STARTING FROM THE HISTORY OF MATHEMATICS IN LATE MODERN ITALY (XVIII-XX CENTURIES): FROM PRIMARY SOURCES TO MATHEMATICAL CONCEPTS

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ABSTRACT

The workshop intends to propose didactic paths that use materials derived from primary historical sources, accessible to secondary school students, integrated with laboratory activities. Its focus is the history of mathematics in Italy, from the eighteenth to the twentieth centuries. The workshop is divided into four parts: 1. The dissemination of infinitesimal calculus in Italy, by comparing two different approaches, the Leibnizian paradigm of the "differentials" and the Newtonian conceptualization of the "ultimate ratios of vanishing quantities". 2. The science of waters as the main field of applied mathematics. 3. Re-launching Italian education and research after political unification. 4. The geometry of paper folding and the resolution of problems of third degree. All the materials related to this workshop, including slides and some supplementary worksheets, can be found on the website of Mathesis Ferrara at the link: http://dmi.unife.it/it/ricerca-dmi/mathesis/materiali-esu-9

Introduction

History of mathematics represents a useful tool for learning mathematics and in the last decades the researches in the field of mathematical education have had a great development both nationally and internationally. As regards Italy, from a legislative point of view, the *Indicazioni Nazionali* for upper secondary school (2010) underline the importance of "connecting different mathematical theories with historical problems that had originated them". The aim of this paper is to present to the teachers a variety of approaches and examples of activities that can be used in their classroom. All these approaches share the

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same general idea, that is to use materials derived from primary historical sources in mathematics. The focus is the history of mathematics in Italy, from the eighteenth to the twentieth centuries. We present topics which can be a starting point for reflection and activities aimed mostly at high school students, in an interdisciplinary context. We suggest supporting our educational proposals with practical activities that students can easily reproduced in the classroom, like geometric constructions with dynamic softwares as GeoGebra, or hands-on activities, such as some simple examples of paper folding.

1 The dissemination of infinitesimal calculus in Italy

The searching for the tangent to a given curve and the squaring of figures were the main problems, to which the invention of infinitesimal calculus was able to give an answer. Between the sixties and the eighties of the 17^{th} century, both Leibniz and Newton came to the invention of infinitesimal calculus through two different methods. Starting from the definition of “differential” Leibniz gave the rules of the homonymous calculus and its main applications. It corresponds to taking the difference between two infinitely close values of the variables. These “differences” (or “differentials”) represent the fundamental parameters for the description of the curves. Starting from the problem of the squaring of figures, Newton suggested a kinematic method and used quantities, named “fluxions”, for which he gave specific rules of calculus, absolutely similar to Leibniz’ differentials. According to a mechanical vision of geometry, Newton considers the variables, that he called “fluents”, as quantities whose value increases or decreases with continuity. The instantaneous speeds are called “fluxions”.

These two different approaches to infinitesimal calculus can provide interesting didactic suggestions. Starting from the reading of selected extracts from the works of two Italian mathematicians that followed respectively Leibnizian and Newtonian approaches, i.e. Maria Gaetana Agnesi and Joseph Louis Lagrange, students can be guided to the discovering of the main ideas at the base of differential calculus that they studied during their schooling.

4.3 A Leibnizian approach in the first Italian treatise on Analysis: Maria Gaetana Agnesi’s *Instituzioni Analitiche*

Maria Gaetana Agnesi's treatise *Instituzioni Analitiche ad uso della gioventù italiana* (1748) represents a significant example of the circulation of Leibniz's approach in Italy.

Born in Milan in 1718, the author is one of the first female mathematicians in history. She devoted the second volume of her *Instituzioni* to differential and integral calculus, conceived as the analysis of "differences in variable quantities, of whatever order those differences may be" (p. 341).

The "versiera" is a special cubic curve associated with Agnesi's name thanks to her description within the *Instituzioni* (problem III). This curve has several teaching potentials and it represents a privileged starting point to introduce the use of original sources in the classroom. The didactic path – suitable to different grades of secondary school – can be articulated in the following phases.

1. Historical introduction to Agnesi and her time.
2. Construction of the versiera by points starting from Agnesi's original problem, where it is required to find the locus of points in the plane that satisfy a certain proportion (upper secondary schools), or rhyming instructions (lower secondary schools).
3. Dynamical construction of the versiera, with GeoGebra.
4. Derivation of the Cartesian equation of the versiera and reflection on the cognitive pivot of dependent/independent variables (only for upper secondary schools).
5. Guided reading of the preface of the *Instituzioni* and debate about the gender gap in science and the role of women in society relying on Agnesi's words.
6. Production of an artwork which includes the shape of the versiera, starting with the doodle that Google dedicated to Agnesi on the occasion of the 296^{th} anniversary of her birth.

