TEACHING SIMILAR TRIANGLES IN HISTORICAL PERSPECTIVE

Polimnia ANDREADOU, Konstantinos NIKOLANTONAKIS
University of Western Macedonia, Florina, Greece
pandreadou@sch.gr, knikolantonakis@uowm.gr

ABSTRACT

This study presents a research designed in order to investigate the way that the implementation of history of mathematics in the teaching process affects the understanding of the concept of similar triangles by the students. It was carried out during the 2020-21, and twenty-one 14-year-old students participated in it. This didactical intervention helped students to acknowledge not only the importance of the concept of similar triangles, but its historical evolution as well.

1. The integration of the History of mathematics in the teaching of the similarity of triangles.

Many researchers have referred to the benefits of using the History of Mathematics in teaching. The reasons why incorporating history is useful have been categorized from time to time and presented in various articles. Fried (2001) summarized these arguments into three categories. The first has to do with the fact that the history of mathematics makes mathematics more human by connecting the study of mathematics with human motives. The second is that it stimulates students' interest by giving variety to the approach of different concepts and by showing the role of mathematics in society. The third category refers to the fact that history provides knowledge about concepts, problems and problem solving.

1.1 Ways of using the history of mathematics in teaching similar triangles.

In this particular teaching intervention, we made an approach to teaching the similarity of triangles, inspired by the historical evolution of the concept. The highlights of this development were identified and given to students through a sequence of problems adapted to modern teaching contexts. Thus, in this way as reported by Tzanakis and Thomaidis (2000), history offers a deeper and more comprehensive understanding of the specific subject.

Also, an authentic historical source was used, namely an excerpt of J. Errard's book *La géométrie et pratique générale d'icelle*. In this excerpt, Errard describes the construction of an instrument for measuring inaccessible distances as well as how measurements are made on a flat surface.

According to Jahnke et al. (2000), three concepts best describe the teaching effect of using authentic sources: The concept of replacement, the concept of reorientation and the concept of cultural understanding.

In the first case, the integration of history offers something different from the usual, because mathematics is seen as an intellectual activity rather than a set of knowledge and techniques. In the sense of reorientation, in understanding a historical text, we remember that concepts were invented and did not just emerge. According to the cultural understanding, mathematics is placed in a specific scientific and technological context of the history of ideas and societies. By studying this passage, the students had the opportunity to understand the social need that led to the construction of Errard's tool. They could also understand that the concept of similarity of triangles is not only limited to the techniques of calculating the sides of a triangle but has important applications in problematic situations of everyday life.

1.2. The Errard's historical instrument and its role in teaching similar triangles.

An instrument in mathematics is the result of an invention and its use can certainly give more knowledge about it than a simple description of it in words (Barbin, 2016). The introduction of cultural artefacts into the classroom also strengthens the connection between school mathematics and the knowledge of everyday life (Bussi, 2000). In addition, it is important that the construction of artefacts helps the interaction and communication between students who work in team to achieve the goal (Bussi et al., 2014).

For the above reasons, therefore, in addition to studying the passage in which Errard's instrument is described, the students had the opportunity to construct and use it.

2. The research

2.1. Purpose and research question.

Our research intervention study the similar triangles. It was implemented in the 2020-2021 academic year to twenty-one students in ninth grade. The purpose of the research was to study how the integration of the History of Mathematics in the teaching of similar triangles, affects the degree of understanding of the concept by the students.

The research question was: In what way does students' dealing with historical problems and constructing/using a historical instrument affect their understanding of the concept of similar triangles. The research tools were: seven work sheets, a cognitive test and researcher's notes during the intervention. We gave one teaching hour to each worksheet.

2.2. The intervention.

In the first worksheet, as an introduction to the concept of similar triangles, we prepared activities based on the Thales' conclusion that "multiple gnomons have equally multiple shadows". Students worked in a GeoGebra file (Fig.1) and found the equal ratio of the homologous sides of two right triangles. After that, they compared the angles of the two triangles, and they concluded that they are equal. In that way, they came up with the definition of similar triangles.

