HISTORY OF MATHEMATICS IN BRAZILIAN SECONDARY SCHOOL TEXTBOOKS

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ABSTRACT

This paper presents some results about the integration between the History of Mathematics and the teaching of mathematics in Brazilian secondary education textbooks.

1 Introduction

The integration between the history of mathematics (HM) and the teaching of mathematics has been widely discussed in recent decades. Several works (e.g. Furinghetti, 2020; Saito, 2018; Fried, 2014) exemplify how this discussion has developed, the conceptions of educators who are dedicated to this integration and the intrinsic tensions between these two areas, history and education, which have different objects, objectives and methodologies.

Our research group CHEMat (*Coletivo de História no Ensino de Matemática/Collective of History in Mathematics Teaching*) studies the integration between HM and mathematics teaching. CHEMat was created in 2017 and is composed of university and school teachers, and undergraduate and graduate students. This paper emerges as a result of these discussions, on understanding which HM reaches Brazilian schools.

In Brazil, basic education is a governmental responsibility. Therefore 82.6% of primary and secondary students are enrolled in public schools (INEP, 2022). The *Ensino Médio* is the Brazilian secondary education level. It consists of three years and it attends students from 14 to 18 years old. It is the last stage of Brazilian school education.

Considering that usually textbooks are the main reference for teachers and students, they play a fundamental role in Brazil due to the PNLD (National Book and Teaching Material Program). The program ensures that all public school students have access to textbooks. Taking these factors into account,

CHEMat has engaged in answering the following question: which HM reaches teachers and students of basic education from PNLD 2018 textbooks?

2 Metodology

To map the presence of HM found in textbooks, we developed a data collection instrument as an online form implemented with the *Google Forms* web application. Each appearance of HM in textbooks is considered an *historical insertion*. Each insertion was analyzed through 60 questions, organized into 12 groups. The questions identify the insertion; indicate its position and layout in the book; identify historical periods and territories, historical characters, documents and research/teaching institutions; describe its iconography; the contents of the chapters in which it is placed, as well as the contents mentioned therein; identify whether or not mathematics and history may be separated in the historical narrative; indicate the proposition of math exercises, historical investigations or other types of activity; identify the references; and investigate the types of historical narrative, the didactic function of insertion (Carlini & Cavalari, 2017), and themes about the uses of history in teaching (Fried, 2014).

After a pilot study carried out between August and September 2020, the collection instrument was adjusted, allowing the group to start collecting data from 8 textbooks collections approved in the PNLD 2018 in March 2021. In this paper, we present the partial results of the study carried out until May 2022 from the collection of data of 4 collections: Luiz Roberto Dante's *Matemática: Contexto e Aplicações*; Eduardo Chavante & Diego Prestes' *Quadrante Matemática*; *Matemática:* Iezzi et al.'s *Ciência e Aplicações*; Joamir Souza & Jacqueline Garcia's *Contato Matemático*.

3 Data analysis

When we went through the 4 collections indicated, each of them with 3 volumes, thus totaling 12 books, we collected 215 historical insertions. In the next paragraphs, we present some partial results regarding the historical periods, territories and historical characters mentioned in the insertions, the themes of Fried (2014) and the didactic function of Carlini and Cavalari (2017).

We begin our analysis by highlighting the *when*, *where,* and *who* elements. It is important to emphasize that a historical insertion can tell episodes from different periods and regions. For example, a single entry might begin by quoting Babylonian tablets from Antiquity, then an Arabic mathematical practice in the Middle Ages, and end with a work in Modern Europe. Thus, the numbers presented account for all these occurrences.

3.1 Whens and wheres

It is quite revealing that the HM of textbooks highly values Antiquity (111) and Modernity (146), but little Medieval (16) and Contemporaneity (27). And even within this scope, the emphasis on events that occurred in Ancient Greece (60) or in Modern Europe (126) is overwhelmingly greater than all the others *when* and *where* identified.

We also note that the territories/periods *Spain, Portugal and other countries in Iberian Europe (from the 16th to the 19th century)* and *Colonial America (from the 15th to the 18th century)* did not receive any mention. This absence is intriguing given that the historical formation of Brazil is directly associated with Portugal. And even advancing to the 19th or 20th centuries, there is no mention of episodes or characters from the HM from Brazil. This is an alarming absence when it comes to textbooks written for Brazilian students, since it can lead the student to think that Brazil does not produce mathematics. We suppose that it is due that the European HM from Modernity to the present is more abundant in the historiography than the history of other territories and periods, including Brazil. Consequently, this excess on one side and absence on the other are not a choice of textbook authors alone.

3.2 Whos

In the next item, we counted 145 characters in a total of 372 mentions. Of these, we have 74 characters, next to half of the total, mentioned only once. As for the most quoted characters, we found 12 names, listed in Table 1. Considering what was observed above, it was to be expected an absence of names of Brazilian characters. Another deserving highlight is the absence of women, reaching all territories and all times.

