

ELEMENTARISATION OF MATHEMATICS FOR UNDERGRADUATES BY INTEGRATING HISTORICAL STANCES WITH TRIGONOMETRIC FUNCTIONS TOWARDS FOURIER METHODS.

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As mathematics advances, a mathematical idea once considered complex become elementary in the sense that it becomes fundamental for higher mathematics. From the perspective of undergraduate mathematics education, a mathematical idea becomes essential for upper-level courses for applied mathematics and STEM majors. Building on Felix Klein's conception, the elementarisation provides a dynamic perspective describing the history of mathematics and its relationship of mathematics education and undergraduate mathematics. The concept of hysteresis, as emphasized by Klein and Schurbring, addresses the process of elementarisation as mathematical ideas become more elementary and foundational based on recent advances and practices with mathematics and its applications. A mathematical idea or practice such as combining trigonometric functions becomes elementary in building and applying mathematics for its further development of mathematics and its applications. New stances on a mathematical idea are gained as these ideas are advanced, recontextualized, reclarified, and redefined in the advancement of mathematics and mathematical sciences. Through historical restructuring of applied mathematics found in upper-level courses in STEM disciplines, students are compelled to grasp a higher conception of a mathematical idea or a practice, for example Fourier methods related to trigonometric functions in their courses such as control systems, signal analysis. The hysteresis orients learner to build a higher stance on a mathematical concept such as trigonometric functions to understand the progressive restructuring of its elements in building mathematics as experienced in the historical development of mathematics and its applications in STEM disciplines. As mathematics advances while applying its concepts and practices, these mathematical practices, such as related to trigonometric functions, are reclarified through their progressive restructuring reorienting mathematical ideas facilitating their advancement and applications. This paper provides an exemplification of the elementarisation process by presenting, discussing and analyzing multiple historical stances on trigonometric functions including circular, hyperbolic, elliptical, com-

plex, finite, higher dimensional spaces, and geometric algebra. The historical approach behind Fourier analysis stance will be emphasized with its orientation towards trigonometric functions as bases of function spaces. Adopting the concept of hysteresis on trigonometric functions, renewed foundational developments on trigonometric functions will be argued to enter the undergraduate mathematics and school mathematics after this process of elementarisation. The implications of delayed hysteresis and neglect will be discussed from the perspectives of engineering mathematics education and physics education. Recommendations for learning trajectories of elementarised mathematics will be presented for their integration in core undergraduate mathematics courses for STEM majors. This hysteresis process informs the curriculum restructuring by revisiting what fundamental mathematical ideas and practices are for the advancement of mathematics and mathematical sciences. This process puts the undergraduate mathematics for STEM disciplines into a productive relation with the progress of mathematics and its fundamental applications in STEM disciplines.

REFERENCES (10pt fonts, spacing 15 pt)

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