RANDOM WALKS IN THE HISTORY AND EPISTEMOLOGY OF MATHEMATICS

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ABSTRACT

The random walks in the title go from eleventh-century Chinese mathematician Shao Yong's Xiantian diagrams, illustrating the genesis of the Yi Jing hexagrams, up to Darwin's tree of life and the contemporary "eternal symmetree" of Stanford physicists in cosmology. They also involve the old Chinese 5-element cycles via random whole numbers. We present illustrative examples of classroom activities inspired by these random walks, with humanistic as well as scientific students, including pre-service and in-service teachers, where we implement an experiential way of teaching. Our final discussion takes into account some caveats regarding Whig history.

1 The Yi Jing, Shao Yong, Bouvet and Leibnitz

Our first random walk starts with the launching of the Jesuit Mission to China (1552-1715) by François Xavier. Among the Jesuits travelling to China ca. 1700 was mathematician and astronomer Joachim Bouvet (1656 – 1730). While cleverly trying to find bridges between Chinese culture and Christianity, a prerequisite to evangelisation, Bouvet stumbled upon Yi Ying, (Book of Changes), the ancient Chinese oracle. Recall that Yi Jing hexagrams (as answers to questions asked) were selected by a traditional random procedure (Wilhem/Baynes, 1950), which provided equally likely Yang (unbroken) and Yin (broken) lines for the hexagrams.

Shao Yong (1011-1077) was an extraordinary Chinese thinker, whose complex and metaphorical thinking blended Confucianism, Taoism and Buddhism, with a unique twist of mind. He has been described as an alien, of another place and time (Birdwhisttell, 1989, p. 52). His main concern was *change* in the Universe, rather than matter and substance, *processes* rather than *structures*. He developed an iconic and philosophical approach to Yi Jing, as a combinatorial encoding of the varieties of change in nature and in human society (Birdwhistell,

1989). Notice how his rectangular and circular Xiantian diagrams (Fig. 1) provide a *synthetic* view of the 64 hexagrams.

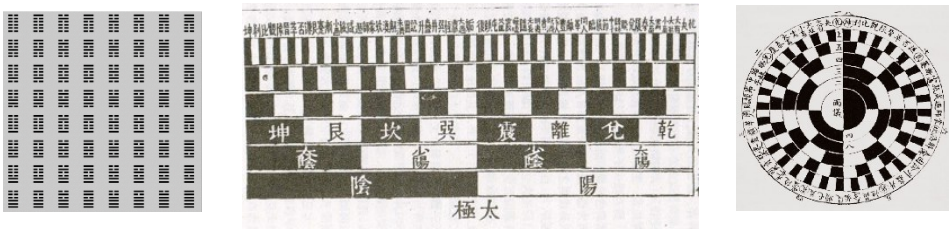

Figure 1. Shao Yong's 8x8 square and Xiantian diagrams (Marshall, 2010)

Bouvet discovered the isomorphism between Shao Yong's 64 hexagram sequence and the numbers 0 to 63 written by Leibnitz in the binary way, enabling Leibnitz to trace back his construction of the binary system 4 500 years ago embodied in the Yi Jing hexagrams. This resonated with his dream of linking Chinese "natural theology" and his Christian faith.

We have here a remarkable epistemological phenomenon: the convergence of an ancient Chinese insight and a more recent European one on the binary nature and dynamics of the Universe, which was a common concern of Shao Yong and Leibnitz. (Aiton & Shimao, 1981; Birdwhistell, 1989). Xiantian is very likely the first explicit avatar of the binary tree in human civilisation. It appears as a pictorial "explanation" of the generation, through binary branching, of the hexagrams, where black=Yin, white=Yang (cf. Needham, 1956, pp. 276-277). Shao Yong appears then as a forerunner of Darwin: both share the remarkable idea of fathoming the current state of a system as the outcome of a *branching process* unfolding in time. Shao Yong saw the 64 hexagrams as the outcome of his Xiantian and Darwin, the sundry living species on earth, as coming out of the Tree of Life (Gould, 1997).Interestingly Xiantian surfaces much later again, in the West, as the "eternal symmetree", a discrete combinatorial model for a multiverse in eternal inflation in cosmology (Harlow et al., 2012). See Fig. 2.

Figure 2. Three-step causal tree and causal future of *a*, for *p=2*.

2 The Chinese five element cycles and random numbers

After the Chinese Five Element Theory (Wu Xing), dating back to ca. 400 BC (Needham, 1956, p. 242), the world changes according to the five elements' *generating* or *overcoming* cyclic relationships (ibid. pp. 253 ff). See Fig. 3. Generating Interactions may be worded as: Wood fuels Fire; Fire forms Earth; Earth contains Metal; Metal carries Water; Water feeds Wood. Overcoming Interactions may be worded as: Fire melts Metal; Metal penetrates Wood, Wood separates Earth; Earth absorbs Water; Water quenches Fire.

