# LEARNING MATHEMATICS, LEARNING ABOUT MATHEMATICS IN PRIMARY SCHOOL: A LONGITUDINAL PILOT EXPERIENCE ON THE HISTORICAL PERSPECTIVE

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## ABSTRACT

Since 2006, action research at Roma Tre University (Italy) has explored the introduction of the history of mathematics into primary education. Topics from the history and anthropology of mathematics were integrated into mathematics courses in the Master's Degree in Primary Education (Gil Clemente, Millán Gasca, 2016). This encouraged both students and in-service teachers from the Primary School Math Research Lab to explore how these historical aspects could be presented to children.

Following a suggestion by Karen Fuson in 2012, a history of mathematics learning path was developed for integration into the existing mathematics curriculum. As part of this broader research, a five-year pilot study (2018–2023) was conducted across all five grades of primary school in Rome. Contents were selected based on a structured learning path (Millán Gasca et al., 2017) and included children's books on the history of mathematics. Assessment focused on standard learning outcomes in mathematics and history as well as "flourishing" personal growth outcomes (Millán Gasca 2016; see Su 2020).

## *1. Theoretical background and pedagogical framework*

Two main research questions were addressed:

- Is it suitable to introduce aspects of the historical evolution of mathematics to children? Can it be cognitively accessible, age-appropriate, and scalable across age groups? What added value does it bring to learning mathematics beyond basic numeracy in primary school? What general educational benefits are expected?
- Is it feasible to integrate history of mathematics into primary curricula and classroom practice (Tournès & Moyon, 2018)? Can it offer educational settings and approaches for children's initiation into mathematics? These questions relate to whether history of mathematics should be used as a tool or pursued as a goal in itself (Jankvist, 2009).

Two counterarguments concerning the suitability and feasibility of this approach address the alleged 'erratic sense of the past,' especially in early grades

(Clark et al., 2019, p. 5) and a possible complication involved in addressing numeracy skills while devoting attention to "sophisticated" aspects of cultural contextualization. What is at stake is thus the more general question of primary comprehension. Yet since long ago Piagetian views on deficiencies in the logical abilities of young children have been overcome in studies on cognitive competence when reasoning occurs in an embedded context—one that aligns with their lived experiences: children possess logical abilities, but these emerge only when the context is meaningful and the communicative intent is clear (Donaldson 1978). Kieran Egan, a historian and scholar in education, describes children's comprehension of causality (in time) as what holds stories together and moves them along (Egan 1997).

Mathematics in primary school is usually associated with practical skills with numbers. Our framework instead addresses the overarching question "what is mathematics?" which is often set aside, thus demanding children to silently accept it as school subject. Such a perspective on mathematics in the early school grades as a gateway to a cultural universe "belonging to each of us as human being," crucial in the access to the scientific attitude (Eshach & Fried, 2005), were already present in Friedrich Fröbel and still more distinctly in Mary Everest Boole's "preparation of the child for science". Charles-Ange Laisant's views on "mathematical initiation" (v. Millán Gasca 2015, Magrone, Millán Gasca, eds 2018), and Federico Enriques's reflection on precocity and the need to combine utilitarian (functional) and formative goals (in the sense of *paideia*, see Fried 2018) since early childhood (Enriques 1938) also reflect this perspective.

Central to our approach is the primacy of human experience to unveil conceptual meaning of elementary objects and relationships having ancestral roots-one, successive, more than, straight line and betweenness, geometrical congruence -(Guzmán 2007); to start the exploration of the *terra incognita* of mathematics as a sphere of human activity and expression (Doxiadis 2003); as a journey in which mathematics contributes to human growth by providing avenues for beauty, joy, and discovery (Su 2017).-At early ages, the teacher's role is to nurture a sense of wonder: "our purpose is to expose the wonder and ingenuity of counting. Our first task, then, is to show that what we take for granted, what is such a casual routine these days, is a product of amazing achievements" (Egan, 1986, p. 77).

Current emphasis in HPM domain-related literature on the importance of monitoring and disseminating implementation in the classroom regarding history of mathematics topics (Clark et al., 2019; Chorlay et al., 2022) is particularly critical in primary and early childhood education, where topics on the history of mathematics are still rarely.

