MATHEMATICAL ARTISTRY FOR JUSTICE:

A workshop on bringing episodes in the history of mathematics to life in the classroom by means of theatre, incorporating a short play set in ancient China

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ABSTRACT

This paper describes the theoretical motivation and practical mounting of a workshop in which participants could experience and reflect on the value of theatre in bringing mathematical ideas and history to life in the classroom. All participants were involved in the production and enactment of a short pre-scripted play, requiring minimal preparation. This was followed by discussion, small group work, and feedback. The play, given in full below, evokes the cultural context within which a Chinese mathematical practitioner may have operated about two millennia ago, and suggests how the Rule of Excess and Deficiency for equation solving could have been inspired and recorded, as Problem 17 in Chapter 7 of the *Jiu zhang suan shu (The Nine Chapters)*. Nine representative scribes/mathematicians from different cultures make cameo appearances, giving the names for 'the unknown' used in equation-solving in their time. The dialogue is accessible and lively, laced with humour and emotions, portraying the excitement of mathematics-making. Based upon primary sources, the play can inspire interest in authentic contextual history of algebra while motivating the learning of algebra.

1. Introduction

Using theatrical devices as communication tools is an art as old as humanity. Dialogue form has been used for a long time to communicate philosophical, mathematical and scientific ideas, for example, by Plato, Galileo, Alfréd Rényi, Donald E. Knuth, Imre Lakatos, and many more (Rényi, 1967; Knuth, 1974; Lakatos, 1976). However, I believe the power of dialogue and live theatre is neglected in current curriculum-driven, time-constrained educational systems.

Videos and films with mathematical, historical and biographical themes might be excellent, but to really win the attention of young people and engage their hearts and their minds it is hard to beat personal involvement in the enactment of theatre. To watch children involved in plays, or live theatre games, or improvisational drama, is to see them come alive! This paper aims to encourage the creation and use of such devices to awaken a love and appreciation for mathematics and its story, and to counter the boredom felt by so many learners.

Live theatre can encourage imaginative entry into the minds and emotions of historical characters, sharing their concerns and their excitement. This is not easily achieved by merely reading their writings. Indeed, the process of generating a dialogue based closely on primary source material and correspondence can be a tool for uncovering and displaying hidden motives, inspirations and emotions, for the researcher as well as the educator (Hitchcock, 2012). Creating dialogues to dramatize the history of mathematics can be approached in various ways. Rényi's *Dialogues in Mathematics* features Socrates, Archimedes and Galileo, and sets out to be 'lively and vivid' and 'comprehensible to non-specialists', while presenting problems 'in their full complexity.' Rényi felt this aim was achievable only by means of the dialogue form. To preserve historical fidelity, he sought to emulate the writing styles of his three protagonists, while assuming some poetic license and 'avoiding anachronism as far as possible.' In contrast, Lakatos's *Proofs and Refutations* deliberately dissociates his characters in dialogue from any historical counterparts, naming them 'Alpha', 'Beta', 'Gamma', etc. He thus contrives a degree of separation between the 'concept story' and the 'human story', indicating in footnotes the historical counterpoint to the classroom dialectic. Designing scripts for live theatre brings its own challenges. Using original sources ensures some authenticity, and there is a natural fascination in different cultural contexts. But this has to be carefully balanced with the requirements of good theatre and accessibility. The tensions are not easily negotiated, and the balance must be adjusted for different levels of education. Scripting a play, one has to take liberties and make guesses. What's recorded and preserved in primary sources is usually the final product of prolonged thought, informal correspondence and debate. At best, it captures the bare bones of the human story behind the mathematics. By 'filling in' some of the missing conversation – the vast tracts of unrecorded communication – we can aim to reconstruct or 're-incarnate' the flow and ancestry of ideas in communities over time. The challenge is to weave together, by the art of the dramatist, the primary source fragments and try to complete the puzzle – to capture with the natural flow of a good story-line something of the elusive and complex historical dialectic behind the reported fragments. For more on the tensions and challenges of achieving this, see Hitchcock, 1992, 1996, 2000, 2023; the last has further references.

The aim of the workshop reported here was to experience and reflect on the ways that the devices of story, narrative, dialogue, drama and theatre can enliven mathematical ideas and history in the classroom, with minimal resources and preparation. Participants were challenged to mount a pre-scripted play from scratch – with casting, rehearsing and production all taking place in about an hour. In the second hour, plenary discussion took place, and then discussion in small groups followed by feedback. Sections 2-4 of this paper give the detailed structure of the workshop and indicate how the play was introduced to the participants. To convey the flavour of the live workshop, and to show how the production can be managed with students in practice, the text gives an almost verbatim account of some of what was said to the workshop participants.

