EUCLID'S PARALLEL POSTULATE AND A NORWEGIAN TEXTBOOK IN GEOMETRY FROM EARLY 19TH CENTURY

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ABSTRACT

Bernt Michael Holmboe (1795–1850) was teacher at Christiania Kathedralskole in Norway from 1818 until 1826, after that he was professor at the University of Christiania. Holmboe wrote textbooks in mathematics, and he was very influential in the development of school mathematics in the first half of the 19th century in Norway. Holmboe's presentation of the subject matter was traditional and "Euclidean". The focus of this paper will be to elaborate on Holmboe's approach to the problematic topic of parallel lines, and the parallel postulate. Holmboe attempts to give a proof of the parallel postulate, and this proof is presented in detail.

1 Introduction

This paper will show how the parallel postulate from Euclid's *Elements* (Euclid, 1956) was presented in a textbook in geometry written by Bernt Michael Holmboe from Norway (Holmboe, 1827). The study goes backwards in theorems, step by step, to see how Holmboe's argumentation leads up to a theorem which is to be understood as the parallel postulate.

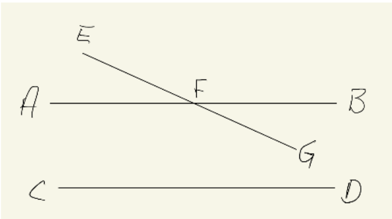
There will first a brief background of Euclid and his well-known parallel postulate, and a short presentation of the Norwegian mathematician and textbook author Bernt Michael Holmboe. The main focus will be to elaborate on Holmboe's approach to parallel lines, and the parallel postulate. Holmboe attempts to give a proof of the parallel postulate, which will be presented in detail. Issues addressed in this paper, and earlier research about the textbook in geometry by Bernt Michael Holmboe, may be found in Christiansen (2009, 2010, 2012a,b, 2015). All translations from Norwegian and Danish-Norwegian to English are done by the author.

2 Euclid's parallel postulate

The Elements by Euclid (Euclid 1956) has for over two thousand years been the exemplar for classical geometry, and for the axiomatic and deductive structure of pure mathematics. The impact of the Elements on modern textbooks in plane geometry is obvious, and it was common for some countries to use books 1 through 6 as textbooks before making their own textbooks.

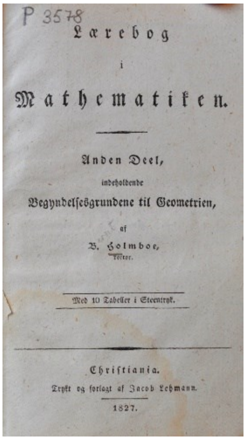
Among Euclid's postulates, the *parallel postulate* occupies a special position, because it does not have the same intuitively obvious character as the others. Therefore, there was long disagreement as to whether this was an axiom or a theorem. Many attempts were made to prove it, but only through the construction of the non-Euclidean geometries in the beginning of the 19th century was it definitively established that such evidence could not be made.

Proclos lived 410–485 AD and wrote a commentary to Book 1 of the Elements. Before giving his proof of the parallel postulate, he examined an argument which is similar to the one about Achilles and the tortoise where he is continouisly halfing the distance between the two lines to



show that it is impossible for them to ever meet. In his proof, the lines AB and CD are parallel, and the line EFG cuts through AB, then EFG will also cut through CD. His argument is that FB and FG are two straight lines from the point F, and the distance between them will increase, which is correct, but without the parallel postulate we cannot assume that the distance between AB and CD does not also increase (Euclid, 1956; Gray, 2008).

3 Holmboe as author of textbooks



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Lærebog
i
Mathematiken.

Anden Deel,
indeholdende
Begyndelsesgrundene til Geometrien,
af
B. Holmboe,
Lector.

Med 10 Tabeller i Steentryk.

Christiania.
Trykt og forlagt af Jacob Lehmann.
1827.

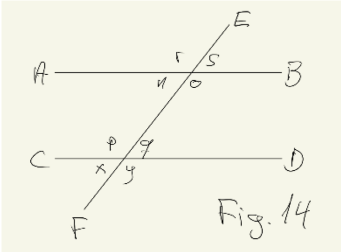
Bernt Michael Holmboe (1795–1850) was teacher at Christiania Kathedralskole in Norway from 1818 until 1826, and after that he was professor at the University of Christiania until his premature death in 1850. As a young man, he became the teacher of Niels Henrik Abel, and Holmboe is today best known as the teacher who discovered Abel's genius and became his first benefactor. Holmboe wrote most of the textbooks in mathematics that were used in the learned schools in Norway between 1825 and 1860, and he was a very influential person in the development of school mathematics in Norway in this period. He wrote textbooks in arithmetic, geometry, stereometry, trigonometry, and higher mathematics.

The textbooks in geometry came in four editions, but only the first two in Holmboe's lifetime. The differences between these two editions are not large, some corrections of errors, and a few re-arrangements of statements. His textbook in geometry were – with one exception – used in all the learned schools in Norway in this period. Holmboe's presentation of the subject matter was in many ways very traditional and "Euclidean".

4 Holmboe and the parallel postulate

It is the belief of this author that Holmboe is aiming to give his students an intuitive proof of a corollary in his textbook that corresponds to the parallel postulate. This chapter will present the line of theorems and corollaries needed for the proof of this, to show Holmboe's reasoning. Holmboe does nowhere in his textbook mention Euclid or the parallel postulate. All quotations are from Holmboe (1827).