## *2. Exploring the introduction of history of mathematics in primary school. Action research methodology*

Action research methodology follows a cyclical structure involving feedback and iterative improvement regarding educational objectives, teaching methods and the choice of historical content (see Table 1).

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|---|---|
| Early observational phase 2009-2011 | Gathering of field data, through single teacher interviews, focus groups, and scattered school sessions by in-service and prospective teachers (the origin of numerals in Mesopotamia, ancient Greek mathematicians, De Castro 2011/2022, Usai 2012) |
| Field work A (2012-2016) | Design of prototypes of educational activities and itineraries (preschool (5 years old)/primary school grades 1-5)<br>• by schoolteachers belonging to the community of practice of the Roma Tre research lab on mathematics in primary education<br>• by prospective teachers in the Roma Tre Department of Education practicum |
| 2017 | Design of a learning path in history of mathematics for Italian primary school (Mazzitelli, Millán Gasca, Neri, Spagnoletti Zeuli 2017, see Table 2) |
| Field work B (2018-2023) | Longitudinal study of a single primary school class group (Rome, 2018-2023) |

Table 1. Roma Tre project on history of mathematics in primary school. Action research diagram

The findings and conclusions from Fieldwork B presented here are intended to further improve the learning path (Table 2). Data gathering and analysis follow Max van Manen's phenomenological methodology, which focus on the individual and community lived experiences (Van Manen, 1990). Raw data includes reflexive writing (by researchers, teachers, trainees), “close” interviews, anecdotes, focus groups on single issues, audio and video in the classroom, children’s individual productions (utterances, logbook, worksheets, collective posters, artefacts, oral and written tests), and general discussions with pupils.

| Grade | Historical/cultural topics | Related contents |
|---|---|---|
| 1 | Prehistory: The origins of number words (spoken language)<br>Numerals in other languages<br>Prehistory: The origins of geometry (shaping materials, the symbolic representation on cave walls)<br>From prehistory to the invention of writing: written numeration | Number words, figures, decimal positional numeration system<br>Geometrical objects and relationships. Shapes.<br>Addition, additive decomposition of numbers |
| 2 | Mathematics in Egypt: hieroglyphic numbers; land surveyor.<br>The measurements of the ancient peoples of the East: length and its units.<br>The Indians invented the writing of numbers that we use today<br>Numbers in China and Central America<br>Hypatia of Alexandria | Number symbolism, additive decomposition of numbers<br>Primordial/primitive concepts of geometry. Polygons: their elements<br>Geometrical bases of measurement<br>Number symbolism, positional principle, base 10.<br>Zero<br>Calculations and measurements in the world, numerical symbolism: base 10-20<br>Women and mathematics |
| 3 | The calendar and the ages of history<br>Babylonian astronomers<br>The Greek mathematicians: Pythagoras.<br>First approximation to the invention of the metric system. | Ordinal numbers, the centuries as segments on a chronological axis<br>The mystery of cyclic motions and omens<br>Base 60 in the measurement of time and angle<br>Philosophy and mathematics: the search for the principles of the world<br>Connection geometry-arithmetic<br>Conversion of measures and money, measurement systems |
| 4 | Roman numbers<br>Mathematics was born in the ancient Greece<br>The Tale of Herodotus. Thales<br>The Theorem of Pythagoras. Euclid and Archimedes<br>Mathematicians who wrote in Arabic in the countries of Islam<br>Leonardo Fibonacci<br>Chinese mathematicians | Numerical symbolism and additive decomposition<br>Calculations and measurements in technical activities and crafts<br>Our idea of mathematics<br>Relations between segments, between angles, between plane figures, between solids<br>Equalities between areas, the first equations<br>Mathematical demonstrations<br>The circle and $\pi$, the diagonal of the square, prime numbers |
| 5 | Mathematics in the birth of modern science in Europe | The ellipse<br>Proportionality |

| | | |
|---|---|---|
| | Kepler, Galileo and Newton<br>Games of chance: probability<br>The Enlightenment: Euler; Monge.<br>The origins of the metric system and other ancient systems<br>From the first calculators to the modern computer | Mathematics and the advancement of science and technology<br>Projections of solid figures<br>The geometric basis of measurement |

Table 2. A learning path for history of mathematics in (Italian) primary school (Millán Gasca et al 2017). In grades 1-2, storytelling with some elements of general historical context. In grades 3-5, the Italian curriculum starts with prehistory.

