BOARD GAMES IN MATHS EDUCATION

comparing the *Intiateur Mathématique* (1910, Hachette) and Polyminix (2019, Creativamente)

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ABSTRACT

The workshop focuses on educational materials (board games involving composition/decomposition) addressed to children, presenting an outstanding historical case, the 1910 "jeu de petits cubes" by Jacques Camescasse(1869-1941), named *Initiateur Mathématique*: inspired by the Charles-Ange Laisant's (1841-1920) book Initiation Mathématique and produced by the French publishing house Hachette and in the 1930's by CEL Coopérative de l'enseignement laic (on Célestin Freinet's (1896-1966) choice).We will connect its pedagogical and epistemological substratum with that of a recent educational board game produced in Italy, Polyminix, a 2D version of a game inspired by Solomon W. Golomb's (1932-2016)'s polyominoes. The tradition of toys and boxes for elementary learning started in the 18th century, including decomposition and composition and ratios (as in Friedrich Fröbel's (1782-1852) gifts and the learning of the decimal numeration system). Camescasse combined both aspects – also thanks to the cutting-edge technical design and materials – and had a clear outlook on mathematics education connecting arithmetic to geometry and the metric system: «La mathématique est une et… peu divisible», he wrote in the Notice containing the directions for the *Initiateur*, included in the box. The workshop will be divided into two parts: participants will be involved in hands on activities in both.

1. Jacques Camescasse and his Initiateur Mathématique

The first part of the workshop focuses on the *Initiateur Mathématique*, that was presented using a copy "reengineered" and built by Alberto Tretta (Falegnameria Tretta, Gallese near Viterbo, Italy). We recommend beginning with some informations about his author: Jacques Camescasse (1869-1941).

Camescasse's contribution is placed at the turn of the 20th century, when many educational innovation proposals flourished, motivated by the spread of public education and inspired by the interest for the political and cultural value of the school and the renewed trust in the educational value of elemen-

tary mathematics[77]. From the point of view of the teaching of mathematics to children, the practices in use up to that time were mainly related to numerical training; on the contrary, the new initiatives also aimed to spread among the popular classes an educational mathematics that also includes geometry, and an approach that sees the pupils as protagonists, promoting their initiative, understanding, and enjoying mathematics. Jacques Camescasse, in the wake of the pedagogical vision of Charles-Ange Laisant (1841-1920) in his work Initiation Mathématique (1906), develops and patent produced with Hachette the educational material contained in a box, called *Initiateur Mathématique* (1910). He complies to Laisant's call "to the friends of childhood": his game is all about the pleasure of the child and he understands it as a factor of change, an educational aid capable to evolve a fossilized practice, around a few procedures. Laisant, a mathematician and parliamentarian of the radical left who then approached the anarchist movement, had long denounced this situation (Auvinet 2017), referring to Johann Pestalozzi (1746-1827) and Jean Macé (1815-1894), the latter author of the best-seller also translated abroad the *Arithmétique du Grand Papa* (1863). Camescasse was a freemason and esperantist, follower in youth of the French pedagogist Paul Robin (1837-1912) with whom he collaborated in the Prévost orphanage in Cempuis (Oise, northern France). His experience with students in the carpentry workshop is strongly connected to the conception of the game of small cubes of wood that can be assembled thanks to metal slats. Camescasse tried to implement tangibly the advice that Macé gave in the already cited book, "using sacks, boxes, beans and peas" as a first attempt to arrive at what he called "objective teaching": through objects that are seen, touched, moved; as Laisant also wrote -enthusiastic about producing a material more powerful than drawing or beans- *entre les doigts* (Laisant in Camescasse 1916, p. 5).

[77] Camescasse's *Initiateur Mathématique* and his educational contribution was studied by the second author in her master thesis (Panichelli 2021)

2. The workshop

The workshop is recommended for 25 participants, to be held in a class with tables; it is organized in two main phases: first, the participants will put their hands on the reproductions of Camescasse cubes, and then they will move on to the Polyminix game. During both phases, the educational opportunities of these games will be discussed, commenting on experiences that have already occurred with both games, in the classroom with the pupils as well as with teachers in training.