§41 — Theorem 41 is a theorem saying that "[w]hen two parallel lines are intersected by a straight line, then all outside angles are equal to their corre-

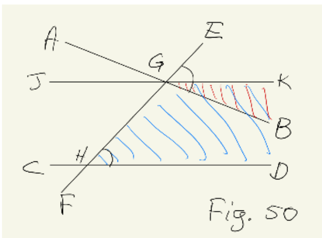


sponding inside angles". In this figure r = p, and so on. This is proved by assuming the opposite and showing that you then will get a contradiction. Theorem 41 in Holmboe's textbook corresponds to Proposition 29 in the Elements (Euclid 1956, vol. I, p. 311). Proposition 29 is also proved by assuming the opposite and using the parallel postulate to prove the contradiction.

Five corollaries follow from Theorem 41, showing all the different relations between the sizes of the angles and let's look at two of them in more detail. Corollary 1 state, among other things that "[w]hen two parallel lines are intersected by a straight line, then inside opposite angles are equal", n = q and o = p, and it is proved by using theorem 41. Corollary 2 states, also among other things that "[t]wo straight lines that are intersected by a third straight line are not parallel in the following situations" ... and one of those situations is "[w]hen the sum of a pair of inside angles is not equal to 2R [two right angles]". That is $n + p \neq 2R$ or $o + q \neq 2R$.

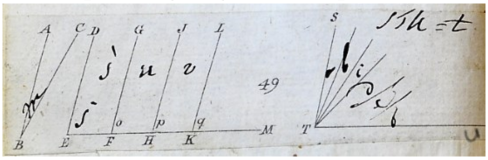
This is the parallel postulate, and it is also proved by using the conditions stated in Theorem 41.

§40 — Theorem 41 is proved by using Theorem 40 which states that "[w]hen two parallel lines are intersected by a third in such a way that an out-



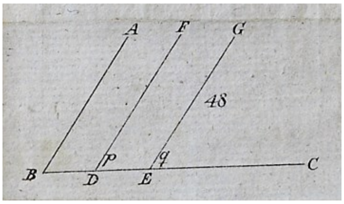
side angle is greater than the opposite inside angle, then the lines are not parallel". If EGB > EHD then AB is not parallel to CD. This proved by a result from Theorem 39 that states that if all these lines are prolonged indefinitely, then will the plane KGB be larger than the plane KGHD, and therefore must AB cut CD.

§39 — Theorem 39 simply states that "[t]he plane between the legs of an angle is always larger than the plane between two straight lines on one side of a transversal when corresponding angles are equal and when all lines are pro-



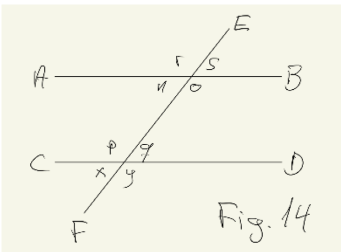
longed indefinitely". This is proved by constructing an angle that contains the angle ABC so many times that it becomes larger than the angle DEM. We have STU > DEM, and <STU = n ·ABC. On the line EM you then make a number of congruent copies of DEFG as shown, and very briefly, n · ABC > DEM > n ·DEFG, which entails that the plane ABC is greater than the plane DEFG.

§38 — Theorem 38 is a necessary theorem simply stating that when BD = DE and the angles B, p, q is equal, then the planes ABDF and FDEG are con-



gruent. Paragraph 37 is just an exercise showing how to construct a parallel to a given line through a given point.

§36 — That brings us to Theorem 36 which says that "[w]hen two straight lines are intersected by a third such that one outside angle is equal to its corre-



sponding inside angle (r = p), then these two lines will not intersect no matter how long they are prolonged at both sides". The proof for this is that if for instance A and C meets, then these lines together with the transversal EF will form a triangle and then will r > p which contradicts the condition.

After this theorem Holmboe presents the definition of parallel lines which is practically the same as Euclid's – "Two straight lines in the same plane which does not meet when prolonged infinitely to both sides, are said to be parallel to each other, or the one is parallel to the other".

Holmboe's backward line of theorems end in the fact that the sum of angles in a triangle equals 2R, which is equivalent to Euclid's parallel postulate.

5 Conclusion

There is no focus on parallel lines as a topic in the Norwegian primary school today. The definition used is that two lines that never meet are parallel to each other. One of the learning objectives in 4th grade is to "explore, describe and compare properties of two- and three-dimensional figures using angles, edges and corners", and one of the learning objectives in 6th grade is to "describe properties of and minimum definitions of two- and three-dimensional figures and explain which properties the figures have in common, and which properties distinguish them from each other". In 9th grade, two of the learning objectives are to "explore the properties of different polygons and explain the concepts of formality and congruence" and to "explore, describe and argue for relationships between the side lengths of triangles". (Kunnskapsdepartementet, 2019)

Pupils today need of course understanding of parallel lines for these objectives, and for using digital tools, but the topic of parallel lines is not emphasized as it was almost 200 years ago. Modern pupils understand what parallel lines are, but they may be lacking the awareness that the concept has tremendous difficulties. In Holmboe’s textbook (Holmboe, 1827) the second chapter is called "About two straight lines intersected by a third", 35 pages long, and the third chapter is called "About parallel lines", 29 pages long. Later there is a chapter five called "About straight lines' relationship to each other" which is 13 pages long. The entire textbook is 158 pages.

Holmboe's textbook in plane geometry was a typical textbook for his time, but this way of presenting the subject matter was challenged (Christiansen, 2012a). Textbooks of today represents a much more pedagogical and didactical correct way to present the subject matter, rather than strict demands on rigour in concepts and methods that characterised textbooks from the 19th century and earlier.

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