## *3. The longitudinal study (2018-2023): contents, educational methodology, and findings*

Over a period of five years (2018-2023), a pilot experience was developed in a primary state public school class in the city centre of Rome, Italy, consisting of 19 children from different socio-economic backgrounds, including 7 with various special needs, 3 of whom were non-native speakers. The first author was the mathematics teacher (from grade 3 also Italian language teacher). In addition to incorporating the history of mathematics, a second pilot experience was developed, on the integration of dance, music, art and storytelling for an aesthetic experience of mathematics (Millán Gasca, A. M., & Neri Macchiaverna, F., in press). Problem solving and math talk were pivotal in the overall educational methodology of mathematics, especially with regard to the teacher's *stance* (George Pólya): to *dramatize* with children the process of discovery, encouraging curiosity and reflection, supporting their reasoning without taking over, and valuing mistakes.

The introduction of history of mathematics was implemented thanks to an integrated methodology: stories in selected picture books (see below), workshop or *atélier*, group work, oral discussion and debate, individual logbooks to document their learning and related mathematical concepts, production of material exhibitions, and stories. During the reading, the teacher focused on expressive reading through the use of pauses, clear pronunciation, modulation of voice, movement within the space, and interaction with the children. During the reading, to foster mimesis, the teacher emphasized moments when questions were posed and asked the children to reason and respond. The reading of each book could last more than one lesson. In workshops or ateliers and during group work, the key points of the stories were re-

worked both at the narrative and conceptual levels: questions and exercises (sometimes involving concrete, even self-made, tools) were proposed on the emerging mathematical topics (table 2).

Figure 1 From left to right: Children build “fingers and balls” inspired by the book *Uri, the little Sumerian*; Group work: three girls decide to use the kit to work on horizontal abacus.

During discussions and debates, children shared their responses to questions, problems, or exercises. Starting from the collective discussion, they then wrote their logbooks to record and remember what they experienced.
In grades 1-2, the logbook was initially designed by the teacher: it served to reconsider the classroom experience, to recall and sequence the various phases of the narrative—often rich in temporal and causal structures—, to became aware of mathematical concepts to engage in reflective thinking inspired by hands-on activities.
As the children progressed in their handwriting skills, they were provided only with guiding questions to help them narrate their experiences; further, they independently created graphical expressions—combining words, drawings, diagrams—to represent their learning exploration. The logbook included exercises. In grades 3-5, pupils conducted research through selected books from the classroom library, presenting their findings through “lectures” on topics such as π, the Rhind papyrus, and biographies of mathematicians. Thus, writing and oral presentations were synergical to encourage reflection and understanding.

Thus, somatic-mimetic, as well as mythic and romantic understanding, were put into play, following Egan’s perspective on primary understanding and his storytelling-based approach to lesson planning (“What is the most important thing about this topic or story? Why should it matter to the children? What would engage them affectively?” Egan 1987).

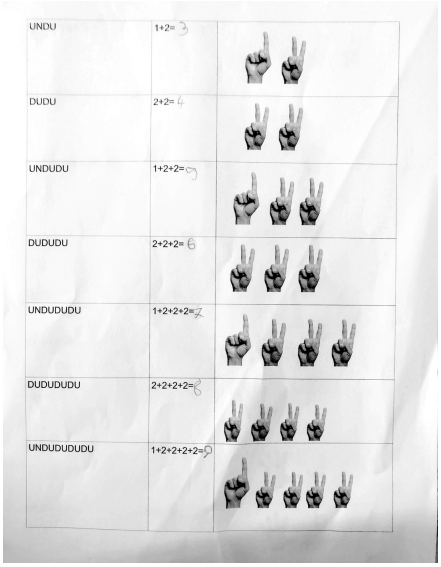


Figure 2 Grade 1 logbook: An exercise on the number words invented in *Awa teches numbers*, involving addition, and representation of number with figures and fingers.

