

# THE DEFINITIONS OF THE CONCEPTS “CIRCLE” AND “RHOMBUS” IN GREEK MATHEMATICS TEXTBOOKS USED IN PRIMARY EDUCATION FROM 1830 TO THE PRESENT DAY

Garyfallia Alexandridou<sup>1</sup>, Yannis Thomaidis<sup>2</sup>, Konstantinos Nikolantonakis<sup>3</sup>

1. Primary School of Chortiatis, Thessaloniki, Greece, g\_alexandridou@hotmail.com

2. School Advisor (retired), Thessaloniki, Greece, gthom54@gmail.com

3. University of Western Macedonia, Florina, Greece, knikolantonakis@uowm.gr

## ABSTRACT

In this paper, the definitions of the concepts “circle” and “rhombus” are studied and recorded, as they appear in Greek mathematics textbooks used in primary education from the beginning of the 19th century to the present day. The study concludes that the traditional teaching method in the past was based on the approach of geometric concepts through the teaching of stereometry. The formal definitions of the concepts were included in the older textbooks, while today there is an absence of definitions of the basic geometric concepts, as teaching is done in an empirical way, emphasizing on manipulative and pictorial material. A correlation is also made between these definitions and those recorded in the course of the evolution of mathematics for the above notions, highlighting aspects of their historical development, with an ultimate aim to their further usage in teaching and learning.

## 1. Introduction

Mathematical definitions are fundamental to the axiomatic structure that characterizes geometry and play a key role in the development of deductive reasoning and proof skills (Mariotti & Fischbein, 1997). They are the most important means of conveying the meaning of mathematical concepts, the basic tools of mathematical language used for written and oral communication in teaching and learning mathematics (Shir & Zaslavsky, 2001). In addition, definitions help to distinguish between critical and redundant elements so that one can clearly see a mathematical situation and can distinguish examples from counterexamples of a concept. In this way, deeper conceptual understanding is achieved and the use of correct mathematical terminology is reinforced (Morgan, 2005).

School textbooks are a fundamental teaching tool that is used in the educational process, being an important resource for students, as they are among the most important factors influencing their learning opportunities in mathemat-

ics, while at the same time serving as a means of guiding the teacher. Moreover, mathematics textbooks as a theme of research have continued to receive rapidly growing attention internationally. Investigations on the development of the exposition of a given concept over a certain period contributes to understanding the meaning attributed to the concept in a given community in that period (Schubring & Fan, 2018).

Therefore, recognizing the importance of the role of definitions in mathematics education and given that textbooks are the main source of educational opportunities for students to teach and learn mathematics, this study presents the historical development of the definitions of the concepts of the circle and the rhombus in Greek mathematics textbooks from the beginning of the 19th century to the present day. Our points of interest are the way of approaching the above primary concepts of geometry, including the definitions used, in terms of their nature and type. The study takes into consideration whether the definitions presented are “genetic”, as they show how the concept is constructed, or whether they are given on the basis of the main characteristics of the concepts. Furthermore, it is examined whether the definitions are “hierarchical” or “partitional”. According to DeVilliers (1994), the classification of any set of concepts depends on the process of creating a definition of the corresponding concepts. A “hierarchical” definition allows the inclusion of more particular concepts as subsets of the more general concept. In a “partitional” definition on the other hand, the concepts involved are considered disjoint from each other (i.e. squares are not considered rectangles).

From the study of school textbooks, the definitions of the above concepts from five (5) textbooks from the period 1830-1982 were recorded and selected to be presented in this paper, as well as the definitions contained in the mathematics textbooks used from 1982 to 2006. Moreover, this paper also presents the way of teaching the geometric concepts of the circle and the rhombus as suggested by the current Curriculum, mathematics teaching guidelines and teacher's books.

## ***2. School mathematics textbooks from 1830 to 1982***

### **2.1 Fatseas, A. (1870). Compendium of geometry A.M. Legendre**

This Geometry begins like Euclid's first book, first introducing the "definitions" of the basic concepts and then proving the "propositions" that follow.

The circle is defined as: "the shape produced by the rotation of a line about one of its ends in a plane" (p. 18). This definition is genetic, as it shows the 'birth' of the circle from the rotation of a straight-line segment, which remains in the same plane, around one of its ends.

The way a circle is constructed is also contained in Heron's definition of the circle, where it is explicitly mentioned: "Circle is the figure described when a straight line, always remaining in one plane, moves about one extremity, as a fixed point until it returns to its first position" (Kipouros, 1995, p. 33).

Also, according to Spinoza, the most appropriate definition of a circle is "a figure constructed from a line by holding one end of the line in place while rotating the other end" (Goudeli, 2015, pp. 7-8).

The rhombus is defined as the quadrilateral "having sides equal and angles unequal" (p. 4). It is noted here that the rhombus is not part of the family of parallelograms but is separated from the other concepts as a subcategory of quadrilaterals. Therefore, the definition is partition and is given based on equality of sides and angles of quadrilaterals.