Section 5 consists of the script of the play, precisely as given to the participants. It is designed to evoke the context and flavour of an ancient culture in which mathematics emerged in response to everyday problems but was gradually becoming an art in own right. The action is introduced by a narrator and a mathematics teacher, who discuss the emergence of ancient techniques for equation solving. The scene is set in the Office of Community Advice and Arbitration in a Chinese town two thousand years ago. The administrator receives an agitated client, and responds to her anxiety about the purchase of a piece of land of mixed quality. The play is based upon the posing and solving of Problem 17 in Chapter 7 of the *Jiu zhang suan shu (The Nine Chapters)*; some of the dialogue is based on the *Sunzi suan jing (Mathematical Classic of Master Sun)* (references are given in footnotes to the play). This play also showcases the rich variety of cultures that contributed to the development over thousands of years of the art of ‘finding the unknown.’ In this way it can provide background and colour for what the modern curriculum calls ‘solving equations’, where the standard techniques are the fruit of a long multi-cultural adventure of human thought, but are commonly encountered by learners with no associated historical narrative. The words and the mathematics are elucidated by the narrator and the mathematics teacher, with the aid of slides in which equations are transcribed into modern notation.

This play is designed for use with teacher trainees and well-motivated secondary school learners; simpler scripts are needed for younger learners. Similar plays of mine have been enacted, with warm response, in many summer-school and conference settings, and published individually in various places (references may be found in Hitchcock, 2023). The mathematics education market seems to be focused on rigidly curriculum-oriented texts; it is hard to find or to publish theatrical resource collections. But I believe a great opportunity is being lost. Therefore, this paper urges the need for more theatrical resources for enriching the learning of mathematics, and provides a model script, while describing how such plays can be easily produced to great effect.

2. *Welcome, synopsis and aim of the workshop*

Welcome! Here's an overview of our 2-hour workshop:

- Welcome & introduction – 5 minutes
- Allocation of production parts & casting – 15 minutes
- Rehearsing in corners – 15 minutes
- Performance of the play – 30 minutes
- Reactions & critique – 15 minutes
- Discussion in groups – 20 minutes
- General feedback and conclusion – 15 minutes

In this workshop we will see how theatre can bring mathematical ideas and history to life in the classroom. The workshop will make the case and I hope be inspirational too, by demonstrating theatre in action, involving all of you in the production. Then we will reflect together on what we have been part of, critique the play, and share any similar experiences. After that, we will discuss in smaller groups how to use biographical and primary source material to generate different plays and dialogues for various classroom contexts, and how you might exploit the power of theatre in your own teaching. So, workshop aim in summary: *observe, enact, critique, create!*

A personal disclaimer: I have had plays like this one enacted, in whole or in part, at a number of ESUs, HPM meetings, and other conferences, and some of my plays have been used by others, in classrooms or maths-camps. But I am an academic and, though passionate about the idea of using history and theatre for mathematical enrichment in schools, I have had limited opportunity to mount plays in real classrooms. Therefore, I am not the expert here! I hope some of you may be, or may become so. My challenge to you is to become co-creators of an exciting educational tool and art-form! I want you to run with the ball, catch a vision, translate your own love of mathematics and its history into magic theatrical moments in your own classrooms. It's not so daunting – to demonstrate that is a major aim of this workshop. We'll put this play on now with minimal fuss, minimal props and rehearsal, and (I hope) have lots of fun. Pretend you are a child again, with no inhibitions!

3. *Introduction to the play*

The play is designed to motivate the learning of algebra as well as inspire interest in authentic contextual history of algebra. To achieve this, it needs, above all, to be entertaining! So, the dialogue should be as lively as possible, with all appropriate emotions and humour, bringing out the drama, the anxiety and the excitement. The play is introduced by a narrator, MARIA, and a modern Western mathematics teacher, EMMY. The curtain goes up on a scene in the Office of Community Advice and Arbitration in a Chinese town, two thousand years ago. The administrator YU-LIN is at his desk, and an agitated client, YUNG CHANG, enters. She fears that her husband may have made an unwise purchase of land for growing rice and millet, and seeks possible redress. In addition to these four major parts, there are nine small parts to play: representative scribes and mathematicians from different cultures speak briefly, giving the names for 'the unknown' used in equation-solving in their time. The actors may use some artistic aids – dress, hats, beards, moustaches, mannerisms – towards achieving some cultural authenticity, but it is important to be respectful to the various cultures and not to caricature them in potentially offensive ways.

