MATHEMATIZATION OF FLUIDS MOTION

An example from Hydrology

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ABSTRACT

Recognizing the contributions in research on teaching and learning of mathematics of change, we present a first approximation to the process of the mathematization of fluid motion from a practice-centered approach. To study this process, we carried out an analysis of original sources in the area of Hydrology, from where we infer some variational practices.

1 Introduction

In research on teaching and learning of mathematics of change (see Kaput, 1994) we find at least two approaches focused on variational aspects. On the one hand, the Covariational Reasoning framework (Carlson et al., 2002) developed on the notion of mental action, and on the other, the Variational Thinking and Language research line adopts a social point of view and it's the basis of the Socioepistemological Theory (ST) (Cantoral, 2020; Cantoral & Farfán, 2004). Our project arises within the last approach, where specific mathematical practices have been identified as fundamental in the mathematics of change, such as *comparison, seriation, estimation*, and *prediction* (Cantoral et al., 2018). Recognizing in the literature that most contributions in this research approach are being developed around the mathematization of a particle motion (see Cantoral et al., 2018), our project seeks to extend it by studying the process of the mathematization of fluid motion and begins analyzing original sources searching practices that accompany this process. In a later phase of the project, the results of this one will base a design research that promotes variational practices in university students.

2 Theoretical and methodological considerations

The ST prioritizes practices that accompany work with mathematical objects, interested in their role in the construction of mathematical meaning (Cantoral, 2020). For this, a sociocultural posture is adopted, recognizing different forms of mathematical knowledge as valid, among them, *scientific*, *popular,* and *technical* (Cantoral et al, 2018). For this reason, studies are carried out in various scenarios (school, historical, and professional, among others).

According to a practice approach, practice is conceived as organized nexuses of activity composed of actions, which are executions of bodily doing and saying (Schatzki, 2001). In the ST focusing on their mathematical character, these practices are organized in a nested model composed at the first levels by *actions, activities*, and *socially shared practices* (see Cantoral, 2020). In the methodological phase, these practices are identified with analytical questions: *what is done and said*?, *how is it done and said*?, and *why is it done*?, (see Cantoral et al., 2023).

Here, we present a synthesis of the analysis of technical knowledge (Hydrology) in a historical setting, to identify variational practices that accompany the mathematization process of groundwater motion.

3 Preliminary results

As a first approximation to the mathematization of fluid motion, relied on Freeze & Cherry (1979), we analyze mathematical practices on the Darcy's experiments developed for the water supply of Dijon (reported in 1856).

First, we identify the consideration of a straight circular cylinder filled with sand with a cross-section with area A, hermetically closed at each end with tubes allowing water inlet and outlet and equipped with a pair of manometers at a distance Δl from each other (see Fig. 1a). We interpret [what is done?] *geometrize* the phenomenon and *recognize variables* involved; [how is it done?] *using geometric* shapes and *establishing a reference system*. Immediately, a *specific flow rate* as $v = Q/A$ (where Q is the volume of water per time unit) and $\Delta h = h_2 - h_1$ (difference of heights at the manometers) are defined. We assume [what is done?] *constructing new comparison* ways; [how is it done?] *establishing other variables* composed of more than one magnitude.

Darcy's experiments showed that v is directly proportional to Δh when Δl is held constant, and inversely proportional to Δl when Δh is held constant. This relationship known as Darcy's Law describes groundwater flow in porous media and can be written as $v = -K\, dh/dl$ (where K is the hydraulic conductivity, a soil property). We interpret [what is done?] *establishing a rela-*

tionship between variables; [how is it done?] *measuring and comparing the change of variables*.

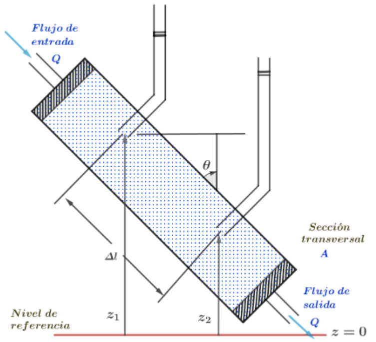

Figure 1a. Darcy's Law Experiment

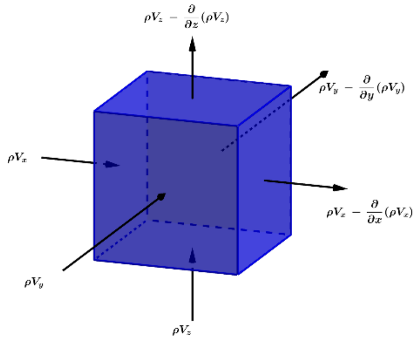

Figure 1b. Elemental control volume

This law is used to establish the equations for a steady-state flow in an isotropic homogeneous medium (the conductivity K is constant and independent of the measuring direction). Consider an elemental control volume (see fig. 1b) and establish the continuity equation $-\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial(\rho v_y)}{\partial y} - \frac{\partial(\rho v_z)}{\partial z} = 0$. We identify [what is done?] *geometrizing* the phenomenon; [how is it done?] *using* geometric shapes, *establishing* a reference system, and *using* physical principles. Then, incompressible fluid is considered where density is constant, $-\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} - \frac{\partial v_z}{\partial z} = 0$, and replacing v_x, v_y, y v_z by its corresponding Darcy's Law we obtain the steady-state flow equation through an anisotropic saturated porous medium: $\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$. Later, for an *isotropic homogeneous medium* $K_x = K_y = K_z$ y $K\ (x, y, z) = C$, the equation is reduced to the Laplace equation: $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$. Here, [what is done?] *Simplifying the differential equation*; [how is it done?] *using* physical principles to *constantifying variables*].

In synthesis, a *geometrization of the phenomenon* is made by considering a straight circular cylinder (rectangle in fig. 1a) and a cube (fig. 1b), a *reference system* is constructed for *comparing magnitudes*: in the first case (fig. 1a), between heights at the manometers and distance between them; and second case (fig. 1b), between amounts of mass entering and leaving the elementary control volume. Also, *physical principles* are used to *express the relationships between magnitudes* in way of *differential equations*: in the first case, the proportional relationships are expressed in the form of Darcy's Law; and in the second, the Laplace equation. Finally, variables are *constantified* using physical principles, i.e., *mathematical expressions are reduced* by considering certain variables as constants, for example, by limiting the flow to a steady state or a homogeneous and isotropic medium.

4 Conclusion

In this process of mathematizing of fluids motion in the case of groundwater, based on the nested model (Cantoral, 2020) we recognize as actions: *geometrizing of movement phenomena*, and *constructing a reference system*; as activities: *measuring and comparing magnitudes*; as socially shared practices: *constantifying* variables using of physical principles to arrive to differential equations that *relating* variables and describe the behavior of phenomena.

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