From the point of view of gender differences, Agnesi appears to be a forerunner of the times: this aspect can be significant for educational guidance, making Agnesi's scientific studies and mathematical commitment an example in the eyes of the students. Furthermore, such an activity includes several factors of didactic effectiveness: historical framing of a mathematical theory; reading of original sources; passages between different domains of maths (from discrete to continuous, from Euclidean to Cartesian plane, from geometric properties to the analytical equation, …); interdisciplinary links (literature, art, …).

4.4 Newtonian conceptualization of the "ultimate ratios of vanishing quantities"

In his studies on the foundations of infinitesimal calculus, Lagrange gives great importance to the calculus of differences. In his treatise, *Principj di Analisi Sublime* (~1754) before exposing the differential calculus, Lagrange gives an algebraic exposition of the calculus of finite increases. A crucial point is the passage from "finite increases" to "infinitesimals". In the second part of the *Principj*, the author develops the algebraic calculus of finite differences. The differential calculus determines "the ultimate ratios of the difference dy/dx, i.e. the ultimate terms to which the general ratios of the differences continuously approach, while these continuously decrease". Lagrange's notation is that of Leibniz, but the approach is Newtonian. Students can be guided to the concept of "limit" through the reading of Lagrange's words.

The ratios of the differences are named ultimate ratios of the differences, considering them in the point where they are going to vanish. Actually these ratios are not ratios of any real differences, since it's supposed that each of them has become equal to zero. They only express the ultimate terms, to which the general ratios of differences continuously approach, while they continuously decrease. These ratios are also named first ratios of the differences, because they can be seen as limits from which the general ratios of the differences, considered as rising to receive continuous increases, begin (see Borgato 1987, p. 129, author's translation).

Browsing the pages of the *Principj* students can also find observations about geometrical constructions, that can be easily reproduced in the classroom. Starting from the reading of original source, students can know and repeat the passages through which Lagrange determines the equation of a particular curve (for instance, the so-called "Conchoid of Nicomedes") and try to reproduce it with the help of GeoGebra.

2 The science of waters as the main field of applied mathematics

According to the classical tradition hydrostatics and science of waters represent two different disciplines. The foundations of hydrostatics can be found in Archimedes' treatise *On floating bodies*, while the science of waters was mainly studied in an empirical way. Italy boasts a long hydraulic tradition and the contribution given by mathematicians in this field has been quite relevant until the 18^{th} century.

We suggest a possible educational activity for students of the first two years of high school in order to present the topic related with mathematics applied to the science of waters from an historical point of view. As regards mathematics, the *Indicazioni Nazionali* for Scientific High School underline the importance to learn at the end of their schooling "a historical-critical point of view of the relationships between the main items of mathematical thinking and the philosophical, scientific and technological background". Following this general idea and taking into account some relevant original historical sources, the proposed educational activity will deal with problems about equilibrium and motion of fluids. The involved subjects are Maths, Physics, History, Italian and Latin (if provided), so the activity has an interdisciplinary nature. It can be divided into the following parts, each of them corresponds to about two hours of lesson:

- The concept of fluid; Physical quantities and their definitions: density, pressure, flow rate, speed.
- Statics of fluids (Hydrostatics): Stevin's law; Pascal's principle; Archimedes' principle.
- Dynamics of fluids (Hydrodynamics): Continuity equation; Bernoulli's theorem and its applications; Torricelli's law.

The activity starts defining some useful physical quantities (absolute and relative density, specific weight, pressure), then we can enunciate the so-called "Stevin's law", that was firstly published in 1586. The law expresses the pressure applied by a fluid inside an immersed body in function of the deepness h of the body, of the gravitational acceleration g and of the density ρ of the fluid. A simple consequence of the Stevin's law is the principle of Pascal: "The pressure at a point in a fluid at rest is the same in all directions", that was discovered by the French mathematician Blaise Pascal and published posthumously in 1663 (*Traitez de l'équilibre des liqueurs*). The chapter about Hydrostatics can be further studied from an historical perspective through the reading of the propositions 3-7 (Book I) of Archimedes' treatise *On the floating bodies*, that express the conditions for which a body balances, floats or sinks. In particular, "a solid lighter than a fluid will be so far immersed that the weight of the solid will be equal to the weight of the fluid displaced". Two original sources, respectively in Latin and in Italian language, can be presented in the classroom to present Archimedes' discovery. Vitruvius in his treatise *De architectura* (I century b.C.) quoted the problem of the crown, as a possible origin of the principle of Archimedes. Galileo, instead, proposes to use the so-called "bilancetta", an instrument that allows to directly weigh the bodies in the water, using the Archimedean concept of specific weight (*La bilancetta*, 1656).