4. *Production and casting*

We will now allocate parts, to be displayed on slides, colour-coded. Don't be shy to volunteer or to nominate somebody, so we can move on. We will need stage-crew, light & sound crew, five directors, four main parts, and nine smaller parts. Colour-coded name cards are handed out as we go, to hang around necks. Nobody needs to memorise their part – scripts will be provided.

Actors! As soon as you have been cast, go to different corners (according to colour coding) with your directors, and start rehearsing. Directors! Help the actors focus on projection of voices, use of body language, emotional colour, entries and exits. At the same time, consider what simple costume or head-wear might be effective for artistic purposes, for distinguishing characters and for enhancing authenticity. A few items are laid out on a table, and some props may be handed to people as they are cast. Remember: while we want to celebrate the diversity of cultures involved in the development of mathematics, it is important to portray your characters respectfully. Meanwhile, the stage-crew prepares the set, and sound crew selects and plans music Actors and directors can talk to the stage crew about where to come on, where to sit/stand, etc. This should all be achieved in 15-20 minutes.

STAGE CREW: Set up chairs, desks, tables, anything you can think of.
SOUND & LIGHT CREW: Work out how and when to adjust lights and play some Chinese music at beginning and end of Chinese scene.
FIVE DIRECTORS: Coach actors, using directions in the script and rehearsing selected passages.

- For MARIA & EMMY (full script)
- For YU-LIN & YUNG CHANG (full script)
- For Egyptian Scribe, Babylonian Scribe, Diophantus
- For Arabic, Indian & Chinese mathematicians
- For Italian abbacist, European mathematician, German cossist

FOUR MAJOR PARTS (full script)

- MARIA, narrator, historian of mathematics
- EMMY, co-narrator, mathematics teacher
- YU-LIN, mathematician-administrator in the Office of Community Advice and Arbitration in a Chinese town, some two thousand years ago
- YUNG CHANG, a female client

NINE SMALLER PARTS (partial scripts)

- EGYPTIAN SCRIBE (2000 BCE)
- BABYLONIAN SCRIBE (1800 BCE)
- DIOPHANTUS OF ALEXANDRIA (3rd century)
- ARABIC MATHEMATICIAN (9th century)
- INDIAN MATHEMATICIAN (12th century)
- CHINESE MATHEMATICIAN (13th century)
- ITALIAN ABBACIST (14th century)
- EUROPEAN MATHEMATICIAN (15th century)

- GERMAN COSSIST (16th century)

5. ***THE PLAY: Mathematical Artistry for Justice***

PROLOGUE

[*The narrator MARIA introduces the scene, assisted by EMMY, a twenty-first century mathematics teacher. They are seated at a small table front stage, to one side of the stage, where they remain throughout the play. The 'curtain' will rise upon a scene in the Office of Community Advice and Arbitration in a Chinese town, two millennia ago, with administrator YU-LIN seated at a desk, and his counting board beside him.*]

MARIA: Hello! My name is Maria, and I am your storyteller. I want to invite you to come with me to China about 2000 years ago. And this is Emmy, a mathematics teacher, who will accompany us.

EMMY: Hello! I will try to explain what the Chinese mathematician is doing, and relate it what we might call doing algebra today. Algebra has had a very long history, and changed character greatly over time.

MARIA: Let's start with the big picture. The subject we call algebra emerged in ancient Egypt and Mesopotamia, more than four thousand years ago. It began with procedures for solving problems about finding proportions, quantities, lengths and areas –

EMMY: – That'll be what we now call solving first degree equations and second degree equations.

MARIA: Right – except that their methods were often what we would call geometrical! The ancient methods have been preserved in texts – a few papyrus rolls in Egypt, lots of baked clay tablets in Babylonia – that were mostly used for training students in the Scribal Schools. These methods arose first out of quite practical considerations, but they became in the end objects of study for their own sake.

EMMY: Then there are even more complicated equations that arise when you are interested in volumes!

MARIA: That's right! Like: how should a granary be designed in order to hold enough grain, or rice, for the village until the next harvest? How big should a cistern be, or a dam or a canal, to carry sufficient water for the community? And what if you wanted to double or triple the capacity? What dimensions should those bricks be to ensure one man could carry a brick? And how big could a column, or a column base, be before transportation became impossible?

EMMY: And what if you wanted to build a massive wall, like the Chinese did, to protect your country from invaders?

