

# HISTORY OF MATHEMATICS IN PORTUGUESE TEXTBOOKS OF VICENTE GONÇALVES (1896-1985) AND JOSÉ SEBASTIÃO E SILVA (1914-1972)

Hélder PINTO<sup>1</sup>, Cecília COSTA<sup>2</sup> and Helmuth R. Malonek<sup>3</sup>

<sup>1</sup> Piaget Institute, RECI and CIDMA, Portugal, [helder.pinto@ipiaget.pt](mailto:helder.pinto@ipiaget.pt)

<sup>2</sup> University of Trás-os-Montes e Alto Douro and CIDTFF, Portugal, [mcosta@utad.pt](mailto:mcosta@utad.pt)

<sup>3</sup> University of Aveiro and CIDMA, Portugal, [hmalon@ua.pt](mailto:hmalon@ua.pt)

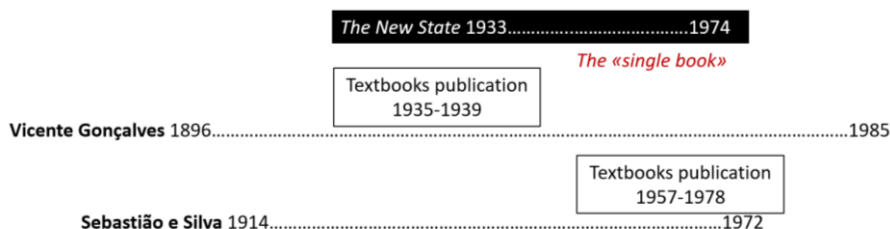
## ABSTRACT

Vicente Gonçalves (1896-1985) and José Sebastião e Silva (1914-1972) were two Portuguese mathematicians' university professors who have written textbooks for undergraduate levels. Those books became famous by their mathematical accuracy but also by their notes about History of Mathematics. In this study we identify and analyse in detail the characteristics of those historical notes.

## 1 Introduction

During the time of the Estado Novo («The New State», synonym for the dictatorship installed by A. Salazar in Portugal in 1933 and lasting until 1974), two Portuguese mathematicians and university professors – Vicente Gonçalves (1896-1985) and José Sebastião e Silva (1914-1972) – wrote mathematical textbooks for high school. This was unusual in Portugal at the time. Only few textbooks by J. Sebastião e Silva were co-authored by the high school teacher Silva Paulo.

These textbooks are from distinct periods: those by Vicente Gonçalves are from the beginning of the Estado Novo (published between 1935 and 1939), while those by J. Sebastião e Silva are from the end of the dictatorship (1957-1978) (Fig. 1).



**Figure 1.** Time scheme

The textbooks written by Sebastião e Silva were, in general, the *livro único* [single book]. Being *livro único* means that they were the only books that were

allowed to be used to teach mathematics in all of the country's schools. The approval period for a single book was five years, during which the authors could propose, in new editions, changes that they considered important. On each copy offered for sale, it would appear on the back cover, the mention "OFFICIALLY APPROVED AS A SINGLE BOOK", with the indication of the Government Diary in which it was instituted. In addition, they were numbered and approved by the Ministry of National Education (Almeida, 2011).

Several of Vicente Gonçalves and Sebastião e Silva textbooks have the particularity of containing notes about the History of Mathematics, a fact that was uncommon in high school education in Portugal. The aim of this study is to identify and analyse in detail the characteristics of those historical notes.

Vicente Gonçalves and J. Sebastião e Silva were important members of the mathematical community of their time; it's possible to find more information about them, for instance, in (Costa, 2001) and (Guimarães, 1972). Their works were widely disseminated in Portugal and are still recognized today for their scientific accuracy. Given their importance in the Portuguese panorama, we consider pertinent the description, study and understanding of this option of inserting History of Mathematics' topics in the educational context through textbooks. These examples of using History of Mathematics are interesting because they are consistent with the goals of the HPM group although they are from an earlier period (mid-twentieth century).

## ***2 Methodology***

The corpus of analysis was constituted by the textbooks for Mathematics Secondary Education, five written by Vicente Gonçalves and three by Sebastião e Silva, two of them in co-authorship with Silva Paulo. The categories considered were the ones established in (Fauvel, 1991): (i) as brief historical notes; (ii) as introduction of new concepts; (iii) as a pedagogical tool; (iv) as a content referring to some historical facts; (v) as resource of exercises or examples.