The chapter about Hydrodynamics will focus on two main themes: the continuity equation; the theorem of Bernoulli and some important consequences.

The continuity equation states that for an ideal and steady motion fluid $\rho A v = \text{constant}$, where ρ is the density, while the quantity Av is the (volumetric) flow rate and represents the volume of fluid that crosses the section in a unit of time. From this equation we can obtain the so-called "law of Castelli", which was formulated by the Italian mathematician Benedetto Castelli (*Della misura dell'acque correnti*, 1628): "the sections of the same river discharge the same quantity of water in the same times, even if the sections are unequal", i.e. the flow rate is constant.

Bernoulli's theorem, which owes its name to the Swiss mathematician Daniel Bernoulli, expresses the principle of conservation of energy. Many consequences can be deduced from this theorem. From an historical point of view, we can quote the law of the efflux. Thanks to the analogy with free fall, Evangelista Torricelli finds a useful model to explain the relationship between speed, pressure and depth (*De motu aquarum*, 1644).

To conclude our activity, we can give an overview of some particular devices for measuring the speed of water at different depths, distinguishing between "fixed water gauges" and "moving water gauges". We can take inspiration from the rich iconographic apparatus present in some of the most important eighteenth-century Italian treatises: F. D. Michelotti (*Sperimenti idraulici*, 1767-71); T. Bonati (*Delle Aste Ritrometriche e di un nuovo Pendolo per trovare la Scala delle Velocità di un'Acqua corrente*, 1799).

3 Re-launching Italian education and research after political unification

After political unity in Italy, in 1861, there was a strong resumption of mathematical research and education. On the one hand, a connection with the most advanced sectors of European mathematical research, on the other, a colossal commitment to the creation of a national education system. As for the teaching of mathematics at secondary school level, there was, as in the rest of Europe, a return to synthetic geometry, without the admixture of algebra. The original Euclidean text was initially revived but later new texts were produced and a new didactic proposal emerged to blend plane and solid geometry and introduce new results in elementary geometry, the fusionism (Borgato 2016).

The proposed educational activity consists of a moment of in-depth study and reflection on the root axis theory, with the aim of highlighting the potential of fusionism. The discussion of this theory by the fusionist approach turns out to be particularly effective and gives the opportunity to show some of the strengths of the movement, such as the simultaneous discussion of plane and solid geometry topics and the simplification of some proofs of plane geometry theorems by means of stereometric considerations. The reference text for the development of the curriculum is the original historical source *Elementi di Geometria*, by Lazzeri and Bassani, in the second edition of 1898. The work is available in a digital version: http://mathematica.sns.it/opere/169/; special reference will be made to Chapter III (Systems of Circles and Spheres) of Book III.

The educational activity is designed for students in the second two years of high school and is expected to last 4/6 hours. During each lesson, students, divided into small groups, will read and analyze the texts of selected theorems directly from the original source. The activity will be guided by a worksheet designed to encourage discussion among group members and help them identify the hypothesis and thesis of the proposition. They will also be asked to elaborate the statement again using simpler language. The discussion will extend to the whole class, under the guidance of the teacher, and will continue with the analysis of the proof supported by previously prepared GeoGebra constructions. The most significant constructions will be made by the students themselves in the lab, under the guidance of the teacher or in a discovery activity in pairs.

All the activities included in the educational pathway (selected theorems, worksheets, GeoGebra constructions) are available in Italian on the University of Ferrara Department of Mathematics website at the following link: http://dm.unife.it/matematicainsieme/fusion/index.html. As an example, one of the theorems that will be analyzed in the pathway is given below.

Theorem - Given three circles in a plane, such that their centers are not in a straight line, the three radical axes of these circles, taken two by two, pass through the same point.