## **2.2 Karagiannidis, A. (1906). Practical geometry for the 4<sup>th</sup>, 5<sup>th</sup> and 6<sup>th</sup> grades**

The book is divided into three parts. In the first part, the rhombus is defined as "the parallelogram which has all sides equal to each other" (p. 14). This definition is hierarchical, as it includes the rhombus in the family of parallelograms. It is also stated that all squares are both rhombuses and rectangles because they have both sides equal to each other and right angles. This classification is consistent with the modern hierarchical classification of quadrilaterals.

The second part begins with the study of the sphere. In this section there is a definition of the circle as the "flat surface, one point of which is equidistant from all the points of the line to which it ends. This point is called the center of the circle" (p. 24). This definition is in accordance with Euclid's definitions (15) and (16), where it is stated that "a circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another. And this point is called the center of the circle" (Exarchakos et al., 2001, p. 91). Thus, the term "circle" here also means the circular disc, while as stated a little further on, the

"line on which the circle ends is a single curve and is called the circumference" (p.24).

### **2.3 Michaelides, E. (1946). The Little Geometer - Practical Geometry**

In this book, in the section on rectangular parallelepipeds the rhombus is defined as "the parallelogram that has all four sides equal" (p. 27). This is followed immediately by instructions for constructing a rhombus and it is clearly stated that a square is not a rhombus. It is worth mentioning that, of all the geometry books studied for this paper, this book is the only one that contains the term "rhomboid" by which is meant the oblique parallelogram, just as in Euclid's Elements. Specifically, it is stated that "the oblique parallelogram is a rhomboidal shape (the shape that pieces of baklava pastry are usually shaped like)" (p. 28).

The circle is studied in the section of the cone. It is approached empirically, as students are asked to copy the base of a cone and the resulting shape is then called a circle. "The circle is bounded by a closed and regular curved line. This closed and regular line is called circumference" (p. 51).

### **2.4 Papadopoulos, P. D. (1952). Practical geometry for the use of pupils of the 5<sup>th</sup> and 6<sup>th</sup> grade of primary schools**

In Papadopoulos' Geometry, the rhombus is studied in the section on the oblique parallelogram and is defined as the oblique parallelogram that has all four sides equal (p. 43).

In the next section, and after a preliminary study of the cylinder, students are intuitively introduced to the concept of the circle. Here "circle is called a flat surface enclosed by a curved line, all the points of which are equally distant from a point on the surface, which is situated in the middle and is called the center of the circle" (p. 64). It appears that by "circle" is also here understood the circular disc.

### **2.5 Kyriazopoulos, A. & Alexopoulos, V. A. (1969). Arithmetic – Geometry of the 5<sup>th</sup> grade of primary school**

In this textbook the study of the rhombus is done in the section of parallelograms where the hierarchical definition given is "a rhombus is an oblique parallelogram which has all its sides equal" (p. 145). The circle is studied in the

next section. As in the previous textbooks, the concept is first approached intuitively and then it is defined. Thus, "circle is a flat segment enclosed by a closed curved line, all the points of which are equally distant from a point called the center" (p. 152).

### ***3. School mathematics textbooks used in the period 1982-2005***

In 4<sup>th</sup> grade, students are asked to construct a circle to lead them intuitively to the conclusion that a circle is the curve formed after a complete rotation of the end of a paper strip around a fixed point.

In the 5th grade, geometric solids are studied first and later flat shapes. The rhombus is included in the family of parallelograms and is then said to have all four sides equal, although this is not a formal definition of the concept. In the next chapter, the concept of the circle and the circular disk is taught. An introductory activity helps students to distinguish the concept of a circle from that of a circular disc.

In grade 6, the lesson concerning the circle begins with some review questions and then the definition of the circle is given: "A circle is a closed curved line whose points are all equally distant from the center, while a circular disk is the flat surface enclosed by the circle" (p.126).

### ***4. School mathematics textbooks used from 2006 to the present day***

In the current mathematics textbooks for all grades of primary school, there are minimum formal definitions of basic geometric concepts. It is also observed that formal definitions of the concepts circle and rhombus are not mentioned in the textbooks. Concerning the teaching of these concepts, it is concluded, that the circle is a concept that is studied very often in all primary school grades, while on the other hand, there is a lack of involvement of pupils with the concept of the rhombus, which appears for the first time as a teaching subject in the third grade. An attempt to emphasize the hierarchical relationships between quadrilaterals is evident from the proposed 4th grade activities, with particular emphasis on students' understanding that the square is a special case of the rhombus and the rectangle. In grade 5, no reference is made to the concept of the rhombus, but a deeper approach to the concept of the circle is attempted, as pupils are introduced for the first time to the terms relating to the main elements of the circle (center, radius, diameter, length,

number  $\pi$ ). In grade 6, some geometric definitions and formulas are given, but these do not include definitions of basic geometric concepts.

## 5. *Conclusions*

From the study of the textbooks, it became evident that the approach to basic geometric concepts, was usually through the teaching of stereometry. It was also observed that while the older textbooks included formal definitions of concepts, today geometry in primary school is approached intuitively.