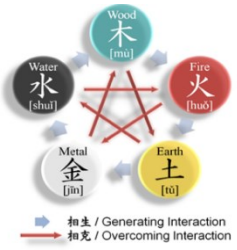

Figure 3. Generating and overcoming cycles in Wu Xing

The mathematical question arises: how could we model in an elementary way these "intransitive" cycles, using just whole numbers, so that the cyclic orderings emerge naturally? One idea is to use *random whole numbers*, i. e. (finitely supported) probability distributions on the whole numbers, which can be embodied in *dice*. A usual dice embodies the random number taking values 1, 2, 3, 4, 5, 6 equally likely. We have a natural order relation among random numbers: A random number *m dominates* (or *overcomes*) another random number *n* if *m* is bigger than *n* "most of the time". Now, to model the 5-element overcoming and generating cycles, we can concoct the following weird dice in Fig. 4 (Grime, 2019). Here yellow is dominated by red, red by green, green by pink, pink by blue and blue by yellow. The opposite cycle goes: yellow – pink – red – blue – green – yellow, where each dice dominates the next.

Figure 4. The Grime dice modelling Wu Xing.

3 Illustrative examples of related classroom activities

Our activities involve the following learners at the University of Chile: (a) First year humanistic students, taking a one-semester mathematics course; (b) Pre-service secondary mathematics-physics teachers; (c) Prospective mathematicians and mathematics-physics teachers taking a history and epistemology of mathematics course; (d) In-service primary school teachers enrolled in a professional development program. Our didactic methodology is to have the students work autonomously on given or self-constructed problems in random small groups of 3 or 4 monitored by the teacher and assistants.

3.1 Discovery approach to Shao Yong's diagrams

How would a *synthetic* (big) picture of the whole binary 64 hexagram sequence look like?

Here learners (d), used to work with concrete material, spent half an hour working in groups, cutting out a paper printout of the hexagrams and trying to put them together side by side. Four of thirty teachers came up with Xiantian like diagrams, after coding Yang as white and Yin as black.

3.2 Archeological exploration of Shao Yong's square

Learners received a copy of Shao Yong's square and tried to make sense of it. They saw the square as an 8x8 matrix indexed by trigrams! Some recognized the binary expansion of numbers by interpreting Yang or Yin in the n-th line as the *presence* or *absence* of 2^n, as Leibnitz did.

3.3 Grasping Xiantian in a glimpse

We showed a glimpse of Xiantian to learners (a), (c) and (d) and asked them to reconstruct it. See Fig. 3 (Soto-Andrade, 2008).

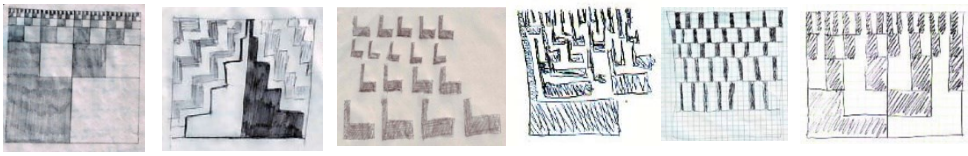

Figure 5. Students' reconstructions of Xiantian

3.4 Numbers as ascendings paths in a rooted binary tree

Learners (c), metaphorised numbers from 0 to 63 as ascending walks in the 6-generation binary tree and went further to the infinite binary tree (Needham, 1956, p. 276) and its *boundary,* rediscovering Hensel's notation of 2-adic numbers as series of powers of 2 (Dickson, 1910).

3.5 A baby avatar of the Chinese 5-element cycle

Our students concocted tetrahedral dice to model the well-known intransitive 3-cycle: scissors, stone, paper. See Fig. 6.

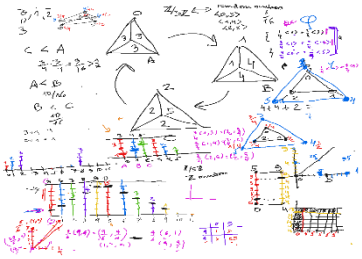

Figure 6. Student production: tetraedral dice to model the 3-cycle

4 Discussion and caveats

In our classroom activities, particularly with students C on a separate historical track (Fried, 2001), we used Shao Yong's diagrammatic mathematical and philosophical insights as *triggers* of autonomous mathematical and epistemological reflexions. As spin-offs of our activities, we noticed that the work the students did in a separate history of mathematics track fed back, in a circular way, to their understanding of previous mathematical contents they were supposed to master. They often reported having understood for the first time contents they had learned in a formal, abstract and disconnected way. An enactivist caveat is required though: we do not see the binary tree as a well-defined mathematical object "standing out there", which is fathomed and *represented* by different cultures in different guises (avatars). To us, the binary tree is a typical Western mathematical construction, which we automatically project onto Xiantian. We intend nevertheless to avoid being "Whiggish" in this historical context (Butterfield, 1931), particularly avoiding the "Brownian mirage" of seeing a direction in our path backwards, which from the perspective of the past is indeed the path of a random walker (Fried, 2001). We try instead to listen to Shao Yong (Arcavi & Isoda, 2007), suspending our preconceptions

(*epoché*) and accept that this may enhance our understanding of "something" that we have constructed as a binary tree nowadays in our culture. Indeed, trees appear often as hierarchical *structures,* contrary to *rhizomes* (Deleuze & Guattari, 1980), but Xiantian suggests a (dichotomous branching) *process*, pertaining to a changing and flowing cosmos, a Taoist view indeed (Birdwhistell, 1989). Epistemological disorientation (Clark et al., 2018) may help us to acknowledge that other cultures have had insights ours is blind to.

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