MARIA: Good example! To build the Great Wall of China over a given piece of terrain, how much stone and how much mortar would be needed? What would it cost to transport it?

EMMY: How many people would be needed for transportation of materials and building the wall? How would you feed them all?

MARIA: How much should they be paid? And how quickly could it be done?

EMMY: The kinds of equations that emerged in the context of volumes are now called third degree equations, or cubic equations. Solving them involves taking cube roots at some stage. In theory, an equation could have any degree, and our modern family name for all of these equations people might want to solve is ‘polynomial equations'. So – did anyone in the ancient world come up with a higher degree equation, fourth degree or fifth degree?

MARIA: Not very often! But the Chinese eventually developed a numerical method for solving cubic equations and then extended it to deal with polynomial equations of any degree, using successive approximations, getting more and more accurate.

EMMY: So, for thousands of years, algebra was mostly about the solving of polynomial equations, with one unknown to be found. But there is another important kind of algebraic problem, isn't there? Finding two or more unknowns which simultaneously satisfy some equations. When were such problems encountered?

MARIA: Over 2000 years ago, the Chinese exponents of the art of *suan* (that means mathematics) had methods for solving *systems of linear equations*, like two equations involving two unknowns, where there are no squares or higher powers of the unknowns. One useful technique was to insert trial solutions. This involved calculating with actual numbers, rather than trying to juggle mentally with unknowns, and at least gave them a very good idea of the size of the true solutions.

EMMY: So, did they not know how to find exact solutions for their unknowns?

MARIA: Oh yes! – they went further and used something they called the Rule of Excess and Deficiency, and it gave them exact values. And all their calculating was done in a brilliant, quick fashion, using a large counting board marked with columns, on which they moved their red and black counting rods.

EMMY: How did they write down and record their calculations?

MARIA: They would write on bamboo tablets with a brush made of rat hair, dipped in ink. Let's go and see how they did it. We go back 2000 years, to a Chinese town and visit the Office of Community Advice and Arbitration.

[*Chinese music plays*]

CURTAIN RISES

[*YU-LIN is seated at a table, writing on bamboo tablets with a brush, which he dips into a small container of ink. Beside him on another table is a large counting board marked with columns, on which are some red and black counting rods about the size of a man's little finger. A woman, JUNG CHANG knocks and enters. She is agitated, and is waving a piece of paper.*]

YUNG CHANG: Good morning, Master Yu-lin! Excuse me! May I consult you, please?

YU-LIN: Good morning, Yung Chang! You are welcome. How are you and your family in these difficult times?

YUNG CHANG: We are well, thank you, all things considered. But there is something that I am very anxious about – it's about a financial transaction ... if I may …?

YU-LIN: Speak on, dear lady! For such matters am I here. Be seated in this chair. Tell me all.

YUNG CHANG: *[sitting down]* Thank you, Master Yu-Lin! It's about some land my husband bought yesterday, for growing millet and rice, and we all went to look at it today – and I am very worried about the quality of the land – *[she breaks down, and covers her face with her hands]*

YU-LIN: Here, take this. *[he hands her a paper handkerchief, and she blows her nose]* Now, my dear, have courage and stay calm, for when the red and black rods on my counting board begin to dance *[he motions towards the board on his desk]*, they may have a happier tale to tell! What can you tell me about this land you have bought?

YUNG CHANG: It is mostly very dry and rocky, and has poor soil. It was very expensive. We had taken out a loan ... I think my husband may have been robbed! He is a very gullible man ... he should have consulted me first. Do we have any legal redress from the justice court?

YU-LIN: Tell me, Yung Chang, did your husband make sure the seller wrote down on the bill his terms? Did he give any details about the quality of land he was offering?

YUNG CHANG: He did say he was asking less on account of the unterraced and non-arable parts of the land. It's down here somewhere ... *[she holds out the piece of paper]* But he didn't say how much was good grain-growing land, and it seems to me that most of it is bad land! I shall not be able to feed my family … we shall have nothing to sell to pay back the loan we have taken. [*she breaks down again and sobs into the handkerchief*]

YU-LIN: Hush, now, it may not be so bad. My calculation art will uncover the truth. Let me see that bill of sale ... *[she hands it to him]* Hmmm ... He states clearly his terms: for 300 coins you get one acre of good land. For 500 coins you get 7 acres of stony land. It says you have purchased altogether 100 acres. What did you pay?

YUNG CHANG: The price was exorbitant – the seller is a wicked, greedy man! He demanded 10,000 coins!