In grade 1, the following two books were read: Anna Cerasoli's *The Great Invention of Bubal* (Cerasoli, 2006) on a prehistoric shepherdess who invents counting methods, and Enrico Giusti's *Awa Teaches Numbers*, based on ethnomathematical research (Giusti, 2019).

Books for grade 2 were: Rafaella Petti's *Uri the Little Sumerian* (Petti, 2025). and Anna Cerasoli's *Pharaoh's Geometry* (Cerasoli, A. 2009) whose main character Ames, is the son of a land surveyor who learns to use ropes for technical tasks. These stories helped the students to grasp the act of counting and cardinality, the symbolic representation of quantities, decomposition, primordial geometrical concepts, and the generations of shapes.

In grades 3-5, following the pupils' interests and needs, the choice was to delve deeper into the already considered historical topics, also in connection with the Italian history curriculum, which usually starts in grade 3 with prehistory, progressing to Greece and Rome in grade 5. We expanded the study of ancient number systems and mathematical achievements from various cultures, including Egyptian, Greek, and Roman civilizations and examined the origin of measurement and ratio thanks to *Mama Khanyi and the Pots* (Vale et al 2018). Further historical topics included in the designed learning path (Table 2) were chosen by the pupils—for example, an in-depth study of Fibonacci's figure through the reading of the book *The Boy Who Invented Zero* (Feni-

ello, A., & Folì, G., 2014). The teacher provided a historical frame of reference and considered the issue of sources.

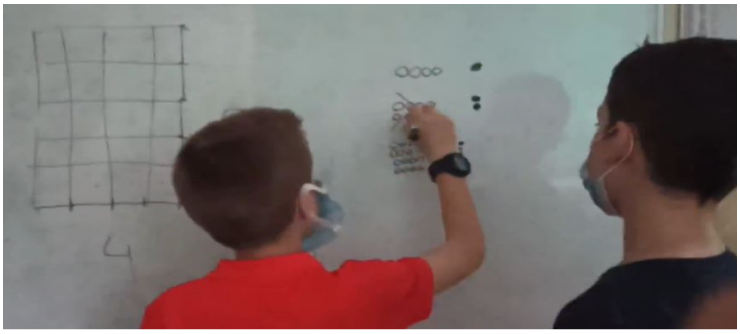

Figure 3 During a lecture on Egyptian mathematics, two children explain the multiplication method.

Writing played a fundamental role in working on the personal elaboration of experiences and the understanding of mathematical concepts.

*Rome, October 14, 2021 (5th grade)*

*Yesterday, together with our teachers, we reviewed everything we have studied and understood about our number system.*

*While re-reading our logbook, we identified three key words: positional, decimal, and ordered. These terms helped us recall all that we have learned together over the past five years.*

*Our system is decimal because we group by tens. But why group by ten? We found a simple explanation: we have ten fingers. Since the time of early humans, our ancestors counted and calculated using their fingers. In this way, fingers were like a natural calculator. That's why even today, it feels intuitive to count in fives or tens.*

*The character Awa taught us that numbers and number systems were born from the body. For example, we remembered that to say "ten," Awa used the word everything, and to say "four," they said one hand minus one.*

*In first and second grade, we also read the story of Uri, who—through his adventures and challenges—helped us understand how useful and essential grouping is.*

*Another important discovery was the invention of number writing. Uri first invented clay fingers, then small balls, followed by tokens, and finally clay tablets on which numbers were inscribed.*

*Our system is also positional: the value of a digit changes depending on its position within a number. All of this is made possible by the invention of zero.*

*Zero has a long history. It was invented by Indian mathematicians, who called it śūnya. In Sanskrit, the ancient Indian language from which many European languages—including Italian—derive, śūnya means "empty." Thanks to zero, we understand when to leave an empty place, and we can distinguish numbers such as 40 from 4 or 400. (...).*

The phenomenological observation and analysis of the lived experience in the classroom led us to reconsidering and adapting the original learning path, because each class is unique and is composed of unique individuals.