Frequency	Name	Frequency	Name
16	Gauss, Carl Friedrich	9	Archimedes

15	Euler, Leonhard	9	Euclid (from Alexandria)
13	Galilei, Galileu	9	Fermat, Pierre de
12	Cardano, Girolamo	8	Descartes, René
10	Leibniz, Gottfried Wilhelm	8	Pascal, Blaise
10	Pythagoras	8	Thales

Table 1 – Most quoted mathematicians

We resent that Brazilian history is still not recognized by Brazilian textbooks. We also regret how the textbooks still portray mathematics as an exclusively male activity. The presence in textbooks of Brazilians and women could contribute to making our books more appropriate to the history of our country, more inclusive and more plural in terms of gender equality (Haubrichs & Amadeo, 2021).

The number of people quoted (145) in textbooks is much higher than that of books, works, primary sources, contexts or institutions (53). This shows how much the textbooks still focus the HM on the people and how little valued is the mathematical production itself. It leads us to think that instead of telling episodes of *the history of mathematics* what is being told are episodes of *the history of mathematicians*, or even *the history of male and European mathematicians*.

3.3 Didactic roles

Let's move on to a more qualitative analysis that considers the didactic role of insertion. Considering the work of Carlini and Cavalari (2017), we tried to classify the insertions according to their didactic functions (Table 2). Such categories are not mutually exclusive. It is possible that a historical insertion can be used as a didactic strategy and, at the same time, seek to elucidate why a mathematical property was developed in such a way.

Category	# insertions	Category description
HM and general cultural formation	121	When historical information brings general knowledge related to mathematics, which does not contribute to the learning of mathematical content.
HM and the elucidation of the 'why'	53	When history shows how, why and under what circumstances certain mathematical knowledge emerged.
HM and didactic	51	When history makes it possible to develop

strategy		some mathematical reasoning
HM and the elucidation of the 'for what'	35	When history shows the usefulness or applications of certain mathematical content (in mathematics itself or in other areas) over time or in a specific period.

Table 2 – Didactic function of the insertions. Source: from the authors and adapted from Carlini and Cavalari (2017, pp. 77-78).

Our attention was drawn to the huge number of didactic function insertions dedicated to the general cultural formation of students (121). It is more than double that of each of the others, including the role that prescribes HM as a teaching strategy (51). It concerns us since insertions of this nature do not have any intention of contributing to meaningful learning. The history is presented as a curiosity, an isolated or simple informative fact.

3.4 Historical orientation

The last set of analysis categories is inspired by Michael Fried's (2014) discussion and his look at the possible uses of history in teaching (Table 3). In the use of our data collection instrument, the three themes were not proposed as mutually exclusive.

Category	# insertions
HM as Motivacional Theme	139
HM as Curricular Theme	59
HM as Cultural Theme	7

Table 3 – Fried's themes classification.

The large number of insertions classified in the motivational theme draws attention. According to Fried (2014, p. 682), this perspective of using HM by educators is quite problematic. The motivational theme does not do mathematics justice, as it assumes that mathematics by itself is not interesting to students; they do not do justice to history either, as they do not recognize it as a field of research, but as a collection of curiosities and anecdotes.

It is interesting to note that 90% of *HM and general cultural formation* insertions are also *HM as a motivational theme*. On the other hand, 78% of the *HM as a Motivational Theme* insertions are also of the *HM and general cultural formation*, and 51% of all inserts are simultaneously of both types. This indicates that there may be a close relationship between Fried's *motivational theme* and Carlini and Cavalari's *general cultural formation*.

Different from the *motivational theme*, the *curricular theme* and the *cultural theme* take more seriously the task of integrating the mathematical contents, the didactic exposition and the historical context. Between these two themes, there is a predilection for the *curricular theme*. This result is consistent with the fact that the *curricular theme* is usually associated with an anachronistic view of mathematical concepts. This conception of the history of textbooks seems to us to be associated with the fact that the authors rely on HM books based on a traditional historiography, mainly on Carl B. Boyer's *History of mathematics* and Howard Eves's *An introduction to the history of mathematics*. Both of them are very popular in Brazil and have several editions in portuguese. Consequently, we have an insignificant number of insertions as a *cultural theme*, since this theme presupposes a perspective on history that is more aligned with current historiography, which still has very little circulation between the textbooks authors.

4 Conclusions

The results presented here reveal that the integration between history and mathematics teaching in school textbooks is still very superficial. We positively acknowledge the authors' initiative to include HM in their books. This is revealed by the significant number of historical insertions. However, most of the insertions present a naive way of how history could be integrated into teaching: It prefers an anecdotal history or mere curiosity instead of conceiving history as a didactic or significant strategy of the student's cultural formation and mathematical learning.

We believe that this portrayal of HM in textbooks may be associated, on the one hand, with the historical sources that the authors use and with the way mathematical contents are traditionally presented in textbooks. Inserting history as a motivation to present the mathematical content as it is done in textbooks or inserting mere historical curiosities in isolated boxes throughout the chapters is a way of appropriating the HM, but without significantly affecting the exposure of the content. Integrating HM into teaching could demand a reformulation in the exposition of contents, in the resignification of mathematical concepts, in the nature of the exercises, and ultimately in the mathematics curriculum itself.

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