## ***3 Findings***

Three of the five textbooks by Vicente Gonçalves have historical remarks: (Gonçalves, 1937a, 1937b, 1939). All the three textbooks by Sebastião e Silva have historical remarks: (Sebastião e Silva & Silva Paulo, 1968a, 1968b) and (Sebas-

tião e Silva, 1978), an after death compilation. Considering Fauvel's categorization (1991), the textbooks of both authors have:

i) **brief historical notes** (many as footnotes):

This notes focus mathematicians' biographies, introduction of notations, and the creation and evolution of mathematical symbols. Next we present some illustrations<sup>52</sup>:

*"Algebra comes from the Arab word al-jabr, reduction, decomposition (see further on no. 137). The al-jabr is one of the operations treated by Al-Kwarizmi in his work Al-jabr w' al moqabalah, translated in the XII century by Leonardo de Pisa and followed in Western Europe till the XVI century."* (Gonçalves, 1937a, p. 240)

*"The sign  $x$  appears for the first time in 1631, in Clavis Mathematicæ published by Oughtred; the dot appears, on the same date, in Artis Analyticæ Praxis, by Harriot. Descartes simply joined together the factors (as in the text)."* (Gonçalves, 1939, p. 62)

*"The word "cartesiano" is derived from "Cartesius", the Latin name of the great French mathematician and philosopher Descartes (1596-1650), one of the greatest thinkers of all time. It was Descartes who introduced this method of representation, founding analytic geometry, which will be studied in more detail in 7<sup>th</sup> grade."* (Sebastião e Silva & Silva Paulo, 1968a, p. 122)

Both Vicente Gonçalves and Sebastião e Silva gave prominence to ancient Portuguese mathematicians and explain, with certain detail, the innovation of their works. The notes concerning Pedro Nunes and Daniel Augusto da Silva, should be highlighted.

ii) **contents referring to some historical facts:**

These textbooks contain several sections entirely devoted to different topics of History of Mathematics. In the following are presented some of the contents treated.

- Chapter Metric System, in (Gonçalves, 1937a, p. 148);
- Numbers Representation, in (Gonçalves, 1939, pp. 26-31), see Fig. 2;

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<sup>52</sup> All the quotes presented in this paper were translated by the authors from the Portuguese original text.

- Distribution of prime numbers, in (Gonçalves, 1939, pp. 169-170);
- Origin and evolution of the concept of fraction, in (Gonçalves, 1939, pp. 196-200);
- Mensurable and incommensurable quantities, in (Sebastião e Silva & Silva Paulo, 1968a, pp. 71-74);
- The infinity and infinitesimals, in (Sebastião e Silva & Silva Paulo, 1968a, pp. 180-184), see Fig. 3;
- History of Algebra, in (Sebastião e Silva & Silva Paulo, 1968b, pp. 211-220).
- History of Logarithms (and slide rule), in (Sebastião e Silva & Silva Paulo, 1968b, pp. 276-281).

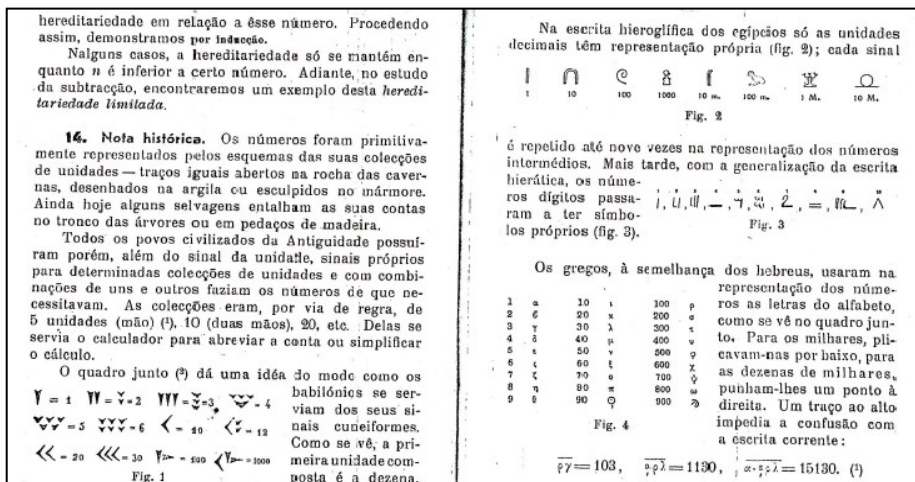


Figure 2. Historical section about numbers in (Gonçalves, 1939, pp. 26-27)

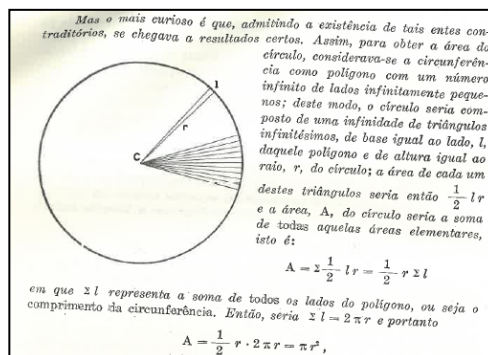


Figure 3. Part of the historical section in (Sebastião e Silva & Silva Paulo, 1968a, p. 182), where were used infinitesimals to find the area of the circle.