Regarding the definitions of the concept of the circle, it appears that in most textbooks the definitions have common elements with Euclid's definition. The only textbook that contains a genetic definition of the circle is Fatsea's Compendium of Legendre's Geometry (1870), which describes how the circle is constructed (like Heron's and Spinoza's definition).

It is also observed that the meaning attributed to the term "circle" changes over the years. Euclid's definition establishes the circle as the surface delimited by its circumference, as is similarly observed in school geometry textbooks until 1982. Today, the term "circle" corresponds to the circumference of the circle.

Regarding the concept of the rhombus, it is worth mentioning that out of the 23 older textbooks studied, the concept of the rhombus is mentioned in only 9 of them, which suggests that the rhombus was not a priority subject in the teaching of geometric concepts at primary school level. The most common definition that appears is "a parallelogram that has all four sides equal", which is hierarchical as it places the rhombus in the family of parallelograms. In two cases, however, there are two partition definitions based on the equality of sides and angles of quadrilaterals, which are identical to the definition of the Elements. The term "rhomboid" appears only in one textbook, referring to the oblique parallelogram, as in Euclid's Elements.

In concluding this paper, it is considered necessary to point out that the role of definition in mathematical thinking is somehow neglected in today's primary school mathematics textbooks and curricula. It is not clear whether this is because it is taken for granted or simply because it is overlooked. We must, however, remember that there are contents in which reference to a formal definition is essential for the correct performance of exercises.

## REFERENCES

- Goudeli, K. (2015). *Introduction to Spinoza's philosophy*. Association of Greek Academic Libraries, National Metsovio Technical University of Athens, Athens.
- De Villiers, M. (1994). The Role and Function of a Hierarchical Classification of Quadrilaterals. *For the learning of mathematics*, 14(1), 11-18.
- Exarchakos, Th., Kontogiannis, D., Ntziachristos V., & Tsigoni T. (2001). *Euclidean "Elements". Modern rendering with introduction, explanations and commentary. Volume I: The geometry of the plane, books I,II,III,IV,V,VI*, Athens.
- Fatseas, A. (1970). *Compendium of geometry by A. M. Legendre*, Athens. Retrieved from <http://e-library.iep.edu.gr/iep/collection/browse/item.html?code=86-84215&tab=01>
- Karagiannidis, A. (1906). *Practical geometry for the 4th, 5th and 6th grades*, Ilia N. Dikaïou Eds, Athens. Retrieved from. <http://e-library.iep.edu.gr/iep/collection/browse/item.html?code=01-18796&tab=01>
- Kipouros, Ch. (1995). *Heron Alexander's - Names of Geometric Terms - Geometric*. Retrieved from <http://www.hms.gr/apothema/?s=ss&i=2>
- Kyriazopoulos, A., & Alexopoulos, V. (1969). *Arithmetic - Geometry of the 5th grade of Primary School, (1st Edition)*, Athens: OEDB. Retrieved from. <http://e-library.iep.edu.gr/iep/collection/browse/item.html?code=03-20555977&tab=01>
- Mariotti, M. A., & Fischbein, E. (1997). Defining in classroom activities. *Educational Studies in Mathematics*, 34(3), 219-248.
- Michaelides, E. (1946). *The Little Geometer - Practical Geometry. (6th Edition)*, I.D. Kollaros & Co. (Eds), Athens. Retrieved from <http://e-library.iep.edu.gr/iep/collection/browse/item.html?code=03-20560573&tab=01>
- Ministry of Education & Pedagogical Institute (2004). *My mathematics 4th grade. Part One*, Athens: O.E.D.B. Retrieved from <http://users.sch.gr/vaskitsios/katsba/dim/biblia/index.htm#6>
- Ministry of Education & Pedagogical Institute (2004). *My mathematics 5th grade. Part Two*, Athens: O.E.D.B. Retrieved from <http://users.sch.gr/vaskitsios/katsba/dim/biblia/index.htm#6>
- Ministry of Education & Pedagogical Institute (2004). *My mathematics 6th grade. Part One*, Athens: O.E.D.B. Retrieved from <http://users.sch.gr/vaskitsios/katsba/dim/biblia/index.htm#6>
- Morgan, C. (2005). Word, Definitions and Concepts in Discourses of Mathematics, Teaching and Learning, *Language and Education*, 19 (2), 102-116.
- Pedagogical Institute (2003). *Interdisciplinary Unified Framework of Curricula - Analytical Curricula of Compulsory Education*, Volume A, Athens.

- Papadopoulos, P. D. (1952). *Practical geometry for the use of pupils of the 5th and 6th grade of primary schools*, Athens: P.Dimitrakos. Retrieved from <http://e-library.iep.edu.gr/iep/collection/browse/item.html?code=20-01513&tab=01>
- Schubring, G., Fan, L. (2018). Recent advances in mathematics textbook research and development: an overview. *ZDM Mathematics Education* **50**, 765–771. <https://doi.org/10.1007/s11858-018-0979-4>
- Shir, K., & Zaslavsky, O. (2001). What constitutes a (good) definition? The case of square. In M. van den Heuvel -Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education*, 4, 161–168. Utrecht, Netherlands: Utrecht University.