Proof - Let c_1, c_2 and c_3 be the three given circles in an α-plane, so that the three centers are not in a straight line. Let three equal spherical surfaces S_1, S_2 and S_3 of radius greater than the radii of the three given circles pass through them, what is always possible, and let O_1', O_2' and O_3'be the centers of these spheres. The planes perpendicular to the segments $O_1'O_2'$, $O_2'O_3'$ and $O_1'O_3'$ at their midpoints, cut the alpha plane according to the three radical axes of the pairs of circles c_1c_2, c_2c_3 and c_1c_3, and moreover pass through the same straight line r (Theorem 161, Cor.) which is the geometric locus of the points equidistant from O_1', O_2' and O_3'. Obviously, the intersection cannot be parallel to the alpha plane because, if it were, the three radical axes would have to be parallel to each other and thus, contrary to the hypothesis, the centers of the three given circles would be in a straight line. So the line r meets the alpha plane at a point, which is common to all three radical axes.

Figure 1 shows the scan from the original source and at the following link 216_Teorema - GeoGebra you can open the GeoGebra construction associated with the proof.

216. Teorema. — *Dati tre circoli in un piano, tali che i loro centri non sieno in linea retta, i tre assi radicali di questi circoli, presi due a due, passano per uno stesso punto.*

Essendo c_1, c_2, c_3 i tre circoli dati in un piano α, in modo che i tre centri non sieno in linea retta, si facciano passare per essi, ciò che è sempre possibile, tre superficie sferiche eguali S_1, S_2, S_3 di raggio maggiore dei raggi dei tre circoli dati, e sieno O_1', O_2', O_3' i centri di queste sfere. I piani perpendicolari ai segmenti $O_1'O_2'$, $O_1'O_3'$, $O_2'O_3'$ nei loro punti di mezzo, tagliano il piano α secondo i tre assi radicali delle coppie di circoli c_1, c_2; c_1, c_3; c_2, c_3, ed inoltre passano per una medesima retta r (§ 161, Cor. 1°), che è il luogo geometrico dei punti equidistanti da O_1', O_2', O_3'. Evidentemente la intersezione r non può essere parallela al piano α, perchè, se lo fosse, i tre assi radicali dovrebbero essere paralleli fra loro (§ 40, Teor.), e quindi, contrariamente all'ipotesi, i centri dei tre circoli dati sarebbero in linea retta. Dunque la retta r incontra il piano α in un punto, il quale è comune a tutti e tre gli assi radicali.

Figure 1. Theorem 216, Lazzeri and Bassani (1898, p. 188)

4 The geometry of paper folding and the resolution of problems of third degree

This workshop refers to the use of paper folding to construct the solutions of third-degree problems, introduced by T. Sundara Row in 1893 and then completed by Margherita Beloch (1879-1976); she dealt with this topic in the courses for prospective high school mathematics teachers (courses of "complementary mathematics") at the University of Ferrara in the first half of the 20th century.

As is well known, some elementary geometric problems (trisection of the angle, duplication of the cube, squaring of the circle) are impossible to solve with the classic instruments ruler and compass, but they can instead be solved with paper folding techniques. The necessary material to realize this workshop is: sheets of paper and a pen to mark points and the folds, when needed. The paper, therefore, is the tool with which the demonstrations are carried out and also the physical place where the demonstration takes place. This makes paper folding workshops even more engaging and challenging.

The activity is addressed to high school students who know what a parabola is (focus, directrix), what a polynomial is and what it means to look for its roots; it will be up to the teacher to decide how much to go deep and develope the formal demonstrations. The workshop can also be offered to university students of different study courses (perhaps going more into the detail of the demonstrations), and, vice versa, to younger students, restricting to the visualization of the parabola with first activity.

The method of resolution is illustrated by Beloch very briefly (Beloch 1934); in the essays (Borgato & Salmi 2018, Magrone & Talamanca 2018, downloadable on the web page indicated in the abstract) a lot of details can be found.

To start the workshop, we recommend to give some news about Margherita Beloch (see, for example, Magrone 2021): she was a leading exponent of the Roman school of mathematics led by Guido Castelnuovo (1865–1952) and Federigo Enriques (1871–1946) as well as founder of the school of mathematics at the University of Ferrara - where she was full professor from 1927 to 1954 - and one of the first Italian women mathematicians to become a university professor. Her research interests ranged from algebraic geometry to more applied and pioneristic topics such as photogrammetry and röntgenfotogrammetry. Then, to enter in the realm of paper folding geometry, project on a screen the pictures and definition of the basic folds of origami geometry, observing possible similarities with Euclid's axioms.

