# A PIONEER EDUCATIONAL AID FOR THE LEARNING OF THE FIRST NOTIONS OF GEOMETRY: JULES DALSÈME'S *MATÉRIEL-ATLAS* (1882)

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## ABSTRACT

This workshop is inspired by the work of the French normal school teacher Jules Dalséme (1845-1904) and is addressed to primary and lower secondary school teachers (future and in-service ones) and it also stems from Ilaria Zannoni's master's degree thesis and internship work, (Zannoni 2018), addressed to 4th to 7th grade pupils, entitled "Geometry as the beginning of scientific thinking: observing, moving, comparing, drawing, representing". Dalséme's *Éléments de Takymétrie (géométrie naturelle): à l'usage des instituteurs primaires, des écoles professionnelles, des agents des travaux public, etc* (1880) presents an original way to teaching geometry to children, going beyond the simple numeracy and proximate to synthetic Euclidean geometry. His goal was to encourage and make science more accessible for everybody, in the spirit of the intentions of the French Third Republic state education. Participants are engaged in hands-on activities taken from the educational aid *Matériel-Atlas* (1882), to calculate the volume of an object commonly found in construction sites.

### 1 *Who was Jules Dalséme? An intermediate figure in the XIX century French national mathematical community*

It is appropriate to start the workshop with some information on the figure of Dalséme, in order to understand his proposals to facilitate the general access of mathematics to pupils and to citizens (primary and vocational schools). Jules Dalsème (1845-1904)[44], a French engineer trained at École Polytechnique, devoted most of his life to children's education in mathematics and to the training of future teachers. In 1869 he left his just started military career to devote himself to that of a teacher, starting as a *répétiteur de mathématique* at

[44] A more accurate insight on Dalséme and his cultural profile will appear in the article (Magrone, Millán Gasca, & Zannoni, 2023)

Collège Chaptal in Paris; in 1872 he became a mathematics professor in *École normale d'instituteurs de la Seine* and in 1874 he published his first work *Premières notions de géométrie*, (Hèment & Dalsème 1874), in collaboration with Felix Hément, his former teacher. This first work was adopted as a text in the schools of Paris and marked the beginning of Dalsème's activity as a writer of textbooks for primary, secondary, normal and vocational schools. Dalséme lived in a period of great changes that affected public education: the minister Jules Ferry's (1832-1893) then new laws were the foundation of the conception of the state school system, no longer conditioned by religion and were the basis for a more democratic and progressive society.

Dalséme's *Éléments de Takymétrie* (Dalséme 1880) is inspired by the geometry of the arts and crafts in order to propose an introductory, intuitive geometry, possibly propaedeutic to demonstrative Euclidean geometry; his work is part of a broader current of thought, which saw in science education a way to help society to improve, and individuals to become "reasoning citizens" (Millán Gasca 2015; Magrone, Millán Gasca, & Zannoni 2022). Several editorial proposals in this framework chose a *fusionist* approach (Barbin & Menghini 2014, Karp & Schubring 2014, Menghini 2015), putting together the 2d and 3d geometrical shapes which follow similar rules for the calculation of area, perimeter, and volume. He was directly inspired by the *pont et chaussée* engineer Édouard Lagout (1820-1885), who in 1857 invented the takymetric[45] method which aimed to quickly train uneducated personnel at the time when he was chief engineer in the building of the Adriatic railway in Italy (Fig 1 left). Dalsème intended to bring the takymétric method (ready, immediate, exact) as  teaching for primary school teachers, maintaining the main mechanism features: the terminology of the construction yard, which relates to the ancient tradition of the mathematics of arts and crafts, and, in this specific case, to the geometry involved in the construction works; the use of "educational aids" (addressing both intuition and understanding) and the combination of those flat and solid shapes whose measure is calculated according to similar rules (fusionist approach).

Dalsème's proposal is that of an everyday geometry, which is intuitive, and make it accessible to primary education as well as to vocational schools.

[45] From the Greek *tachys*, meaning fast, quick, and *métron*, measure: "Fast measure". See also (Leme da Silvia and Moyon) for the diffusioni of Lagout's takymétrie.