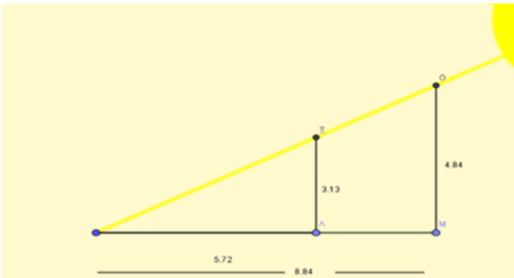

Figure 1. An image from student's work on GeoGebra

After that, the students had to write down the differences between similar and equal triangles. They also had to draw a similar and an equal triangle to one triangle designed in their worksheet, using geometrical tools.

We noticed that the students pointed out the differences and easily constructed the shapes. They seemed to have understood the stability of the ratio of the homologous sides of similar triangles. However, they met difficulties in describing the process they followed to design the two shapes. Only four students made an appropriate description. Some students could not find the appropriate words for their description, while others only mentioned the definitions of equal and similar triangles and then sketched the triangles.

In the second worksheet students had to deal with two tasks. In the first task students had to draw, without using geometrical tools, two similar triangles in the plane and two similar triangles in space. Students draw similar triangles in the plane successfully, but it was difficult enough for them to imagine and make such shapes in space. Unfortunately, students of that age are not familiar with the shapes in space or with the concept of perspective in general.

In the second task students worked on a 3-D GeoGebra file (Fig.2). At first, they had to find that the two triangles are similar, using the definition of similar triangles. Then they could move M and N point (Fig.2) and they noticed that the corresponding angles are still equal. In that way they concluded that two triangles with equal corresponding angles are similar.

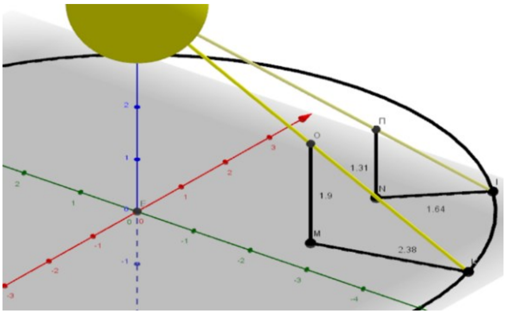

Figure 2. Another image from student's work on GeoGebra

In the third worksheet, students studied an excerpt of an original historical source from Plutarch's work "Morals". They tried to understand the way that Thales measured the height of the Great Pyramid of Giza, thus understanding the need for finding a way of measuring inaccessible heights. After that, they were given the appropriate distances and they had to find the height of the pyramid and describe the procedure (Fig.3).

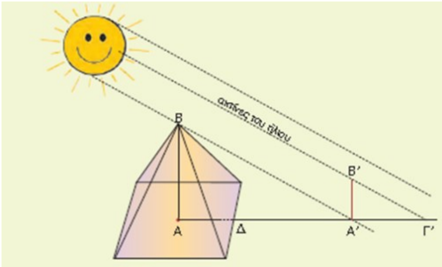

Figure 3.

The discussion between the students about the method followed by Thales in measuring the height of the pyramid, stimulated their interest and most of them managed to calculate it. They used the criterion of similarity to prove that the two right triangles were similar and then they found the height using the equal ratio of the homologous sides. However, only 11 students managed to describe verbally that procedure.

Two examples of the descriptions of the students are given below:

"He used the similar triangles which have proportional sides and equal angles. Thales found the ratio of the shadows and thus managed to find the ratio of the rods, i.e. the height of the pyramid."

"Thales calculated the pyramid's height using the triangles ABA΄ and A΄B΄Γ΄. Both triangles are formed by the rays of the sun. As we already know, the rays are not far apart, so the angles B and B΄ are equal. The angles A and A΄ are right angles therefore according to the similarity criterion the two triangles are similar." For the fourth worksheet, we used an original historical source translated in Greek, some excerpts from the book, *La géométrie et pratique générale d'icelle*, by the 16th century French engineer Jean Errard. In this book, Errard describes the design and construction of an instrument for measuring inaccessible lengths, based on the theory of similar triangles (Fig.4). The aim was for students to learn about another way of measuring inaccessible distances and to think about aspects of everyday life that we can use this instrument. The students liked the study of this original source but found it quite difficult to understand the contents of the passage.

