TEXTBOOKS DEVELOPED IN THE PATH OF MODERN MATHEMATICS (1976-1980): A COMPARISON

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ABSTRACT

From 1968 to 1975 the centralized Portuguese school system began with six years of mandatory education (6-11 years old). Those that wanted to pursue education should choose between secondary schools (liceus) and technical schools. From 1976, a unification of the remaining secondary school years was implemented, and the secondary education comprised a General Course - grades 7, 8 and 9 - and a Complementary Course - grade 10 and 11. The establishing of the Unified Secondary Course required curricular changes, and new mathematics programs are issued. As a consequence, three mathematics textbook collections for the General Course gradually appeared on the market. In the period of our study the ideas of modern mathematics were being applied in Portuguese schools. This text intends to understand different ways to interpret the mathematics programs for Unified Secondary Course, by comparing first editions of these collections of textbooks. We looked at the general structure of the texts, the intended relationship with the reader and the ways in which exercises were proposed. The present text focus on the introduction of the grade 7's topic 'Numerical equations of the first degree in Q' and identifies differences and similarities in the approach to this concept. Results show that two of the collections were organized to promote the active participation of students. Observing the introduction of the topic, there are similarities in the approaches of two of the textbooks that used inverse operations to solve equations whereas the other used scales. It stands out that only one textbook refers to the equivalence of equations. Regarding the exercises, two of the textbooks have an Exercise Book and a Worksheet Book, respectively, to support students' study. The third includes a list of exercises on solving an equation as activities for the students. The study was based on a documental analysis of a descriptive and interpretative nature, with a historical perspective. Our sources were legislation, programs, and textbooks.

1 Introduction

From 1968 to 1974, the centralized Portuguese school system began with six years of mandatory education (6-11 years old), followed by parallel branches for secondary education: the liceus and the technical schools. The 25th of April Revolution that occurred in 1974, overthrew the dictatorial regime and restored democracy in Portugal. Later, the structure of the education system began to change. A major alteration was the elimination of the two education-

al tracks that existed for secondary education and the creation of the Unified Secondary Course, in 1976. The unification was considered a means to balance educational opportunities for all students. The immediate result was the alteration of designation, the Liceus and the Technical Schools became Secondary Schools. The secondary schooling encompassed two parts: the lower secondary (7th – 9th grade) and the upper secondary (10th – 11th grade).

In Portugal, the textbook approval system established one single officially approved textbook for all school disciplines since 1947. After the 1974 revolution, the teachers at each school decided on the adoption of textbooks. They chose among textbooks produced by private publishers (Almeida, 2007).

In the panorama of mathematics education, the reform of Modern Mathematics attempted major changes in content and methods for teaching mathematics, in many countries. This curricular movement brought to the agenda a proposal that gains different interpretations in the countries that participate in this renewal. There are many journals and books that circulate and are translated, disseminating the new modern mathematics. In each country the movement is received and incorporated according to its culture and its specificities (Moon, 1986). The reform took place in all levels of education from primary to higher education in most countries of the world. From the mid-1970s onwards, other curriculum options were developed internationally, and reform was declining (Furinghetti, Matos, & Menghini, 2013).

The Modern Mathematics movement affected Portuguese school mathematics culture in two major ways, paralleling international trends. Firstly, there were major changes of the representation of what constitutes appropriate mathematical content. Secondly, mathematics was believed to be a major driver for social and economic development (Matos, 2009). In Portugal, the events concerning this reform can be divided into three intertwined periods: the beginnings, from 1957 until 1963, in which the flow of new ideas can be detected; experimentation, from 1963 to 1968, during which the new ideas were implemented in classrooms; and dissemination, from 1968, that saw the gradual generalization of the reform to all students. From 1968 until 1974, programs were sequentially modified to incorporate the new ideas as these pupils progressed through the cycles. After gradual changes, the new math programs had been replaced in early 1990 (Almeida e Matos, in press).