*… la méthode intuitive […] se propose d'agir sur les sens pour pénétrer jusqu'à l'esprit et s'adresse aux facultés intellectuelles par l'entremise d'objets matériels, d'images visibles, de faits, méthode dont l'application et les avantages peuvent s'étendre bien au-delà du cadre modeste de l'enseignement primaire et qui peut se résumer d'un mot la mise en éveil incessante des facultés d'observation. […] La répétition des choses qui se comptent fit naître les premiers linéaments du calcul. Les idées de forme et de grandeur, issues de tant d'objets environnants, engendrèrent les premières conceptions géométriques. (Dalsème 1889, 5).*

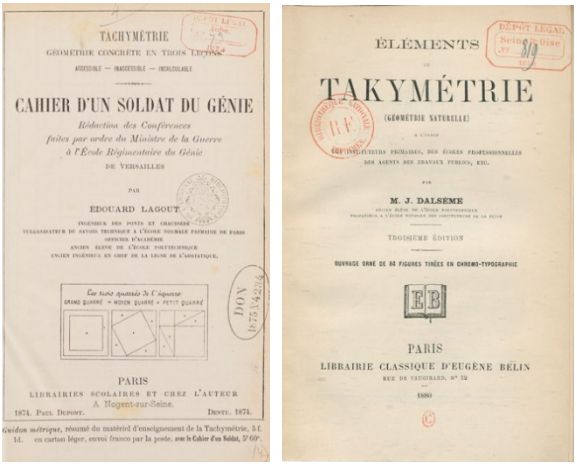


**Figure 1.** From left to right the front covers of (Lagout 1874) and (Dalséme 1880)

## 2 A hands-on workshop from a XIX century cut-out atlas

The workshop is based on the educational aid contained in the album *Matériel-Atlas de takymétrie* (Fig. 2) which was probably inspired by the richness of the educational materials associated with the takymetry books devised by Lagout.

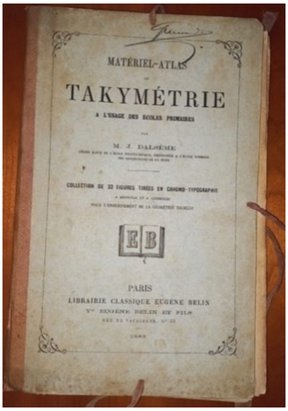


**Figure 2.** Dalseme's Atlas front cover, source : "*Matériel-Atlas de takymétrie à l'usage des écoles primarires*. Collection de 32 figures tirées en Chromo-typographie à decouvrir et à assembler pour l'enseignemente de la géométrie usuelle". Paris : Librairie classique Belin. Educational aid, designer Jules Dalsème, producer Librairie classique Belin, 1882.

The collection of bi-colored cardboard cards (12 cards, not bound, 14x25cm.), which had to be cut out, folded, and assembled, is a forerunner of the albums we find today in stationery shops. The workshop was designed for 20 participants, to be held in a classroom with tables and chairs; participants had scissors, tape and we suggested that they work in pairs. This workshop was set up according to the experiential *Anfomam* workshops, for the training of pre-school and primary school teachers, see (Lizasoain Iriso, Magrone, Millan Gasca, et al., 2022). In particular this workshop belongs to the category TMb: "*Playing with* primary school math activities. The participants place themselves simultaneously in two worlds, as if they were children, while at the same time they have the perspective of a teacher" (TM from the Spanish: *taller de matemáticas*). We use a comparison of teaching methodologies, educational aids, and show how they evolved throughout history in order to achieve as a main purpose the diffusion of mathematical culture to the *multitude*[46]. It combines historical elements, manual activity, and comments on what was previously done in the classroom. We showed and discussed some images of Ilaria Zannoni's experience with primary school children, taken

[46] The word *multitude* is intended as in (Price 1994)

during her internship, addressed to 4th up to 7th grade pupils; the activities were inspired by (Dalséme, 1880). In Fig. 3 a picture of the decomposition of a cube into three pyramids is shown, which she did together with her pupils.

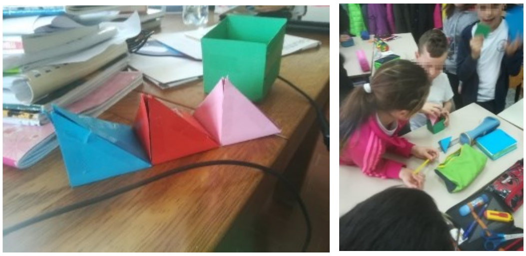

**Figure 3.** Activities in the classroom. Pictures by Ilaria Zannoni

Participants were provided with copies of the Atlas pages relating to the "tas de cailloux" (pile of pebbles), a geometric figure which, according to both Dalséme and Lagout (both mention this object) encompasses an entire geometry course (Fig. 4).