The first hands-on exercise consists in implementing the fold O5 repeatedly; this leads to folding the envelope of the tangents of a parabola. At this point, start a “mathematical conversation” with the participants about why the displayed curve is an actual parabola (see Magrone & Spreafico 2022). Next step consists in realizing the fold O6, also called “Beloch’s fold” (Borgato & Salmi 2018, Magrone & Talamanca 2018): the line obtained is the common tangent to two parabolas (Fig. 2, right).

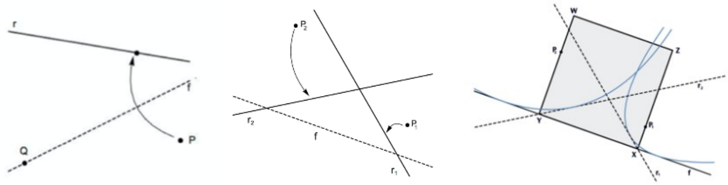

Figure 2. From left to right, Fold O5, fold O6 or Beloch's Fold, Beloch’s square and the two parabolas with their common tangent. Pictures are taken from Magrone & Talamanca 2018

This fold leads to the solution of third-degree problems by the application of the method invented by Edward Lill (1830-1900), a graphical way to find the real roots of polynomials. Beloch’s fold enables a graphical construction, which leads to find a “resolvent path” envisaged in Lill’s method (for details see again the above mentioned papers). The solution is physically constructed with paper, in particular a certain angle is found, and the trigonometric tangent of this angle gives the numerical value of the desired root.

Conclusions and feedbacks from students

In this paper, we have presented a variety of activities, focused on the use of primary sources of history of mathematics in teaching mathematics. Some of these activities have been already tested in the classroom and have had interesting feedbacks.

A part of the parabola folding workshop was tested by P. Magrone in 2022 in a pilot experiment together with M. L. Spreafico, with two parallel groups of students (two groups of 75 students, for details see Magrone & Spreafico 2022). The activity included both the visualization of a parabola as an envelope of tangents, obtained with paper folding, and further exercises on focal properties and coordinates (not considered in the present paper). A well-known topic as the parabola curve, widely studied in high school, is "rediscovered" through an activity such as origami, and looked at with new eyes;

conic curves are proposed at school almost exclusively from an algebraic point of view, seen almost exclusively as equations and the geometric definition is often neglected, while it is essential from both an historical and epistemological point of view. The action of folding embodies the geometric definition of the parabola, so the attention is on the geometric locus. As soon as each student has ended the folding, the teacher asks to prove that the curve that appears is actually a parabola. This question puzzles the students, they are not used to solve this kind of problems, where they have almost nothing but a sheet of paper marked with folds. The search for the answer forces them to think deeply on the geometric definition of parabola and to recall the gestures that lead to the creation of the curve. The (anonymous) tests on the satisfaction of the activity that we administered at the end showed how much the hands-on session was appreciated, so much so that many students suggested us to propose it again to their colleagues for the following year.

Starting from the 2019-2020 school year, the activity on Agnesi's Witch has been experimented on multiple occasions, both at lower and upper secondary school levels. In total, ten classes and over 200 students were involved (Scalambro 2022). From the feedback of people who took part in the activity, it emerged that the use of historical sources contributes to promoting a narrative approach even in an “abstract” discipline like mathematics, encouraging students’ engagement. Some peculiar aspects surfaced from the experimentation of this laboratory in lower secondary school. Firstly, the importance of developing curiosity and interest towards the subject, also addressing topics that may seem too “high” or complex. Secondly, there is a risk often encountered in daily teaching practice of spurring students to study mathematics exclusively in relation to its usefulness. Teachers involved instead emphasized how such historical approach contributed to contextualize the topics of study and to dismantle – at least in part – the idea of mathematics primarily linked to its practical implications. Finally, embracing the conception of mathematics as an activity that can be described as a long conversation over the millennia, the aspect of identification with the presented character is essential. In this case, it is important to keep in mind that Agnesi is a woman and a precursor of her time, thus representing a significant example to students regarding the love and dedication for mathematics, as well evidenced by the excerpts from the *Instituzioni Analitiche* presented during the activity.

Other activities have been designed, preparing both a selection of primary sources and some supplementary worksheets, that can be a useful guide for teachers.

All the materials prepared for our workshop can be freely consulted online on the website of Mathesis Ferrara at the link: http://dmi.unife.it/it/ricerca-dmi/mathesis/materiali-esu-9

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MULTIMEDIAL SOURCES

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