These initial considerations give an insight on the context of our study. The end of the single officially approved textbook, allowed the publishing of textbooks that provide their author(s) interpretation(s) of the mathematics programs for secondary education established in 1976. We traced three mathematics textbook collections developed for the lower secondary (7th, 8th and 9th grade), the name of each collection is *Compêndio de Matemática*, *Eu e a Matemática* and *M*. It is the purpose of this text to understand the authors' perspectives presented in these collections of mathematics textbooks by looking at the general structure of the textbooks, the intended relationship with the reader and the ways in which exercises were proposed. Here we will focus on the textbooks for the 7th grade and on the introduction of the topic 'Numerical equations of the first degree in Q'. Data analysis followed a proposal by Maz (2005) adapted to the approach of first degree equations. In it, we looked at the historical context, the author, the structure of the work, and the initial approach of first degree equations proposed by the textbooks.

We based our research on a documental analysis of a descriptive and interpretative nature with a historical perspective. The documentary corpus consists of mathematics textbooks for secondary education. We analysed the first edition of the textbooks within the period of our study. Other sources were legislation and mathematics programs.

2 The collections

2.1. Collection 'Compêndio de Matemática'

The authors António Almeida Costa, Alfredo Osório dos Anjos e António Augusto Lopes were teacher trainers and had been active participants in the reform of Modern Mathematics in the liceus. They were authors of the single official approved mathematics textbook for the liceus, published in 1971.

In the preface the authors state that the textbook is their interpretation of the program, responding to a personal stand on the prosecution of the learning objectives for the 7th grade. The program for the 7th grade has seven topics, but the Index of the textbook show only six chapters, that correspond to six of the programs' topics, the topic 'geometric transformations' is part of the chapter corresponding the topic function. The colour of the text and of the titles of sections is black and red, respectively.



Figure 1. Definitions included in a box Source: Costa e Anjos (1976, p. 82)

There aren't workbooks associated with this collection. There is usually a selection of application exercises and their solution at the end of each section of the chapters. There are no references or indication of other sources related to the mathematics topics addressed in the textbook. The presentation of contents, though expository, tries to motivate the student to reflect and guide him or her in reaching conclusions, leaving, for example, blank spaces to complete, and posing questions to the student. The use of boxes helps to stress parts of the text of the various sections (Figure 1).

2.2. Collection 'Eu e a Matemática'

The first author Maria Engrácia Domingos was a certified teacher of liceus, the other authors Mário Cerqueira Correia and Télio T. Fernandes were authors of textbooks for technical schools.

These books are written only in black and do not have bibliographic references or indication of other sources related to mathematical subjects. A glance at the Index show that the books follow the 7th grade programs' sequence of topics, but they also reveal that the authors had some autonomy, in adding extra contents, regarding pre-requisites.

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~	f) $3a + \frac{1}{3}b - 4a - 6b$					
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Figure 2. Example of a Worksheet Source: Domingos, Correia e Fernandes (1976)

In a foreword, the authors describe the collection and provide recommendations to students and teachers. They explain that there is a Reference Book and a Guidance Book, outlining the purpose of these books. The Reference Book is assumed as a source of information, following all topics of the program for the 7th grade. The Guidance Book' function consists of facilitating the acquisition of knowledge, through Worksheets (Figure 2), which correspond to all the topics presented in the Reference Book. There are references to the pedagogical principles in which the work stands that are those of personalized teaching, which is based on adapting teaching to individual differences. The authors leave proposals to support the teacher's choice of classroom practices. There were Worksheets at the end of each topic that allowed students to assess understanding before moving on.

2.3. Collection 'M'

The authors Paulo Abrantes and Raúl Fernando Carvalho were members of a group of mathematics teachers that advocate changes in mathematics teaching and learning. They participated in the foundation of the Mathematics Teachers Association, in 1986, and had an active role in teacher education and in mathematics education.

Textbooks are printed in two colours, black and red, and do not present bibliographic references or indication of other sources related to mathematical subjects. Exercises Books are printed in black only. The books are structured according to the 7th grade program.

In an introduction to their work, the authors assume it was conceived for the student. So, the caution in preparation included: the language used; the examples chosen; the form of initial approach to the concepts; the form of active participation in the study that is proposed to the student ("activities", notes, ...).