2.1 The game box *Initiateur Mathématique* (1910)

The game box *Initiateur Mathématique* contained 1200 cubes, 600 red and 600 white and 144 steel bars. The box was accompanied by a Notice of the author: “Pour être sainement éducatif, tout enseignement doit être objectif, surtout au début. L'abstraction systématique introduite dans l'enseignement, sans préparation objective, est nuisible ». (Camescasse 1916, p. 10)

Figure 1. Left: Camescasse’s box, steel rods, and wooden cubes. Source: Jobbé-Duval 2020. Right: the illustrative booklet on the Initateur Mathématique, 1st edition, 1910. Source: Boutin 2019, p. 155 (possibly photograph of the specimen kept at CNAM)

Camescasse envisaged a first use for very young children to use the cube as building bricks. The game received much appreciation: Alain, a famous philosopher of the time,[78] about Camescasse's cubes :« Je me gardai bien de leur dire quelle était la solution juste car jeter la vérité toute trouvée à ceux qui la cherchent, c'est une mauvaise action et je les renvoyai au jeu de cubes. » (Camescasse 1916, p. 30), and the pedagogue Céléstin Freinet (1896-1966) who, in 1931, decided to bring back the game through his cooperative because he was convinced about the educational value of the game. Despite this since the mid-twentieth century this game has been used less and less and today is kept at the Musée des Arts et Métiers in Paris.

Figure 2. The reproduction of *Initiateur Mathématique* by Alberto Tretta's carpentry. Left: the cubes, made of two different types of wood, and aluminum rods; each cube contains two grooves that allow it to fit inside the bars. Right: the box containing the game. Picture by V. Panichelli.

It is important to handle the cubes with both hands, on a table (Fig. 3).

[78] Pseudonym of Émile-auguste Chartier (1868-1951)

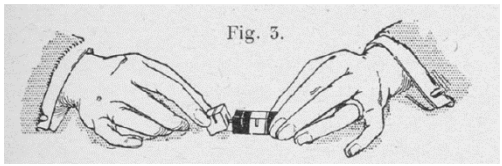

Figure 3. Example of manipulation of cubes for the construction of a row of ten.
Source: Notice sur *l'Initiateur Mathématique* 1916, p. 17

At the beginning each participant will form a bar made by 10 cubes to represent the number ten; then participants will form the hundreds, combining ten bars of ten, working in groups (Fig. 4). We point out how in this teaching material the three components of mathematics are always altogether: numbers, geometry, and measurement; through the small cubes it is possible to introduce tens, hundreds and thousands as well as cm, cm^2 and cm^3. Camescasse himself stated that « La mathématique est une et… peu divisible» (Camescasse 1916, p. 36)

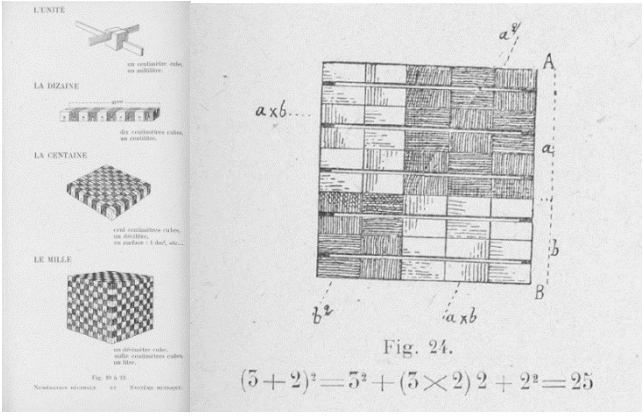

Figure 4. left: the ten, the one hundred and the thousand by cubes. Source: Camescasse 1916, p. 26; right: the square of the binomial represented with Camescasse' cubes. Source: Camescasse 1916, p. 31.

With his cubes we can explore the decimal numbering system, the operations but also the arithmetic theorems and measure, implementing an objective education through a tangible material. The main difference between these cubes and the more traditional blocks used to teach base ten arithmet-

ics, is that the cubes can be assembled and disassembled, avoiding pushing too early children towards abstraction. The most important value of this game is its manipulative dimension that allows us to touch the numbers and the relationships between them. Following Camescasse's words: "Les jeux des enfants guident souvent l'éducateur attentive" (Camescasse 1916, p. 8), our role as educators is to follow our pupil's game.

2.2 The Polyminix board game (2019)

The second part of the workshop, as already mentioned, is devoted to Polyminix board game. Every participant is given a single bag with pieces and cards. The game Polyminix, inspired by the world of geometry with continuous forays into arithmetic, was born in the summer of 2019[79]. The educational potential of the game for geometry or measurement - key aspects of the introduction to scientific thought - is closely linked to its power of attracting students. It can be used from kindergarten (with appropriate simplifications) up to second grade secondary school. It is part of the materials proposed in a national competition for mathematics "Matematica per tutti" designed by ToKalon Association and is included in the teaching guide produced for the Erasmus+ ANFoMAM project[80] (see references).

