IN SEARCH FOR THE EARLY ROOTS OF THE EUROPEAN MODERN MATHEMATICS MOVEMENT

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ABSTRACT

The origins of the European modern mathematics movement are to be situated in the early 1950s within the International Commission for the Study and Improvement of Mathematics Teaching. In 1952, Caleb Gattegno, the actual animator of the Commission, managed to bring the Bourbakists together with the Swiss psychologist Jean Piaget on the theme of "mathematical and mental structures." A connection was established between the structures in the work of Bourbaki and the structures of children's cognitive development as revealed by Piaget. For that reason, it was argued that the didactics of mathematics needed to be rethought from the Bourbaki perspective on mathematical structures. A model for the science of mathematics gradually became a model for mathematics education.

1 Introduction

The 1959 Royaumont Seminar is commonly seen as the official launch of the modern mathematics movement in Europe (Schubring, 2014). The Royaumont Seminar was indeed a decisive meeting: For the first time in history, European mathematicians such as Jean Dieudonné, Gustave Choquet, and André Lichnerowich, who were members of or had a strong connection to Bourbaki, as well as American reformers such as Edward G. Begle, director of the governmentally strongly supported School Mathematics Study Group, were brought together. However, the claim that the New Math originated in the US and crossed the Atlantic in 1959 (Nadimi Amiri, 2017) is unjustified: Already in the early 1950s, ideas were launched in Europe to reorganize the teaching of mathematics according to the model that Bourbaki had developed for the science of mathematics from the end of the 1930s onwards.

The organization that initiated the European reform movement was the International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM), formally established in 1952 after Caleb Gattegno, an Egyptian-born mathematician and psychologist, had paved the way for it in the previous two years. We reconstruct the debates at the first meetings of this group, particularly the meeting in 1952 where the Bourbakists met the Swiss

developmental psychologist Jean Piaget. Debates resulted in the assumption of an alignment of mathematical and mental structures, which became a main argument for the reform of mathematics education in Europe.

2 The debates within CIEAEM in the early 1950s

In April 1950, Gattegno brought together an international group of experts in mathematics, psychology, and education, including some experienced mathematics teachers, in Debden (UK). Although the number of participants was limited, the range of competences had to allow for "a thorough reconsideration of the whole problem of the child and mathematics" (Gattegno, 1947, p. 220). This meeting, followed by two similar meetings in 1951, one in Keerbergen (Belgium) and one in Herzberg (Switzerland), led to the official founding of the CIEAEM in La Rochette sur Melun (France) in April 1952.

In La Rochette, the foundations were laid for the "modern mathematics" movement in Europe. The meeting was not an accidental encounter between top mathematicians and top psychologists/epistemologists of that time; on the contrary, the meeting was carefully prepared by Gattegno who, as a holder of doctorates in mathematics and in psychology, was familiar with the recent developments in these fields, in this case particularly the work of Bourbaki and Piaget. Already at the Debden meeting, Gattegno would have announced: "I will have the Bourbakists, I will have Piaget, I will have Gonseth" (Félix, 1986, p. 26).

The debate was initiated by Dieudonné who outlined Bourbaki's points of view, paying particular attention to the origin and essence of structures in modern mathematical science (Félix, 1986). He argued that structures are by no means artificial constructs that appear out of nowhere; they are "explicitations" of ideas that were already present in the work of great mathematicians of the past, implicitly and under different guises, but which were not yet recognized as such. Their role in mathematical research was clarified by Lichnerowicz: "A structure is a tool that we search for in the arsenal we have at our disposal. It is not at this stage that it is created" (quoted from Félix, 2005, p. 82). Choquet and Lichnerowicz also testified about how they actually used structures in their research (Félix, 1985, 1986, 2005). For the teachers in La Rochette, the vision of the Bourbakists and the way they practiced mathematics was nothing less than a revelation. After World War II, Bourbaki's work

was known to research mathematicians, but most secondary school teachers, even those who had graduated in mathematics, were completely ignorant of this "modern" evolution within mathematics.

During the discussion, Dieudonné emphasized that the Bourbakists were not dealing with questions of a philosophical or metaphysical level, only common logic was used (Félix, 1986). Associations of mathematical structures with extra-mathematical constructs were not suggested by Bourbaki, but they were established by Piaget, who explicitly related Bourbaki's structures to the mental operations through which a child interacts with the world (Piaget, 1955)[81]. More specifically, Piaget identified the fundamental structures and stages of early mathematical thinking, as revealed by (his) psychological research, with the mother structures in the work of Bourbaki:

> *Now, it is of the highest interest to ascertain that, if we retrace to its roots the psychological development of the arithmetic and geometric operations of the child, and in particular the logical operations which constitute its necessary preconditions, we find, at every stage, a fundamental tendency to organize wholes or systems, outside of which the elements have no meaning or even existence, and then a partitioning of these general systems according to three kinds of properties which precisely correspond to those of algebraic structures, order structures, and topological structures.* (Piaget, 1955, pp. 14–15)

Piaget's identification of Bourbaki's mother structures with the basic structures of thinking, implying a harmony between the structures of "contemporary" mathematics and the way in which a child constructs mathematical knowledge, had a straightforward pedagogical implication: The learning of mathematics takes place through the mother structures of Bourbaki, the structures with which 20th-century mathematicians had founded and built their science. Correspondingly, Piaget (1955) asserted that "if the building of mathematics is based on 'structures', which moreover correspond to the structures of intelligence, then it is on the gradual organization of these operational structures that the didactics of mathematics must be based" (p. 32). In other words: A model for the science of mathematics was promoted as a model for mathematics education. Some teachers who participated in the 1952 meeting

[81] Piaget (1955) was the summary of his presentation in La Rochette (1952), as mentioned in a footnote to that book chapter.

in La Rochette immediately adapted their teaching practice to what they had learned (Félix, 1986).

Bourbaki's reconstruction of mathematics from a limited number of basic structures, connecting different branches of this science and underlining its fundamental unity, "supported" by Piaget's theory of cognitive development, would further stimulate the debate within CIEAEM. For the 1954 meeting in Oosterbeek (The Netherlands), the theme "The modern mathematics at school" was chosen. One of the questions concerned the adaptation of curricula "in the light of what we know about modern mathematics and the thinking of the child and the adolescent" (Lenger, 1954–1955, p. 58). According to Frédérique Lenger, it was primarily up to the teacher to introduce something of the spirit of modern mathematics into their teaching.

> *Modern mathematics […] is the result of an awareness of structures and the relationships between these structures. The thinking of the modern mathematician is relational. And it seems to me that relational mathematical thinking can be recreated by a child or adolescent if the teacher is aware of it and if he presents the appropriate situations*. (Lenger, 1954–1955, p. 58)

3 UNESCO enters the scene

In the mid-1950s, a systematic effort to map national developments in the teaching of mathematics at the secondary level was undertaken by UNESCO, in cooperation with the International Bureau of Education led by Piaget (UNESCO, 1956). In an extensive survey of secondary school mathematics, in which 62 countries participated, one of the questions reflected a growing interest in modern mathematics: "to what extent does the evolution of modern mathematics affect secondary education?" (p. 10). The review of the answers of the 62 countries stated:

> *The question as to what precise extent modern developments in the mathematical field have affected the secondary teaching of mathematics was answered by some twenty of the countries. In some cases the reply states merely that those developments were taken account of in the formulation of the secondary mathematics syllabuses. In other cases details are given of definite modifications made or impending in those syllabuses, including the introduction of infinitesimal calculus, coordinate geometry, statistics,*

etc., and added stress on functions, vectors, the calculation of probability, differential and integral calculus, and applied mathematics. (pp. 26–27)

Mathematical structures were not included in this list. Willy Servais, who was a delegate for Belgium at the UNESCO conference in Geneva in July 1956, at which the results of the survey were discussed, did make explicit mention of modern mathematical structures in his report:

> *To what extent can the more abstract […] mathematical structures, discovered within classical mathematics and developed worldwide by today's mathematicians, have a beneficial impact on secondary education? This is a very recent question to which pioneers in many countries are seeking an answer. The results obtained so far hold the promise of a pedagogical innovation.* (Servais, 1956–1957, p. 40)

In general, survey responses revealed an awareness of the need for changes in secondary school mathematics curricula, but little mention of implemented reforms. At the national level in Europe, discussion about a modernization of secondary mathematics curricula emerged from the mid-1950s, particularly in France and Belgium, and from the end of the 1950s (but before the 1959 Royaumont Seminar), some concrete reform initiatives were taken (see, for example, Barbazo & Pombourcq, 2010; De Bock & Vanpaemel, 2018, 2019).

4 Conclusion

At the meetings of the CIEAEM in the early 1950s, especially at the 1952 meeting in La Rochette, the seeds were sown for a structural approach to the teaching of mathematics, i.e., modern mathematics. Bourbaki offered the mathematical rationale and Piaget provided the psychological justification. In the subsequent years, the CIEAEM would continue to play an important role in refining its modern view on mathematics education.

The European modern mathematics movement was certainly not an "import product" of the American New Math. On the contrary, our findings provide evidence for Bob Moon's claim that "a 'wave' of development in the USA 'crossed over' to Europe, although it is oft repeated, may be too simplistic a picture […] a pattern of 'parallel' innovation would be a more appropriate characterization" (Moon, 1986, pp. 46–47).

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