Some of the historical sections were very long and complete; for instance, in the section about the “*History of the Algebra*” were pointed out all the following names (the order of the list respect the order in the original text): Alkarismi, Euclides, Diofanto, Fibonacci, Dedekind, Cantor, Pedro Nunes, Viète, Jordanus, Stiefel, Harriot, Descartes, Del Ferro, Tartaglia, Cardano, Ferrari, Bombelli, Abel, Ruffini, Galois and Liouville; and with the warning: “the History of Algebra does not end here: it continuous, following the history of man on Earth” (Sebastião e Silva & Silva Paulo, 1968b, p. 220).

The study and the analysis of some historical parts from these textbooks can provide current teachers some ideas for introducing new concepts as well for using it as source of exercises or problems.

#### ***4 Final remarks***

This study shows that the uses of History of Mathematics in the textbooks of Vicente Gonçalves and Sebastião e Silva, two very prominent 20<sup>th</sup> century Portuguese mathematicians, are mostly as brief notes and as content referring to historical facts.

The brief historical notes focus on mathematicians (their life and work), and the introduction of notations and their creators, as well as creation and evolution of mathematical symbols. The reference to some historical facts, in various cases is very extensive and complete. There are detailed explanations of various topics that are seldom currently found in a textbook. For example: measurable and incommensurable quantities; the approximation of the number Pi; Zeno's paradoxes; and Zermelo's axiom. There are also very detailed notes on some of the most important figures of all time such as Fermat, Leibniz, Newton, Galois, as well as references to several of their works.

History of Mathematics was not mandatory and it was printed in small letters; in small letters are also printed the most difficult and theoretical exercises that only should be applied to the “good” students. Teachers were advised at the Preface of the book to present these topics only to the more interested students (Sebastião e Silva & Silva Paulo, 1968a and 1968b). Nevertheless, as it was *livro único* [single book], all students had access to the historical information contained in those textbooks.

Note that these historical notes were intended, mainly, to humanize the discipline of mathematics:

“The detailed clarification of certain issues, as well as the insertion of the subjects within the framework of a general culture, which temper and humanize the abstractionism inherent to mathematics, trying to explain it as a historical process – all of this is a considerable enterprise, which can only be attempted in a book.” (Sebastião e Silva & Silva Paulo, 1968a, Preface)

Finally, notice that these historical notes were quite complete and thorough in comparison to those in current textbooks in Portugal. Notice also that there are examples in other countries of the use of History of Mathematics in footnotes (for one example, in 19<sup>th</sup> Century (Spain), see (Muñoz-Escolano & Oller-Marcé, 2021)).

## REFERENCES

- Almeida, M. (2011). O livro único em Portugal. 1947-1974. In L. Saraiva (Ed.), *Livro de Atas: V Encontro Luso-Brasileiro de História da Matemática* (pp. 305-320). Castelo Branco: C. M. Castelo Branco.
- Costa, C. (2001). *José Vicente Gonçalves: Matemático. . . porque Professor!*. Funchal: Centro de Estudos de História do Atlântico.
- Fauvel, J. (1991). Using History in Mathematics Education. *For the learning of Mathematics*, 11(2), 3-6.
- Gonçalves, J. V. (1937a). *Aritmética prática e Álgebra para as 1.ª, 2.ª e 3.ª classes do curso dos liceus*. Braga: Livraria Cruz.
- Gonçalves, J. V. (1937b). *Compêndio de Álgebra para o 3.º ciclo (7.ª classe) do curso dos liceus*. Braga: Livraria Cruz.
- Gonçalves, J. V. (1939). *Compêndio de Aritmética para o 3º ciclo (7ª classe) do curso dos liceus*. Braga: Livraria Cruz.
- Guimarães, A. A. (1972). *Vida e Obra do Professor José Sebastião e Silva*. Porto: FCUP.
- Muñoz-Escolano, J. M., & Oller-Marcén, A. M. (2021). Notas al pie en la Aritmética (1884) de los Coroneles Salinas y Benítez. *Matemáticas, educación Y Sociedad*, 4(1), 17-33.
- Sebastião e Silva, J. & Silva Paulo, J. (1968a). *Compêndio de Álgebra para o Ensino Liceal, 6.º ano* (Tomo 1). Braga: Livraria Cruz.
- Sebastião e Silva, J. & Silva Paulo, J. (1968b). *Compêndio de Álgebra para o Ensino Liceal, 7.º ano* (Tomo 2). Braga: Livraria Cruz.
- Sebastião e Silva, J. (1978). *Compêndio de Matemática para o Ano Propedêutico*. Lisboa: GEPMEIC.