To analyse the qualitative raw data offered by the interviews, the diaries, and the teacher's notes on classroom conversations, we adopted a combination of three approaches: holistic, selective, and detailed (Van Manen, 1990). Several key findings related to children's attitudes and knowledge emerged:

- Most pupils expressed the feeling of being "as if" they were directly facing the problems that the characters in the stories had to solve and "as if" they were living in the same historical context as those characters;
- A strong desire to understand and address the problems and questions emerged. All pupils, including those with special needs, showed increasing curiosity and engagement, their imagination expanded and their creativity was enhanced by the combination of historical narratives and mathematical problems;
- Pupils developed new strategies to explain and narrate mathematical concepts and find new ways to explain their learning;
- Logbooks and interviews show that the pupils were able to create meaningful connections between different mathematical concepts interdisciplinary connection to the subject matter.

Individual tests, based on the assessment objectives selected by the school from the "National Guidelines" for grades 1 to 3 and 4 to 5, were conducted to monitor individual pupil progress. At the end of the fifth grade, out of 19 children (7 boys and 12 girls) , 12 (4 boys and 8 girls) achieved excellent results in mathematics, 2 (one boy and one girl) very good results, 4 (2 girls and two boys) good results, and 1 (one girl) more than sufficient results. The programme contents had been simplified for one student while tests were modified (slightly simplified) and additional time was given for 3 pupils from the third group and for 1 pupil from the first group (just additional time) in accordance with their personalized educational plans. These evaluations also reflect the raw numerical data available from the testing activity and national tests in 5 that were analysed using qualitative phenomenological methods.

### *4. Final remarks*

From a practical standpoint, the pilot experience confirmed that the integration of the history of mathematics into classroom practice is *feasible* through storytelling, an approach that is accessible and scalable across different ages. Pupils showed an intense appreciation of history of mathematics since grade 1, and in grade 3-5 they wanted to turn back to the same historical periods to obtain a better, more mature understanding, before turning to new issues. The pilot experience let us widely explore the issue of *suitability*.

First, considering aspects of the historical evolution of mathematics offers mathematical experience soaked by human intentions and goals, thus enhancing understanding and connections. The stories showed mathematics as an evolving body of knowledge, shaped by human experience, and help children understand the origins and nature of mathematical concepts. This approach can bridge the gap between abstract principles and lived experience.

Moreover, it offers a cultural richness regarding anthropological and philosophical aspects of number and form that extends maths learning beyond basic numeracy: pupils showed a growing awareness of the specificity of mathematical universes as linked to human life but involving imagination and the "invisibile". By placing mathematics within its cultural and historical contexts, pupils gain a deeper and more global understanding of the subject, seeing it not just as a disembodied set of principles/procedures but as a dynamic and evolving "worlds", including speculative aspects. Historical narratives foster an appreciation for mathematics, presenting it as a vivid and immersive activity

Finally, there are overall educational goals involved in the pilot experience, regarding the building of a "scientific attitude" (objectivity, argumentation, research prompted by problems). By exploring figures such as Ahmes, Fibonacci, Al-Khwarizmi, and Awa, actual mathematicians or fictional characters inspired by historical research, pupils deeply shared their mathematical struggles. Engaging with historical narratives nurtures cognitive and emotional development, enabling children to cope with challenges (mathematical problems) involving analytical skills, fostering perseverance and courage. Historical contexts provide an appropriate setting for problem-solving, encouraging students to persevere individually and in a cooperative manner. Learning about the challenges that mathematicians faced led pupils to approach mathematical tasks as opportunities for discovery and growth, thus enhancing their commitment to the subject. In this way, a historical perspective can be the

driving force for presenting mathematical content in primary school. The historical framework adds an aesthetic and feeling dimension to mathematics, inviting students to encounter mathematical concepts through wonder, pleasure, play, fear, curiosity, helping children to relate to these feelings in both the mathematical and broader life scenarios.

**REFERENCES**


Cerasoli, A. (2006). *La grande invenzione di Bubal*. Milano: Emme Edizioni

Cerasoli, A. (2009). *La geometria del faraone*. Milano: Emme Edizioni

Chorlay, R., Clark, K. M., & Tzanakis, C. (2022). History of mathematics in mathematics education: Recent developments in the field. *ZDM Mathematics Education*, 54, 1407-1420.

Clark, K. M. et al (2019). History of Mathematics in Mathematics Education – An Overview. *Mathematica Didactica*, 42(1), 3-28.