Nevertheless, all the students after processing the text information about the Errard's instrument managed to make a drawing of it on their worksheet.

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OR ie desirerois que la composition fust telle: Que deux reigles de leton bien droictes fussent conioinctes en forme de compas comme AB, AC, & de lõgueur chacune d'vn pied & demy ou enuiron, & de largeur d'vn poulce, se mouuans & tournans au centre A, apres que sur la reigle AB, (que nous appellerons these) soit vne graueure pour couler vn demy anneau marqué icy D, lequel enclorra vne autre regle EF (appellée base) laquelle sera de semblable longueur, mais de largeur egale aux autres, ou vn peu plus estroicte, & qui se coulera auec le demy anneau au long de AB, & en sorte que sur son centre E, (qui sera iustement sur la ligne droicte AB,) elle pourra s'incliner & faire tel angle qu'on voudra auec la reigle AB, au long & ioignant les superficies des reigles de la these & de la regle mobile AC, & laquelle neantmoins pourra estre arrestée par le moyen de quelque vis appliquée audit demy anneau.

Cela ainsi composé, & les pinnulles estant mises aux poincts ABC (comme on a accoustumé faire à tous instrumens) Que chacune desdites trois reigles soit diuisée en 300 parties egales, ou en autre nombre tel qu'il plaira. Finalement soit attaché à la reigle de la base vn pied pour soustenir, & sur lequel se puisse mouuoir tout l'instrument de costé & d'autre en telle inclination qu'on voudra, attaché, di-je, à la base, en tel endroit qu'icelle estant en pesanteur egale aux deux autres, puisse seruir de contrepois, afin que le mouuement de l'instrument en soit plus doux & aisé. Ie laisse à discretion la fabricature du pied auec les ioinctures qui y doiuent estre: Seulement i'ay icy depeint audit pied vn globe enchassé dans vne concauité, parce qu'il

Figure 4.

Based on the previous worksheet, as well as on the instructions of the teacher (fifth and sixth worksheets), the students were divided in groups and each group made its own instrument which they used to measure an inaccessible height in the school building (Fig.5). To construct the instrument, they used: three numbered pieces of modelling paper 15 cm long each, a bolt, a plastic sight, a clamp and a pin. Students could move two of three pieces and they had to keep the other one horizontal by using a level thread.

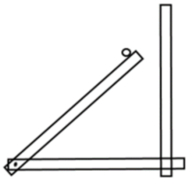

Figure 5.

In the seventh worksheet students dealt with three historical mathematical problems which were all solved by using the similar triangles concept.

The first problem was solved by Thales himself and involved measuring of the distance of a ship from the harbour (Fig.6). In this problem, about half of the students described verbally the process followed by Thales, but found it difficult to prove that the triangles are similar. They could not find the pairs of equal angles, because no one mentioned in his description the parallelism of the two sides of the triangles.

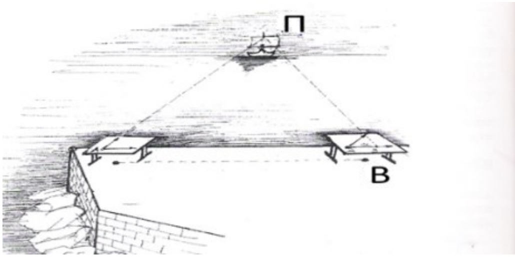

Figure 6.

The second problem was the measuring of the depth of a well, which is included in the Chinese book, *Jiuzhang suanshu*, written between 100 BCE and 100 CE. In this problem, the students very easily recognized the similar triangles and using the appropriate equality of ratios calculated the required depth (Fig.7).

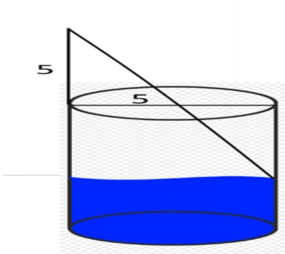

Figure 7.

