. For example, wanting to propose to the learners a reflection on algebraic symbolism, the educator could choose a text by Descartes (in which the symbolism is completely analogous to the current one) or one by Bombelli (which has many aspects of diversity compared to the current one). We think that in the second case the educator can be aware that the discussion with the learners will be broader and will lead to highlighting more historical-epistemological aspects. We suppose that these could concern algebraic symbolism as a tool (choosing Descartes) or as a special form of language (choosing Bombelli). We express ourselves in terms of suppositions, aware of how the educator's personal, different beliefs can influence the classroom discussion.

In the context of a classroom, the educator reacts to the text in terms of an agentic culturally and historically situated positioning through which his subjectivity shows itself polyphonically in the reception of the text that carries its unavoidable estrangement. Pointing at future empirical studies in the HPM literature, the way this subjectivity is showing itself still has to be described.

We have suggested, in Levinasian terms, that there is possibly a Third Party that is engaged in this emerging dialogue when referring to educators confronting with the past, a Third Party that is not the same of the mathematicians nor of the historians. We agree with Fried when he says that “the way teachers of mathematics teach mathematics is not only determined by their understanding of the nature of mathematics, historical or otherwise, but also by their own teaching goals and the kinds of problems they hope their teaching practices will solve” (2018, p.18). This is where the Saussurean perspective, used to describe and to differentiate the mathematician’s and the historian’s way to relate to the past, loses its efficiency. We think that the lack of a sustained discussion around de Saussure’s perspective has obscured the understanding of the empirically observed problems surrounding synchronic and diachronic readings of historical texts, therefore, there is a need that in HPM literature a discussion of these issues to take place⁶.

In this paper, we think about the ideal role of educators. We consider that those who act in their role as educators refer to, for example, the special institution constituted by the HPM study group, a special Third Party that indicates (considering the work of its members) the need for an ethical relation with both the author of the historical text and with the students, to be managed according to justice. However, we problematize the subjectivity of the educators and consider that it may happen that they refer, for example, to the community of mathematicians or that they refer only to some aspects of the HPM proposal. We ask ourselves then if, in this way, the conditions exist for the right coexistence of the ethical relation with the student and the ethical rela-

⁶ This Saussurean perspective on language, focusing on interpretation and relying on the concepts of diachrony and synchrony, has been frequently discussed in HPM literature (e.g., Bråting, 2019; Rodriguez & Lopez Fernandez, 2010). What has been less discussed is the very conception of language offered by de Saussure and its implication for the understanding of the history of mathematics in mathematics education (see Guillemette and Radford, 2022).

tion with the author to be realized. Evidently, other communities of reference (for example that of educators in the broad sense) will be able to provide the prerequisites for this coexistence to be achieved.

Indeed, the school mathematics culture may enter in the scene here, as the mathematics knowledge take on original forms in educational context. From a historico-socio-cultural perspective, the source of mathematical abstraction is to be found in the material production of life and cultural ways to engage with others. It is within the economic, social and symbolic structures (for instance: values, governmentality, historical significances) of the society that the needs to be satisfied in educational terms are defined, mediated by ethical relations with the learners. And it is within these same structures that the “ways” of satisfying these needs are also defined, and to which educators can inspire for getting to take into account the needs deriving from learners’ subjectivities. These “ways” are in fact “ways of doing things”: they appear as “knowledge” (D’Ambrosio, 2006). This is where we could find the genetic roots – focal point of emergence and growing – of the dialogue between voices from the school context and voices from the past, and maybe the focal point of the investigation of the educator way to relate to the past.

Another way to characterize this engagement is to take into consideration the inherent responsibility of the educator, who proposed the text, in the emergent relation between the learners and the mathematicians from the past. The educator experienced relations with both the learner and the mathematicians from the past, to make coexist. To promote these relations, the educator has to make choices that are just. *Justice* becomes the key concept (Levinas, 1991) and has to inspire the answers to questions, such as: In which way can the text support personal learners’ needs? What have I to say to the learner about the author, and the context in which the text has been produced? About the mathematical content of the text, how can author’s reasons and learner’s needs meet⁷? Is the text an opportunity to tackle learners’ difficulties or does it introduce supplementary difficulties? In this case, what educational oppor-

⁷ Let’s consider, for example, that the text might be part of a larger work - such as a book with its own purposes and structure - where it is part of a discourse or a deductive reasoning, instead the learners can find in it the opportunity to deepen a specific content or to experience the “dissonance” between their own and author’s understanding.

tunities can justify an investment of time and resources to tackle these difficulties? Inside what I say to the learners, is there something that can stimulate them in deepening author's works or life context?

In making choices conditioned by justice, the educator focuses on, thematizes and compares different needs of the author and the learner (such as those mentioned inside the previous questions). It is a new perspective, different from that of a relation with an Other (the author or the learner, separately, to relate ethically to, in an unthematized form). It is the prelude of an entity ethically constituted, whose principles are developed from justice and could be even formalized as a system of criteria to be followed in classroom.

The HPM community is a social symbolic structure whose members made choices to try to answer those questions; it is bearer of knowledge, as ways of doing things, regarding original historical documents in teaching and learning mathematics. In any case, it is not excluded that the educators do not refer to each of the different subjects, that is: the learners (as a group or as singles), the author, the members of a community devoted to pedagogy and mathematics. It is not in the present paper that we want to discuss the negative educational consequences deriving from this.

This said, educators' way to relate to the past in educational contexts still needs to be described in theoretical and empirical terms. The idea is not to discredit the mathematician's and historian's way to relate to the past, but to signify how fundamentally different could be the educator's one, and how this investigation could foster new ways to understand how educators conceive the role of the history of mathematics in mathematics education, how they manage and envision the introduction of the history in the classroom and the difficulties they are expecting and living. The pedagogical and didactical role, and potential of the history of mathematics could then be discussed in more refined and intimate dialogue with teachers' and teachers' educators' voices.

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