ANALYTIC NARRATIVE OF STRATEGIC INTERACTIONS THROUGHOUT CONSTRUCTIVIST APPROACH

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ABSTRACT

Mathematical theories based on the concept of rational choice - individual decision theory, game theory, social choice theory - have spread rapidly in many fields: economic, social, political. The mathematically and logically systematic formulation of Game Theory arose as a necessity to explain situations of conflict or interaction between several individuals dating back to earlier times. Past events, in fact, can be reinterpreted through Game Theory models. History becomes, therefore, an "analytical narrative" of events, in which mathematics is the main tool for the formalisation of theoretical models that become universal keys for analysing relationships between agents.

1 The background

Mathematical theories based on the concept of rational choice, such as individual decision theory, game theory, social choice theory, have been widely diffused in the economic, social and political fields. Game Theory received its first general mathematical formulation by John von Neuman and Oskar Morgenstern in 1944. In the last 40 years, Game Theory has become the most important and useful tool when comparing situations where the best action of one agent depends on expectations of what one or more other agents will do, and vice versa. The mathematically and logically systematic formulation of Game Theory arose as a necessity to explain situations of conflict or interaction between several individuals dating back to earlier times. Past events, in fact, can be re-interpreted through Game Theory models, in its classical formulation (games in normal form) or using a tree representation (games in extensive form). History becomes, therefore, an "analytical narrative" (Mongin 2018) of events, in which mathematics is the main tool for the formalisation of theoretical models that become universal keys for analysing relationships between agents.

2 The National Guidelines and the MHS project

The globally integrated laboratory project has been developed in full compliance with the national indications issued by the Ministry of Education, which emphasises that *"[the student] will be able to frame the various mathematical theories studied in the historical context within which they developed and will understand their conceptual meaning. The student will have acquired a historical-critical vision of the relationships between the main themes of mathematical thought and the philosophical, scientific and technological context."*

The Department of Mathematics of the University of Salerno projected an educational path named Liceo Matematico, translated with Mathematical High School Project (MHS) developed in extracurricular hours, whose aim is to recover the interdisciplinary dimension of mathematics in teaching to stimulate reflection on fundamental human and social problems and promote the integration of knowledge.

Researchers designed and tested educational paths and laboratories, among them the one of Mathematics, History and Economics described in the present paper, aimed at restoring to culture that overall vision and to understand how mathematics has influenced historical events or has sometimes been guided by them.

3 The Globally Integrated Laboratory Project

In our work we describe an interdisciplinary activity developed in classes of "Liceo Matematico" project working with a constructivist approach in which students move using their natural learning skills and in which they personally and actively build the appropriate knowledge, while the teacher assumes the role of consultant, assistant and guide (Vygotsky, 1962).

Thanks to the mediation of the teacher, all processes of laboratory activities are conducted by students in first person. They analyse some important historical events, recognizing the essential elements of history to reconstruct the analytical narrative and identify the equilibrium of the model. With the use of narrative analytics, students examine the game of interaction in order to evaluate the outcomes of the history. Students identify the agents: some are individuals, but others are collective actors, such as power groups or electorates. Students reconstruct the preferences of the actors, through the analysis of possible alternatives and the information they possessed at the time of the

choice of strategy. Students examine the expectations they formed and the strategies they adopted, taking into account all the constraints on their actions. In this way we model the processes that led to an outcome. Finally, in a collective discussion, students analyse the equilibrium of the model and that of the actual history.

The activities are developed with various levels of difficulty and different use of the mathematical tools acquired. In particular, from a constructivist perspective, students approached geometry problems solved with the use of experimental geometry software. Students considered the succession of choices and calculated all possible outcomes and associated payoffs. Without having any knowledge of optimisation ('unconstrained' and 'constrained') for functions in several variables, the students constructed solutions to the proposed tasks by analysing the historical and strategic elements of the examined context.

The activities are elaborated with various levels of interaction, a transdisciplinary vision of knowledge for the contextualization of ideas and mathematical discoveries in the historical period, the interaction of knowledge for a metacognitive reflection and the anecdotal description to involve and interest the students (immediate and motivating approach, with little impact on the training framework).

4 The Analytics narrative

Our analysis focuses on some historical events using the analytics narrative approach. Bates and collegues (1998), proposed an original reconciliation of the history with the modelling allowed by mathematical theories of rational choice. With narrative analysis, authors argued that certain historical events raising interpretative and explanatory problems cannot be solved by narratives in classical form and require the use of well-formalised models.