Dalséme introduces the truncated figures in the sixth lesson of (Dalséme 1880); namely, trapezoids and truncated pyramids. In a fully fusionist spirit, he associates, 2d and 3d figures whose measurement rules are similar. The geometric object that encloses the essence of all truncated shapes is the "tas de cailloux":

*Parmi ces volumes, le principal, ou du moins celui que l'on rencontre le plus souvent, nous est offert par les tas de cailloux s'élevant de distance en distance sur les routes et servant à leur entretien [...] C'est la forme que l'on retrouve dans l'auge du maçon, le tombereau du terrassier, le pétrin du boulanger, etc.  C'est aussi la forme des gros poids en fonte.* (Dalséme 1880, p.38).

Lagout shares the same enthusiasm for the didactic power of this object: "Le tas de cailloux résume à lui seul toute la géométrie des figures terminées par des lignes droites et des plans, de sorte que sa règle contient l'ensemble des formules déjà trouvées directement." (Lagout 1874, p. 24).


**Figure 4.** Dalséme argues that the tas de Cailloux is found very often, along roadsides and in many other objects that belong to everyday life. From left to right, top to bottom: “tas de Cailloux”, “l’auge di maçon”, “le tomberau du terrassier”, “le pétrin du boulanger”, a big “poid en fonte”

Figures 5-6 must be enlarged and printed in color[47], and distributed to every couple of participants, in particular: figure 5 left represents the entire shape and one of the corner pyramids, whereas 5 right displays the remaining three corner pyramids; figure 6 left and center shows the 2 small and the 2 big sloped sides, which Dalséme calls “talus de longeur” and “talus de largeur”; finally, the central cuboid is in figure 6 right.

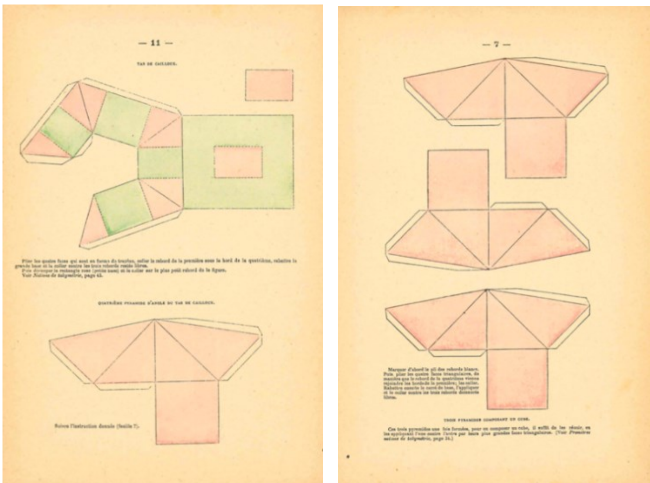


**Figure 5**

[47] The ideal is a 100-120 gr paper (the usual photocopy paper is 80 gr. and it’s too thin)

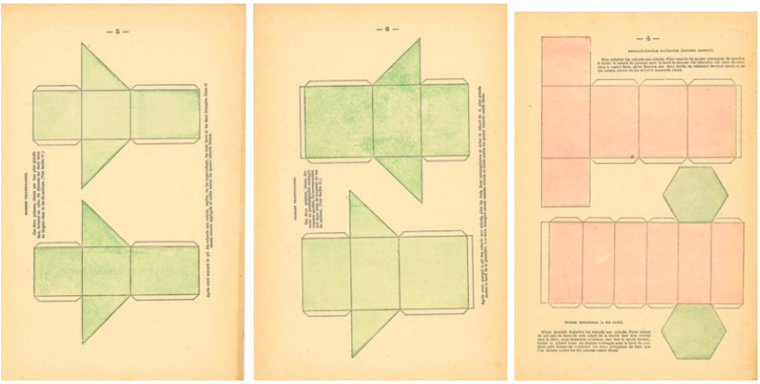

**Figure 6**

First step: cut and assemble the tas de cailloux, as a single object, taken from figure 5 (upper picture); this is necessary in order to have the entire shape in one's hands, as a reference model for the second phase (Fig. 7, left). Second step: cut out from figures 5-6 all the nine parts which will form the entire shape; fold them and assemble them by fixing the single pieces with clear tape (Fig. 7 right).