Em cada uma das seis questões, ape- ten uma das respostas — (a), (b) ou (c),— e correcta. Em cada caso, indica qual e, marcando uma cruz no local corres- juniente do quadro.	AUTO-TESTE 3A 1. O número racional $-\frac{1}{2}$ é solução de apenas uma das equações seguintes. Indica qual:				
(5) (6) (6)	(a) $\frac{1}{2} \times -\frac{1}{4} = 0$ (b) $3 - 2x = 4$ (c) $x + 1 = -\frac{1}{2}$				
	2. O conjunto de soluções da equação $\frac{5(3-x)}{2} = 7''$ é:				
The second second	(a) $\{ \}$ (b) $\{ \frac{1}{5} \}$ (c) $\{ -\frac{1}{5} \}$				
X	3. Indica em qual das alíneas seguintes se encontram duas equações equivalentes:				
	(a) • x + 1 = 2x - 1 (b) • $\frac{x}{2}$ + 1 = $\frac{3}{2}$ (c) • 3 - $\frac{x}{3}$ = 2				
al in the sec	• $2x + 1 = -3$ • $\frac{x}{2} - \frac{3}{2} = 1$ • $x - 3 = 0$				
Compara os teus resultados com a	4. De entre as equações que se seguem, uma é impossível. Qual é?				
 Have do fim do livro. Verifica se já és capaz de: 	(a) $\frac{1}{2} - x = 1 - 2x$ (b) $2(x + 3) - 6 = 2x$ (c) $3(x - \frac{1}{6}) - 1 = 3x$				
 verificar se um número é solução duma equação; 	5. Resolve a equação "2 (x + 6) $-3^2 = 2$ " e procura, nas respostas possíveis,				

Figure 3. Example of a self-assessment test Source: Abrantes e Carvalho (1980, p. 49)

There are two books for the 7th grade, a Textbook, which is a study book for students, and an Exercises Book, was aimed at helping the pupils in the study of mathematics. The Exercises Book includes, at the end of each subchapter, a self-assessment test, through which the students can control their learning (Figure 3). The two books are instruments for the student, which allow them to structure, acquire and evaluate their knowledge.

3 Numerical equations of the first degree in Q

The topic equations represented for students the beginning of a new phase in their study of mathematics since this is a basic concept of Algebra. Furthering the study of numerical expressions, involving numbers and operations with which they contacted previously, now, other expressions involving new symbols and manipulation rules arise, which require a higher level of abstraction. This led me to question the introduction of the topic 'Numerical equations of the first degree in Q', to identify aspects that change in the approach to this concept.



Figure 4. A procedural scheme

Source: Costa e Anjos (1976, p.82)

All the collections start with a motivation problem and address at the very beginning of the chapter the language associated to equations (notion of equation, variable, solution of an equation, ...). The 'Compêndio de Matemática' introduces the solving of equations by using inverse operations of addition and multiplication (Figure 4). And the collection 'Eu e a Matemática' uses an approach based on a procedural scheme like the one above to find the value of the unknown variable that gives meaning to equality. These collection present procedures that can be performed on both sides of the equation, that will lead to practical rules to resolve an equation. The collection 'M' introduces the solving of equations using scales method ("balanças") (Figure 5) to find the solution. It is the only collection which refers to equivalent equations and its rules.



Figure 5. The use of a scale to illustrate the situation Source: Abrantes e Carvalho (1980, p. 58)

4 Final remarks

Our textbook analysis show that the books are structured according to all the knowledge that a student is supposed acquire on the topics proposed in the 7th grade program.

Though we can trace, in all collections, an understanding that students construct their own knowledge, only in two of the collections the authors assume that their work is organized to facilitate and promote the active participation of students in their learning. Regarding the way in which the three works introduce the topic 'Numerical equations of the first degree in Q', there are two different approaches to make equations and their solving comprehensible for students. One the one hand, there is a schematic style that uses inverse operations. On the other hand, there is the use of scales.

Research becomes very difficult when investigating past school practices, because usually only indirect evidence can be obtained, as is the textbooks. So, it would be interesting to study in which extent these collections were used and for what purpose.

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