[YU-LIN scratches chin and twirls moustache, pondering the problem and making notes, while EMMY explains the slide]

SLIDE:

Price of one acre of good land is 300 coins; price of 7 acres of stony land is 500 coins.

Purchased altogether 100 acres. Paid: 10,000 coins.
Unknowns: amount of good land, and amount of poor land.

In modern terms: Let there be x acres of good land and y acres of poor land. Then we know

$$x + y = 100$$
$$300\,x + \frac{500}{7}y = 10{,}000$$

EMMY: [*can improvise if desired*] Here's a modern expression of the problem. We would call it solving a pair of simultaneous linear equations in the two unknowns, x and y, whose sum is the total amount of land. Can you see how the figures on the bill are expressed in terms of those two equations? … Anybody have questions?

YUNG CHANG: *[waits until there are no more questions and EMMY gives her a signal]* Master Yu-lin, can your art really tell us anything about the kind of land we have bought?

YU-LIN: Hmmmm ... Yes, certainly!

YUNG CHANG: That would seem like a miracle to me!

YU-LIN: *[laughs]* Yes, it continues to seem like a miracle of the darkest arts to some of my students! But, Yung Chang, teach your children this: *suan* – mathematics – is a noble art! *[strikes a pose and recites:]*

Mathematics rules the length and breadth of the heavens and the earth; it affects the lives of all creatures; it forms the beginning and the end of the five constant virtues – benevolence, righteousness, propriety, knowledge, sincerity; it brings to birth the yin and the yang; establishes the symbols for the stars and the constellations; manifests the dimensions of the three luminous bodies; maintains the balance of the five phases – metal, wood, water, fire, earth; regulates the beginning and the end of the four seasons; formulates the origins of myriad things.[53]

[53] This paragraph is based closely on the preface to the *Sunzi suan jing (Mathematical Classic of Master Sun)*, trans. in Lam and Ang, 1992, quoted, with Joseph Dauben's commen-

YUNG CHANG: *[clearing her throat]* Excuse me, Master Yu-Lin, I don't want to take too much of your valuable time ...

YU-LIN: *[recollects himself, coughs, and concentrates once more on the bill]* – erm, yes, I am perhaps getting carried away, but what I am saying, Yung Chang, is that mathematics can shed its pure light on many things. I understand you are eager to see mathematics bring forth children close to the *yin* and the *yang* of your own heart right now – the unknown elements of your problem today *[gestures toward the bill in his hand]* ... The unknowns are the number of acres of good, arable land, and the number of acres of poor land. I let those unknown numbers be called by the names *tian*, and *di*. What you desire to know are *tian* and *di*, and that is what my mathematical art will discover.

[He begins to place rods on the counting board and move them about, while MARIA and EMMY speak. YUNG CHANG fidgets during the interlude, looking worried and impatient, and blowing her nose, but also showing interest in what YU-LIN is doing.]

MARIA: Those words mean ‘heaven' and ‘earth'. So, the Chinese name for the basic method of finding one unknown is *tian yuan*, ‘Method of the Celestial Element’. When there is a second unknown involved, it is called *di*, meaning ‘earth.’ There is an old Chinese book on algebra called ‘Heaven and Earth in a Bag’.

EMMY: Our name for the art of finding unknowns is ‘algebra’ – derived from an Arabic word.

MARIA: Algebra has been developing for a long time, with contributions from many cultures. Whatever culture a scribe came from, the unknown would be given a special name. Let’s hear from a few!

[Cameo Appearances; actors should be pre-seated in order of appearance, each getting up and walking across the stage, stopping in the middle to speak;

tary, in Katz, 2007, 297. Although this work is believed to date from some centuries later, it is likely to be drawn from much earlier treatises, and the preface gives a good picture of the reverence in which the ancient Chinese held mathematics.

then (keeping their order) in a couple of minutes they will walk back across the stage to take a bow, one by one, returning to their seats]

EGYPTIAN SCRIBE (2000 BCE): Let the unknown be called *Aha*.

BABYLONIAN SCRIBE (1800 BCE): The unknown shall be called *Line*.

DIOPHANTUS OF ALEXANDRIA (3rd century): I say let the unknown number be called *Arithmos*.

ARABIC MATHEMATICIAN (9th century): The unknown shall be called *shei*, and the unknown square shall be called *Māl [pronounced to rhyme with first syllable of 'starling']*.