Polyminix is based on a choice of 15 polyominoes and playing cards (Fig. 5). There are three types of cards: green (entry level), yellow (intermediate), red (difficult). Each card has a central white squared region, representing the playing area, that is the surface to be covered; the number written on the white region refers to its extension (how many unitary squares, see Millán Gasca 2016, Millán Gasca and Spagnoletti Zeuli, 2015). The white surface must be covered with polyominoes, according to precise indications, contained in the six black rectangles on top and bottom of each card.

[79] Plyminix was designed by the first author of the present paper, with Anna Mazzitelli, a primary school teacher, together with the gaming Italian company CreativaMente.

[80] For additional information about the national competition "Matematica per tutti" see https://matematicapertutti.it/; Anfomam (*Aprender de los niños para formar at los maestros en el área de matemáticas*) project, number 2018-1-ES01-KA203-050986.

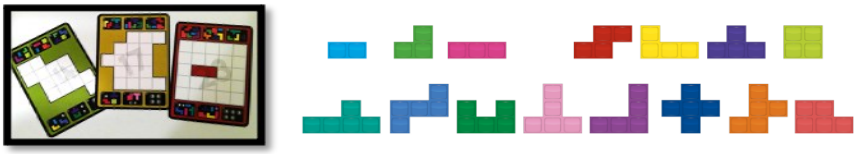

Figure 5 left: Polyminix playing cards; **right:** Polyminix's polyominoes

The workshop consists in proposing some of the game modes represented in each card.

The first game mode "Cover the figure!". A choice of polyominoes which surely covers the white checked part is assigned, the right combination has to be found. In Figure 6, 3 possible solutions to the same problem. Sometimes it happens, during classroom activity, that the solution is hard to find, so someone tries a "partially correct solution" (Fig. 7): the solution involved a polyomino which was not among the allowed ones.

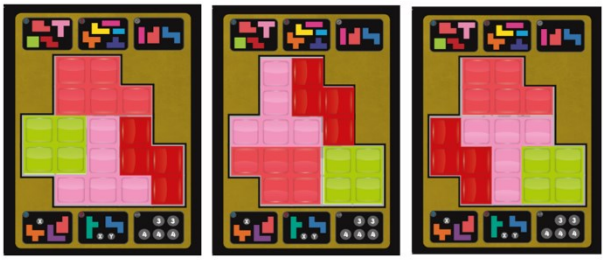

Figure 6. Three possible solutions to the same problem situation.

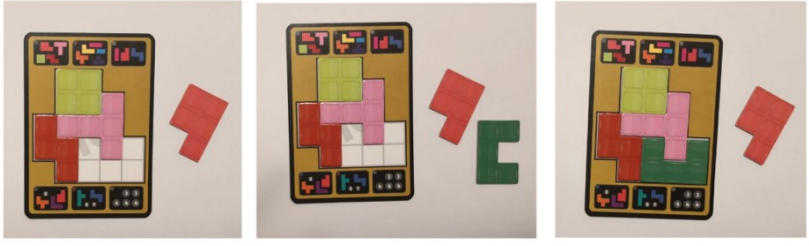

Figure 7. A "partially correct" solution

At this point the participants are offered what was proposed in the classroom by the teacher: a new problem, resulting from the previous partially correct solution. Cover a surface, with the only rule to use the dark green pentamino

(the “C”). In Fig. 8 some possible combinations. This random problem born from an error has more than one solution, a good observation to do with the students: it happens very often that right solutions are not unique.