Using the Analytics Narrative approaches students learned research methods, starting with understanding the fundamentals of selecting cases for study. We traced the historical context in which the events occurred, identifying the actors, the decision points they dealt with, the choices they made, the possible strategies they adopted and those they avoided, and how their choices generated events and outcomes. The construction of the analytical narrative in the class work led us to highlight and focus on the logic of the processes that generate the phenomena we study, in terms of the rational choice theory. Indeed,

Game Theory considers the sequence of choices and emphasizes their importance for outcomes, which reflect the influence of history, the importance of uncertainty, and the ability of people to manipulate and strategize.
Mongin (2017) highlighted that some historical events that raise interpretive and explanatory problems cannot be solved by the narratives in classical form, and thus require the import of models. The *Analytical narrative* thus refers to the change in narrative type that emerges from the use of models that fit the historical events. All economic and social theory was transformed by Nash's ideas, which formalised something that was already recognizable in the historical events. Analytical narratives use qualitative data, such as changes in institutional patterns and decisions made by individuals or groups. What is really surprising for students is the construction of individual and collective preferences behind Economic modelling: People have consistent and stable preferences, and they choose the alternative from the set of feasible alternatives that is ranked highest under these preferences. As pointed out by Brams (2011), disciplines in the humanities (religion, law, history, literature...) represent a world we do not normally associate with mathematical calculations of strategic interaction and rational choice.

5 Activities, some examples

5.1 The war strategies of Napoleon

Among the various activities carried out by the students, the workshop on Napoleon and the search for an optimal point were also presented. The laboratory started from a historical narrative, then dealt with mathematical analysis to explore the repercussions in the economic field.

In the Renaissance period, many attempts were made to design an instrument that facilitates arithmetic calculations and geometric operations, especially to respond to a military need in which firearm technology required more and more precise mathematical notions. Among these instruments we certainly find Galileo's compass. The great French general Napoleon was passionate about mathematics and his mathematical knowledge helped him in his military career. He was convinced that all Euclidean geometry could be described through the compass without the aid of the ruler and put this hypothesis to Lorenzo Mascheroni who answered affirmatively and described it in the volume “The geometry of the compass” that he dedicated to "Bonaparte the Italic"

comparing him to a geometrician. Napoleon in battle used field-compasses to find the central position for attacks. That point was extremely important for organizing assault strategies but, on the battlefield, it was difficult to find through angular triangulation. Instead, it was much easier to rely on field-compasses and obtain it graphically thanks to the "Napoleon's theorem" which states that

The barycenters of equilateral triangles, built externally on the sides of any triangle, form an equilateral triangle.

The proof of the theorem leads to the determination of the geometric position of the Fermat-Torricelli point.

The optimal position of Napoleon can be determined by solving a constrained optimization problem or with analytical geometry and properties of triangles barycenters. The great French general Napoleon was also a great mathematician and it was mathematics that helped him in his military career. One of his youthful passions was the compass. He was convinced that all Euclidean geometry could be described by this simple instrument. In battle, Napoleon used 'field compasses' to find the central position. The determination of this point was crucial for managing assault strategies.

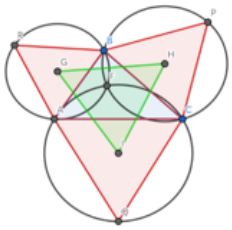

Figure 1. Napoleone's theorem

5.2 Democratic stability under rational political choices after the Civil war in the USA

A person's behaviour is rational if it is in his best advantage, given his information. On March 4, 1865, in the speech of his second presidential inauguration, Abram Lincoln exposed some significant, surprising and uncomfortable positions even for his own political party. He underlined a position far from the rhetoric of warmongering nationalisms, highlighting that God's plans were inscrutable and it was not sure that He would support them and support them

in war. He recognized and affirmed that the cause of conflict was slavery and that the result, initially unintended, has been its abolition. In particular, Lincoln said: “Both parties deplored war; but one would make war rather than let the nation survive; and the other would accept war rather than let it perish. And the war came”.

In the same way, Aumann (2006) recognized that “Wars and other conflicts are among the main sources of human misery”. According to Auman an economic system, a social or individual choice can be viewed as a game, and the incentives of players to make one choice over another when interacting with each other has a degree of complexity and often leads to surprising, sometimes counterintuitive results. Aumann continued:” You want to prevent war. To do that, obviously you should disarm, lower the level of armaments. Right? No, wrong. You might want to do the exact opposite. In the long years of the cold war between the US and the Soviet Union, what prevented “hot” war was that bombers carrying nuclear weapons were in the air 24 hours a day, 365 days a year. Disarming would have led to war.”

6 Conclusion

In the “Liceo Matematico” project, working with a constructivist approach, we developed an interdisciplinary activity to analyse historical events through mathematical models. We examined some important historical events, identifying the essential elements of history in order to reconstruct the analytical narrative and identify the equilibrium of the model. The students were asked to identify the agents (individuals, groups, voters) and reconstruct the actors' preferences by analysing the possible alternatives and the information they possessed when choosing a strategy. Taking into account all the constraints on the players' actions, we examined the possible outcomes of the game, using both techniques from Game Theory and elements of analytical geometry. In this way, we modelled the processes that led to historical events. Finally, we analysed the equilibrium of the model and that of real history.

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