INDIAN MATHEMATICIAN (12th century): In calculation the unknowns are the *bīja*. I call the measure of an unknown quantity, *yāvattāvat*; or, if there is more than one unknown number to be found, I call them by the names of colours, and I write a letter, for short.

CHINESE MATHEMATICIAN (13th century): The name of the unknown element is *tian*; or if four unknowns are present we call them *tian, di, ren* and *wu*.

ITALIAN ABBACIST (14th century): The unknown is called *Cosa!*

EUROPEAN MATHEMATICIAN (15th century): That which we seek shall be called *res*, or *radix*.

GERMAN COSSIST (16th century): We shall call the unknown *Coss*!

SLIDE:

Egyptian	**2000 BCE**	***Aha***	**Heap, pile**
Babylonian	1800 BCE	*Line*	Geometric length
Diophantus	3rd century	*Arithmos*	The Number
Arab	9th century	*Shei, Māl*	Treasure
Indian	12th century	*Bīja,* *Yāvattāvat*	Seeds As much as so much
Chinese	13th century	*Tian,* *Di, Ren,* W*u*	Heaven, Earth, Man, Matter
Italian Abbacist	14th century	*Cosa*	The Thing
European	15th century	*Res*, R*adix*	Thing, Root
German Cossist	16th century	*Coss*	borrowed from the Ita-

MARIA: In the language of Yu-lin and Yung Chang, *tian* means ‘heaven’ – and stands for an unknown number of whatever – people, coins, acres of land.

[each scribe-mathematician walks back across the stage and bows, as MARIA mentions them]

The ancient Egyptians’ word *Aha* means a heap or pile, [waits for scribe to bow] and we guess their calculations were often about grains, loaves, fruits or other food that is stored in piles.

The Babylonians worked a lot with geometrical diagrams, so their Sumerian word for unknown means ‘line’.

Diophantus of Alexandria uses a Greek word, *Arithmos*, which simply means ‘the number’.

The Arabic mathematician’s word *Māl* means ‘treasure’, or valued property.

The Indian’s word *bīja* means seeds – things yet to germinate in the dark earth and emerge into the light! His word *yāvattāvat* means ‘as much as so much’; the letters he spoke of stand for three colours.

The medieval Chinese mathematician still uses Yu-lin’s old word *tian* for an unknown. His names for four unknowns, which I would rather not pronounce, mean ‘Heaven, Earth, Man, Matter’.

The Italians’ word *Cosa* means ‘The Thing’, and the fourteenth century Italian experts at the art of calculating with Hindu-Arabic numerals were called *maestri d’ abbaco*, or abbacists, meaning ‘master-calculators’.

European scholars wrote in Latin for centuries, and the words *res* and *radix* mean ‘thing’, and ‘root’.

The Germans’ word *Coss* is derived from the Italians’ word, and in sixteenth century Germany, algebra was called the ‘Art of the Coss’, and so its practitioners were known as ‘Cossists.’

What do we call the unknown today, Emmy?

EMMY: Well, algebra is full of symbols now. This helps for economy and elegance, but, just like a foreign language, it can be tough at first, for learners.

The unknown can be represented by any letter or symbol. We usually use letters from the end of the alphabet – x, y, z.

MARIA: The Indian way was similar – to use the first letters from three colour names, like r, b, and y, for red, blue, yellow. Of course, they would use letters from their own script and colour names from their own language.

EMMY: If there's only one unknown, we usually call it x, ever since René Descartes, the 17th century French mathematician.

MARIA: It is possible that there are ancient origins in various x-words. The medieval Arabic word, *shei*, for an unknown thing, was translated into Greek as *xei*, spelt X, E, I. And the Greek word for an unknown person is *xenos*, from which we get 'xenophobia' – an irrational fear of outsiders, or foreigners; both begin with an x.

EMMY: I like the fact that when the German physicist Wilhelm Röntgen discovered a wonderful, mysterious, and unknown form of radiation which could help us 'see' inside the human body, he naturally named the rays *X-strahlen*, which we still call X-rays, though Germans call them *Röntgenstrahlen*. And then there is the X-factor – made famous by a certain television programme seeking talented people with that mysterious, undefinable thing that makes celebrities out of unknowns ... *[both laugh]*

MARIA: The quest to discover unknowns is everywhere! Let's go back now to ancient China, and see how Yu-lin is doing with calculating the unknowns for Yung Chang.