**Figure 7.** The tas de cailloux as a single object; the solid assembled with the nine parts

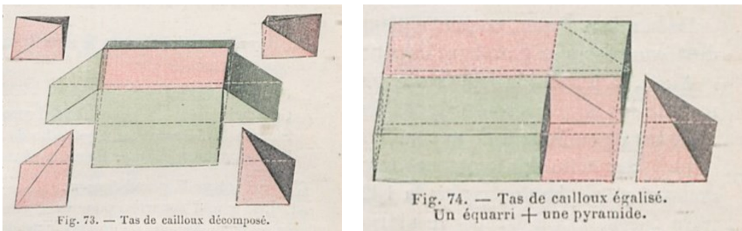


**Figure 8.** The re-assembling of the tas de cailloux, recomposing it in a different way, in order to calculate the volume: on the right it is easy to visualize that the volume

corresponds to that of an equivalent cuboid, plus one of the corner pyramids (Dalséme 1880, 38).

We are following now Dalséme's takymetric reasoning, which he himself describes in pages 38-40 of (Dalséme 1880): in order to measure the volume of a figure, we equalize it to another shape, for which the rule for the volume is known. This process is described with a lot of details, specifying the physical movement to be done: "Je transporte maintenant à droite le talus solide de gauche, mis sens dessus-dessous. Les deux plans de talus, égaux, s'appliquent l'un sur l'autre (comme deux équerres pour former un rectangle). Le tas se trouve ainsi équarri dans le sens de sa longueur" (Dalséme 1880, p 39 and fig. 8). At the end of this procedure, we will obtain two figures, whose total volume is equivalent to that of the *tas*: a cuboid and a pyramid.

Dalséme uses a kind of "spoken algebra": he describes the formulas by words ("multiply", "add", "height" instead of using letters and symbols) and only at the very end of the lesson he writes down a formula with only symbols.

The *tas* is a trapezoid in both length and width, in other words its lateral walls are slanted, and only its ceiling and floor are parallel to each other. So Dalséme proposes to compute its volume by re-assembling the nine pieces in a more regular shape, an "equarri" (a cuboid, Fig 8, right), plus a pyramid. The issue is to compute the right dimensions of the resulting cuboid. We call the two horizontal dimensions of the *tas* "length" and "width" and for each of them, there will be a small one and a big one, since the solid is trapezoidal. By reading carefully pages 38-40 of (Dalséme 1880), or simply observing the geometrical shape, one can deduce that the resulting cuboid has these dimensions:

- The height is the same as the *tas*
- The length is: ½ (big length+ small length)
- The width: ½ (Big width + small width)

The dimensions of the corner pyramid (whose volume must be added) are:

- The height is the same as the *tas*
- The length is: ½ (big length - small length)
- The width is: ½ (big width - small width)

Dalséme's words for the volume of the *tas*: "La 1 /2 somme des longueurs multipliée par la 1/2 somme des largeurs et par la hauteur, plus la 1/2 différence des longueurs, multipliée par la 1 /2 différence des largeurs et par le tiers de la hauteur” (Dalséme 1880, p. 40); then, in formulas:

$$\text{Formule : } \left(\frac{L+L'}{2}\right)\times\left(\frac{l+l'}{2}\right)\times h+\left(\frac{L-L'}{2}\right)\times\left(\frac{l-l'}{2}\right)\times\frac{h}{3}$$

**Figure 9.** The formula for the volume of the tas de cailloux. L= largeurs, l= longeurs; the capital letters mean to distinguish between the minor or major base of the trapezoid; h= height

### *3 Final remarks*

This workshop was attended by about 15 participants, among them university colleagues of different nationalities who deal with the history and didactics of mathematics, and some schoolteachers. Although the subject is elementary geometry, such as the calculation of the volume of a solid with flat faces, the participants showed interest and enthusiasm in carrying out the cutting and assembling activities; hands-on activities are themselves challenging, since they break the usual pattern of the frontal lecture, fostering group work and discussions among participants; the fascination of working with paper and scissors using an educational aid designed more than 100 years ago should not be underestimated. The fusionist approach is an educational aspect that should be retrieved: the geometry books dedicated to schools of every level are nowadays filled with repetitive formulas, which are reproposed, as if they were new each time and therefore to be learned, while instead they deal with already discussed cases. The approach of Lagout and Dalséme of grouping geometric figures that respond to the same calculation rules, such as rectangles and parallelepipeds, cylinders and prisms, trapezoids and truncated pyramids, etc., could be taken as examples to eliminate excess formulas.

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