The third problem was about the “Efpalinos tunnel”, a tunnel constructed in the Greek island of Samos in order to irrigate the city of Samos from a spring high in the mountains (Fig.8). First, the students watched a relevant video and then, based on this, they had to describe how Efpalinos measured the length of the tunnel. They had a lot of difficulty, so it was necessary to watch the video twice and have a discussion with the teacher so that they could be able to describe the process Efpalinos followed using the similarity of triangles.

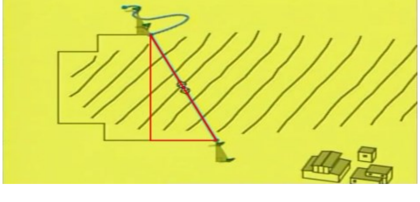

Figure 8.

In the cognitive test, there were four problems. The first two problems (Fig.9) were about two different ways of calculating an inaccessible height. The students had to calculate the height of these two buildings and to describe how they did it. Almost everyone calculated the height using the equality of the ratio of the homologous sides, in both problems. However, in the first shape was difficult for them to find the pairs of the equal angles, in order to prove the similarity of the triangles. Only six students finally proved it. Furthermore, seven (other?) students described verbally the process that someone could follow to calculate the height of the two buildings.

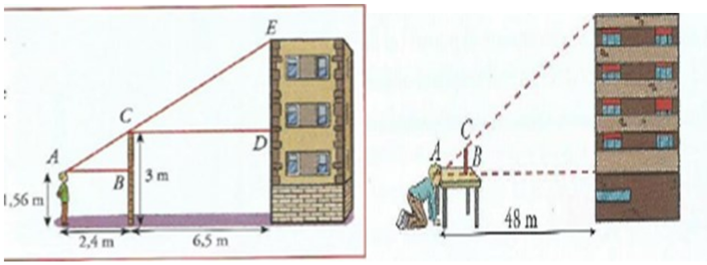

Figure 9.

In the third problem, students were asked to calculate the length of the diving board (Fig.10). 19 students successfully calculated this length. 12 of them used the similarity of the right triangles, that one is contained into the other. They found it easier to find the equal angles.

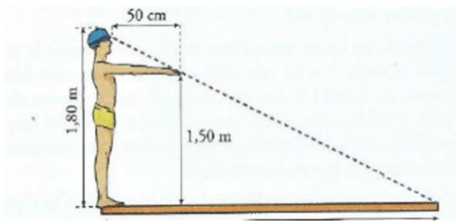

Figure 10.

In the fourth problem, students had to calculate the unknown lengths x and y of the sides of the second triangle (Fig.11). 19 students proved the similarity of the triangles and found x and y, using the equal ratios of the homologous sides.

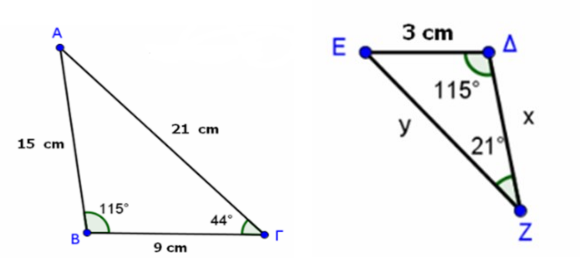

Fig.11

3. Conclusions

The purpose of our intervention was to investigate the extent to which the implementation of the History of Mathematics in the teaching process will help the students to understand better a specific subject. The subject we chose was the similarity of triangles. It was obvious that the enrichment of the lesson with the historical references stimulated the interest and the curiosity of the students and helped students to cooperate with each other and also to create a positive climate about dealing with the subject of similar triangles. The fact that we used a hands-on approach, in particular the students' construction of Errard's tool, contributed to a better understanding of the concept of the ratio of the homologous sides of the triangles, as they had to do the measurement and design process themselves. With this construction and also by solving historical problems, the students understood why the concept of similarity of triangles is important and where it is used, which contributes positively to its understanding. Another thing that we noted during the intervention was that the students struggled in the questions where they needed to develop a verbal proof. They also faced difficulties when they had to transfer the real situation to paper, during the measurement of an inaccessible distance. This research took place during pandemic. The schools in Greece were closed for almost three months. This fact affected the planning of the teaching intervention and limited our available time as it had to be carried out when the schools were open. It would be interesting if this research happened again under normal conditions.

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