Donaldson, M. (1978). *Children's minds*. London: Fontana/Collins.

Doxiadis, A. (2003). Embedding mathematics in the soul: Narrative as a force in mathematics education. 3rd Mediterranean Conference of Mathematics Education, Athens.

Egan, K. (1989). *Teaching as storytelling. An alternative approach to teaching and curriculum in the Elementary school.* Chicago: The University of Chicago Press.

Egan, K. (1997). *The educated mind: How cognitive tools shape our understanding*. Chicago: University of Chicago Press.

Enriques, F. (1938). La matematica nella storia e nella cultura. Bologna: Zanichelli.

Eshach, H., Fried, M.N. (2005) Should science be taught in early childhood?. *Journal of Science Education and Technology*, 14, 315-336.

Feniello, A., & Folì, G.( 2014).*Il bambino che inventò lo zero*. Roma: Laterza

Fried, M. N. (2018). History of mathematics, mathematics education, and the liberal arts. In G. Kaiser et al. (eds.), *Invited Lectures from the 13th International Congress on Mathematical Education*, 85-101. Cham: Springer.

Gil Clemente, E., & Millán Gasca, A. (2016). Integrating history of mathematics with foundational contents in the education of prospective elementary teachers. In L. Radford et al. (Eds.), *2016 ICME Satellite Meeting HPM Proceedings* (427-440). Montpellier: IREM.

Giusti, E. (2019). *Awa Teaches Numbers: Young Awa teaches numbers to her village* (1ª italian ed. 2011) Firenze: Il Giardino di Archimede

Guzmán, M. de (2007). Enseñanza de las ciencias y la matemática. *Revista Iberoameri-*

*cana de Educación*, 43, 19-58.

Jankvist, U. T. (2009). A categorization of the "whys" and "hows" of using history in mathematics education. *Educational Studies in Mathematics*, 71(3), 235–261.

Magrone P., Millán Gasca, A. (eds.) (2018). *I bambini e il pensiero scientifico. Il lavoro di Mary Everest Boole*. Roma: Carocci.

Lizasoaín, I. et al (2022). *Growing with mathematics: Experiential math workshops for education of pre-school and primary school teachers*. Pamplona: Universidad Pública de Navarra.

Magrone, P., Millán Gasca, A. (eds.) (2018). *I bambini e il pensiero scientifico. Il lavoro di Mary Everest Boole*. Roma: Carocci.

Millán Gasca, A. (2015). Mathematics and children's mind: The role of geometry in the European tradition from Pestalozzi to Laisant. *Archives Internationales d'Histoire des Sciences*, 65(175), 759-775.

Millán Gasca, A. (2016). *Numeri e forme*. *Didattica della matematica con i bambini.* Bologna: Zanichelli.

Millán Gasca A., Mazzitelli, A. , Neri Macchiaverna, F., & Spagnoletti Zeuli, E. (2017). Storia e racconto nella Matematica della scuola primaria: Basi didattiche e sequenza operativa. *Annali online della Didattica e della Formazione Docente*, 9, 209-239.

Millán Gasca, A., Neri Macchiaverna, F. (in press). *Ritmi e geometrie*. Carocci.

Neri Macchiaverna, F. (2021). Making the gaze live: Perspectives and encounters between primary school and territory. Two exhibitions on mathematics and science. In A. Poce (Ed.) (2021). *Memory, inclusion and learning experiences in cultural heritage* (181-223). Napoli: ESI.

Petti R. (2025). *Uri il piccolo sumero*. (1ª ed. 2008) Trieste: Il Giardino di Archimede/Scienza Express

Su, F. (2020). *Mathematics for human flourishing*. New Haven, CT: Yale University Press.

Tournès, D., & Moyon, M. (2018). *Passerelles. Enseigner les mathématiques par leur histoire au cycle 3*. Bouc-Bel-Air : ARPEME.

Vale, P., Graven, M., Višňovská, J., & Ford, C. (2019). *Mama Khanyi and the pots: A mathematical story and activity book*. Makhanda, Rhodes University.

Van Manen, Max (1990). *Researching lived experience: human science for an action sensitive pedagogy*. New York, SUNY Press.