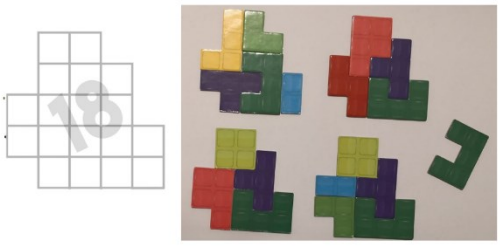

Figure 8. Some possible tessellations of the white squared area

We point out that we can explore a connection between geometry, measure, arithmetic, through composition and decomposition of polyominoes: to cover a figure with different polyominoes corresponds to obtain the same number adding different integers; a clear and tangible application, for example, of the commutative property in arithmetic and of geometric decomposition (Fig. 9)

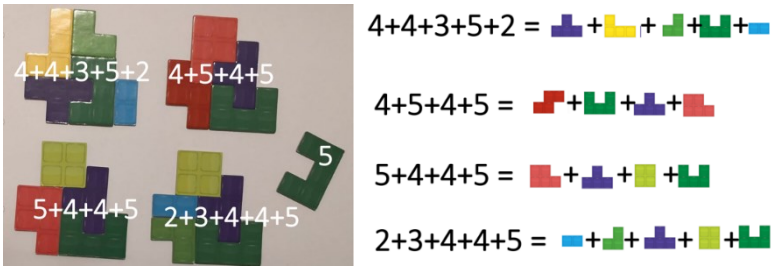

Figure 9. Geometrical decomposition and arithmetic decompositions of the number 18

The second game mode “find the missing piece”. Only part of the needed polyominoes is showed: to cover the figure the player must also find the missing one/ones. The missing polyominoes are indicated with letters X and Y, the unknowns, a first approach to the use of variables. After a first free exploration towards the solution, the leader encourages to proceed by reasoned exploration, possibly going into the world of arithmetics. If the area

measures 18, the given polyominoes add up to 15, the missing polyomino has to be a tromino. (Fig. 10)

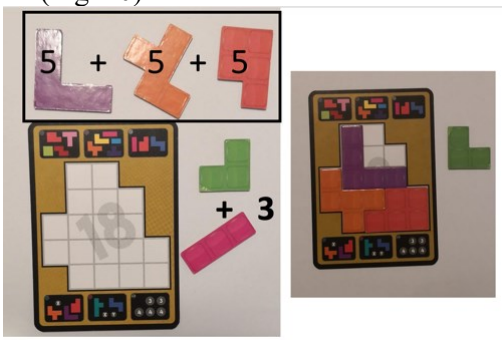

Figure 10. An arithmetic solution, but not a geometric one

Third game mode: "Find the right pieces whose measure is...". The only clue given to the player is a set of numbers, referring to the size of the pieces, the player must choose which pieces, to cover the area.

3. Final remarks

At the end of the workshop, starting from the comparison of the ancient and modern material, there will be a discussion with the participants on what unites both experiences that have been proposed; this discussion aims at bringing out first of all, the principle of composition and decomposition: both games are made of little bricks/cubes, that build and break up. Children can have the vivid perception of assembling and disassembling with both games, forming tens, hundreds and thousands (and back to the single cubes), touching the multiplication, with the cubes, as well as forming numbers and areas with the polyominoes.; this leads naturally to the question of connecting, in composing and decomposing, figures with numbers, geometry and arithmetic. Finally, a discussion about the power of game and playing is very appropriate, starting from some considerations extrapolated from the work of Johan Huizinga (1872-1945) *"Homo ludens"*:

- The game is mainly a free action; it is not real life; it is the awareness of being different from ordinary life.
- The game is a serious thing, but it is not just serious nor amusing, it belongs to an independent, unique and extraordinary category.

- The game is tension, suspense, excited expectation, the chance of a good or bad outcome.

Furthermore, we recall the point of view of Roger Caillois (1913-1978) who, before others, treated the theme of the play from a sociological point of view and after that, play has no longer been interpreted only as a mere entertainment or just a children's activity. Caillois stated that playing is something circumscribed, a space separate from common reality, a magical place where rules and ways of everyday life are suspended; he classified games relying on player attitude: *agon* (competition), *alea* (fortune), *mimicry* (simulation), *ilinx* (vertigo) (see Caillois 1958, Regoliosi 2022). Participants can be asked which categories of attitude they identify with, which ones were brought up during the session.

During game sessions, learning is encouraged and made easier since children develop a real *mimesis* (the aptitude of human beings to imitate, to become like anyone or anything, the capacity to look at the world outside oneself and learn by means of imitation, see Scaramuzzo 2016) within the object and the game, forgetting everything around, immersing completely in the fiction of the game. The small cubes were born in the wake of the battle undertaken by Laisant in favor of the education for children (see Millán Gasca 2015), but they are also born to instruct while having fun, so we can consider Camescasse's cubes as an educational aid halfway between games and didactic material, useful to introduce mathematical concepts and to physically represent mathematical theorems and also to make constructions; Polyminix, born in 2019, was designed with the same purpose.

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