AN INTERDISCIPLINARY APPROACH TO THE SECOND QUANTUM REVOLUTION FOR EDUCATING TO PROBABILISTIC THINKING: THE RANDOM WALK CASE.

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ABSTRACT

Today more than ever we live in an accelerated society in continuous change (Rosa, 2013), an uncertain society in which we have to understand how to manage risk and contingencies. It is therefore pivotal to find new words, a new vocabulary and new competencies that can better help us to grapple with the contemporary society and provide the new generations with them. The physics of the '900, like quantum physics and science of complex systems, has proved particularly rich in this perspective as well as the technological revolutions in progress such as the Second Quantum Revolution at the heart of many investments today. Quantum technologies, exploiting the capabilities of isolating, controlling, and manipulating the single quantum object and its properties, give to quantum physics a new perspective and provide new teaching and learning possibilities. In this contribution, we present an approach to the Second Quantum Revolution and, in particular, an activity on classical and quantum random walk designed to shed light on the ongoing scientific and technological advancement and in its making as well as to touch on some of the most important foundational and epistemological debates such as the differences between the epistemic and ontological probability, and the true randomness that characterize quantum physics.

1 Introduction

The contemporary society, more than ever, is requiring us to face global challenges (i.e., climate change, COVID pandemic) and it is pointing out how important educating to the "logic of uncertainty" is. As De Finetti argues, the main problem lies in the scholastic claim of reducing the whole process of thinking to the trivial logic of Yes or No (De Finetti, 1989). The science of the twentieth century is a rich source of concepts and theories that could promote the shift from deterministic thinking, namely the one that allows us to predict with certainty the evolution of a system given the initial conditions, to probabilistic thinking and the logic of the uncertainty. Together with science of complex systems, chaos theory, and theory of probability, quantum physics provide a new vocabulary and new words (like ontological probability, space of possibilities, uncertainty) that can help to better grapple with the contemporary society.

In particular, among the many societal, scientific and technological challenges, there is the research and development of very new technologies whose functioning is based on the law of quantum physics. As Dowling and Milburn stated, "*We are currently in the midst of a second quantum revolution*" (2003, p.1655). As other revolutions, this Quantum Revolution is not only challenging scientific research but also science education research, requiring it to help to expand the workforce in the field, to create new ecosystems and synergies between universities, enterprises, and schools, to convince new generations to choose STEM careers, and to promote a citizenship's quantum literacy[74].

To contribute to this challenge, within the projects I SEE[75] and IDENTITIES[76], we designed an approach to Second Quantum Revolution and developed a course for secondary school students to value the ongoing revolution as, first, a cultural revolution and its intrinsic interdisciplinarity as a potential way to include students' different kinds of reasoning, tastes and identities. We focused on the interdisciplinarity between physics, mathematics, and computer science with the aims of i. fostering the understanding of basic concepts of quantum physics (quantum state, superposition principle, state manipulation/evolution, measurement, and entanglement), ii. reflecting on differences between Boolean and quantum logic underlying the functioning of computers (e.g., Feynman, 1981; Deutsch, 1985; Wilce, 2002) and iii. exploring the role of ontological probability to promote the development of probabilistic thinking.

In the course, we introduce students to the pillars of quantum physics as a theory (the postulates and the basic concepts), to some relevant quantum technologies and algorithms such as quantum cryptography, quantum teleportation, and the quantum random walk. Emphasis is given to protocols and algorithms since they provide a glance at contemporary challenges and, at the same time, are addressable with a bunch of concepts that embody a paradigm shift (Kuhn, 1969), which unhinged the way we looked at and investigated nature as well as our conception of object: from classical to quantum physics. Furthermore, during the course students are asked to reflect also on the main implication of the Second Quantum Revolution on many

[74] https://qt.eu/app/uploads/2020/04/Strategic_Research-_Agenda_d_FINAL.pdf

[75] https://iseeproject.eu/

[76] https://identitiesproject.eu/

dimensions such as politics, economy, society, environment, education and so on.

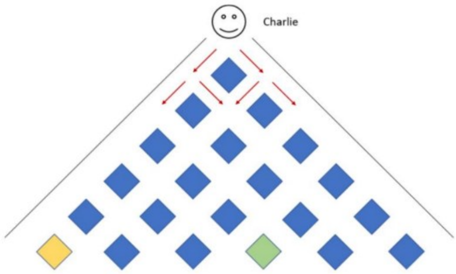
In the following, the random walk activity is presented as a context to reflect with high school students on the intertwining between mathematics, physics, and computer science and how the probability in the quantum case is intrinsic and rooted in the "main ingredients" of the quantum algorithm.