YU-LIN: From these numbers you have given me today, Yung Chang, I am working out how much good land your seller claimed to be including in the sale, and how much bad land.[54]

[54] This is Problem 17 in Chapter 7 of the *Jiu zhang suan shu* (*The Nine Chapters)*. For details, see Katz, 2009, 209–10 and Katz, 2007, 270–72. For detailed working of similar problems using this same method of 'excess and deficit' – a kind of 'double false position', from both the *Suan shu shu* and the seventh chapter of *The Nine Chapters*, see Katz & Parshall, 2014, 83–87; and Cullen, 2004, 79.

YUNG CHANG: I still cannot believe it – those numbers leave me in thick darkness! How can these unknown quantities emerge from the darkness into the light? But I will be very grateful if you can perform the miracle –

YU-LIN: The method we use is not magic, it appeals not to gods and demons but to the science of reason! It is called 'The Method of Excess and Deficiency.' It leads us as we seek the two unknowns of your problem – by reducing the problem to finding one unknown, which we can discover, and thence we can find the other unknown also. Thus, we unify the *yin* and the *yang*, to turn the two opposites into one. We must begin with a supposed solution. Suppose there are 20 acres of good farmland, 80 acres of poor farmland.[55]

YUNG CHANG: I don't believe there is that much good land!

SLIDE:

Method of Excess and Deficiency in modern notation:

Trial solution: Suppose 20 acres of good land, 80 of bad.
Then the total price would be:

$$300 \times 20 + \frac{500}{7} \times 80 = 6{,}000 + 5{,}714\,\frac{2}{7} = 11{,}714\,\frac{2}{7}$$

Thus, the excess over 10,000 is $1{,}714\,\frac{2}{7}$.

YU-LIN: Then that would cost you, hmmm ..., twenty acres at three hundred each, that's six thousand coins; and eighty acres at, hmmm ... [*mutters and surveys the counting board, writing down the answer with his brush*]. Altogether it is 11,714 and 2/7, so there is an excess over your buying price, and it is 1,714 and 2/7.

YUNG CHANG: You mean two-sevenths of a coin? – but that's silly! And you say *excess*? You mean there *isn't* that much good land?

YU-LIN: That's right. Now, if we suppose instead that there are only 10 acres of good land –

[55] Yu-Lin's words and calculations are based closely on the solution in the *Jiu zhang suan shu*.

YUNG CHANG: – Oh no! That's 90 acres of bad land! Surely there cannot be that much bad!

SLIDE:

In modern notation:
Trial solution: Suppose 10 acres of good land, 90 of bad.
Then the total price would be

$$300 \times 10 + \frac{500}{7} \times 90 = 3{,}000 + 6{,}428\,\frac{4}{7} = 9{,}428\,\frac{4}{7}$$

Thus the deficiency under 10,000 is $571\,\frac{3}{7}$.

YU-LIN: Hmmm ... *[mutters and moves rods around on counting board, writing down the answer]*t would cost you 9,428 and 4/7. This time there is a *deficiency* of 571 and 3/7.

The Rule of Excess and Deficit says: Display the assumed rates of good land, and lay down the corresponding excess and deficiency below:

SLIDE:

In modern notation:

amounts of good land:	20	10	$10\left(1{,}714\frac{2}{7}\right) + 20\left(571\frac{3}{7}\right)$
excess and deficit:	$1{,}714\,\frac{2}{7}$	$571\frac{3}{7}$	$1{,}714\,\frac{2}{7} + 571\,\frac{3}{7}$

YU-LIN: Now cross-multiply by the rates, and add the results to get the numerator. Also add the excess and the deficiency to get the denominator. Next, divide the numerator by the denominator ... if there are fractions, reduce them ... *[more muttering and moving rods around counting board]*

SLIDE:

Solution in modern notation: There are x acres of good land and y acres of bad land.

x is : 10 × excess and 20 × deficiency
divided by excess added to deficiency

$$x = \frac{10\left(1{,}714\ \frac{2}{7}\right) + 20\left(571\ \frac{3}{7}\right)}{1{,}714\ \frac{2}{7}\ +\ 571\ \frac{3}{7}} = 12\frac{1}{2},$$

$$y = 100 - 12\frac{1}{2} = 87\frac{1}{2}.$$

YUNG CHANG: What's it saying? What do the rods say?

YU-LIN: The answer is ... [*mutters and keeps moving rods around counting board*] ... Hmmm, this is quite a challenging calculation ... Ah! The answer is twelve-and-a-half – that is how many acres of good land. And so there must be eighty-seven-and-a-half acres of bad land. [*writes answer down*]

YUNG CHANG: Eighty-seven and a half acres of bad land! Oh-oh!!