2 The quantum random walk: unpacking the ontological status of probability in the interplay between physics, mathematics, and computer science.

The random walk activity was designed to explore the aspect carried out in the previous section. To pursue the first aim, we kept the quantum technicalities as simple and clear as possible to foster a deep understanding of the essential physical concepts: the concepts of quantum state and superposition principle, state manipulation and evolution, measurement, and entanglement. To pursue the second one, we compared the classical and quantum random walk algorithms in terms of the logic underlying their functioning. To reach the third one, we fostered students to reflect on the differences in terms of nature of probabilities in the classical and quantum case. The activity, that lasts about 2h, consists of a teamwork activity on the classical version of drunkard's walk problem (Pearson, 1905) and discussion; an introduction to the model of the random walk to scaffold the comparison between the classical and the quantum cases; a collective exploration of the classical and quantum random walk through an interdisciplinary lens (mathematic, physics, computer science) and the introduction of some application.

We start by posing the drunkard's problem:

Charlie, after drinking too much wine, returns to the city of Eve. As soon as he crosses the city gates, a problem arises: he no longer remembers where he lives or the way back. He then begins to walk between the blocks, proceeding randomly and never going back, hoping to find the right way. What is the probability that Charlie will reach his house (green square) at random? Is the probability that he will reach, at random, his friend's house (yellow square) the same? How would you model the problem? How do you model the "randomly proceeding"?



After students solve the problem in a group and collectively discuss the results, we build the model starting from students' solutions. In the building of the model, we mainly focus on the "randomly proceeding", which is modelled by *the coin flipping*, and the Charlie's moving through the city "never going back", which is modelled by choosing a *shift operator* according to which "if the outcome is head Charlie moves one step to the left, if the outcome is tail Charlie moves one step to the right". We build the algorithm as a sequence of coins flipping and application of the shift operator.

We then start to scaffold the comparison between classical and quantum random case by asking students what they expect if Charlie should follow quantum physics and by stressing the different logic of the coin. In fact, the classical coin can assume only two possible values "head OR tail". In the quantum case, we can design a coin that creates a superposition state, namely that transforms a state ($|head\rangle$ or $|tail\rangle$) in a "linear combination of head and tail": $|head\rangle \rightarrow a|head\rangle+b|tail\rangle$, where the square of a and b give the probability of measuring respectively $|head\rangle$ OR $|tail\rangle$. The shift operator remains conceptually the same in the classical and quantum case. Following Kempe's treatment (2003), we introduce to student the quantum random walk algorithm and calculate Charlie's quantum probabilities to reach the yellow or the green house. The mathematical, physical, and computer science dimensions are intertwined, in the classical and in the quantum case, by focusing first on the model and the calculation to obtain the solution and the probability distribution (mathematics).

We pass then to physical examples and applications of the random walk (physics), and to the code in python showing what modelling the problem with a computer and a simulator means. Operationally, students are introduced to different kinds of representations (algebraic, circuital/logical, physical and the coding) both to stress some conceptual revolutionary aspects and to include different kinds of students' understanding and reasoning. By showing the mathematical and computational representations, we pave the way to reflect on the nature of the coefficients *a* and *b* that are incorporated in the mathematical description, intrinsic to the quantum object and we introduce the concept of ontological probability. The physics highlights the physical interpretation of the coefficients, namely the interference phenomenon that leads to an asymmetrical probability distribution in the quantum case. Furthermore, through the computational perspective, we introduce students to another pivotal debate: the problem of generating random numbers. The comparison with the classical case and the impossibility to generate "truly" random numbers allow us also to reflect on the intrinsic determinism that characterizes the standard classical computation and the intrinsic non-determinism of the quantum one. We conclude the activity by showing some application fields of the random walk algorithm, such as research algorithms (e.g., Shenvi, Kempe, Whaley, 2003), decision-making and optimization algorithms, econophysics (e.g., Orùs, Mugel, Lizaso, 2019) and art.

3 *Final remark*

This activity is emblematic of our approach to the Second Quantum Revolution. The random walk activity touches on and show to students some of the contemporary challenges like research and optimization problems. Furthermore, it proved to be a context to take a glance at pivotal epistemological debates that can support students to embrace the uncertainty and probabilistic thinking, promote the development of the logic of uncertainty providing them with thinking tools to navigate the complexity of the present and orient themselves toward a more collectively and individually sustainable future (OECD, 2019).

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