YU-LIN: Unfortunately, as you suspected, most of your land is not arable. But that was reflected in the price, so I think you may have no grounds for appeal. Even by the seller's own calculations, he claims to be selling you only a small amount of good land. It is likely that you will find it is an honest price. Here is my calculation, with my signature. *[he hands her a bamboo tablet; and then gives her back the bill]* And here is your bill of sale.

YUNG CHANG: Thank you!

YU-LIN: But if you are not satisfied, would you like me to send a surveyor?

YUNG CHANG: No, no, thank you! I know how much *they* cost. We have very little money left, and a big loan to repay.

YU-LIN: But remember, dear lady, that this problem had two unknowns – and the other one, the number of acres of good land, is the one you should be most interested in. Twelve and a half acres – that's something!

YUNG CHANG: Yes, you are right. It is not as bad as I feared it might be. And at least my husband has not been cheated. I will try to rent some of the rocky land out to my neighbours for their cows and pigs. Eighty-seven and a half acres, hrrumph! Thank you for your assistance. How much do I owe you?

YU-LIN: Consider it a favour from the sweet goddess Mathematics! – Just bring me a bag of grain at harvest time from your twelve-and-a-half acres of good land!

[they bow to each other]

YUNG CHANG: Thank you, Master Yu-Lin! May your *suan* art prosper! May you discover many a hidden *tian* for the people who come to consult you! And may your wife and your land be fertile … *[EXITS]*

YU-LIN: Farewell! *[smiling as he turns to audience]* A little mathematics goes a long way – what would we be without it? I suppose I should really not be so generous with my skills – my wife would kill me if she found out. *[laughs]* I sincerely hope she does not turn out to be more fertile! But I will please her by bringing home the money the Minister has promised when we finish compiling the manual on mathematics. May my *suan* – my mathematical art – prosper, indeed! Good land and bad land – now, that's an excellent problem to put into the book! I will write it up while it is fresh in my mind ... *[he dips his brush-pen in the ink and begins to write]*

CURTAIN FALLS *while Chinese music plays*

CAST LINES UP AND TAKES A BOW

6. ***Discussion***

After the play, there was plenary discussion and feedback, reflecting on the play and how it might be used in classrooms. Participants were asked for their 'gut responses' to the play, and critique and questions were invited. A lively conversation took place, prompted by such questions as these:

- What were the best moments? What didn't work well?
- What improvements can you suggest, in staging, action, emotion, voice projection?
- Authenticity – how close to stay to the primary source material?
- Can any of you share experiences of using theatre? Or being inspired by theatre? Or seeing young people inspired by theatre/improv/drama?
- How might this play be used for educational purposes at various levels?
- Could you write something similar for different groups or levels of communication; for different balances of personal, historical, mathematical?
- Can personal involvement – actually enacting a character from another world – bring a new dimension into our learning and motivation?

- How can such experiences be encouraged under the pressures of the curriculum?}

The discussion was wide-ranging and positive. There is no space here to attempt more than a very brief summary. As usual, surprise was expressed at how little effort and preparation was needed to produce an effective piece of theatre, and how much fun was to be had in the process. The script was agreed to be appropriate and entertaining. Discussion focused on the issue of making readily available for teachers more resources that are relevant and accessible to learners. The difficulty was faced and discussed of balancing the requirements of art and authenticity, and the importance was stressed of insisting that participants show respect for diverse cultures. It was agreed that physical participation in a play, enacting dialogue between historical characters, is an excellent way of 'getting into the heads' of old mathematicians and feeling something of their emotions.
The participants then broke into small groups, and discussed one or more of the following assigned questions, with each group reporting back briefly afterwards.

1. Think of an episode in the history of mathematics, perhaps set in your own country, that could be enlivened by a play and used in a classroom. Brainstorm ideas for the play you could write.
2. Think of parts of the standard mathematics curriculum that are hard to motivate, and brainstorm ideas for a introducing such a topic with a play.
3. How can we win educators/administrators/heads over to the cause? How can we motivate them to invest time & resources in mathematical drama?

7. Concluding remarks

An important feature of this workshop was the opportunity for each participant to act, help direct and rehearse the actors, or set up stage props, lighting, or sound. The discussions, both plenary and in small groups, showed enthusiasm for the idea of incorporating such experiences in the classroom, as a means of firing up interest in both mathematics and its history. The workshop closed by issuing a challenge: We need more theatre in mathematics education, and we need more resources. Consider creating your own, preferably based on primary sources and staying as true as possible to the history and context.

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