POTENTIAL OF COLLABORATION BETWEEN HISTORY AND MATHEMATICS TEACHERS: EMPERICAL INVESTIGATION

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ABSTRACT

At the ESU-8 conference (Affan & Fried, 2019), we emphasized the rationale of our research concerning the potential and importance of collaboration between history teachers and mathematics teachers in the project of incorporating history of mathematics (HoM) in classroom. In this talk, we present the exact structure of the workshop and a partial empirical result from analyzing the data collected from a questionnaire distributed to 557mathematics and history teachers. Initial categories constructed according to clinical semi-structured interviews, observations, video records and log analysis with 10 teachers who participated in a course based on history of Islamic mathematics are also set out. The goal of the analysis is to identify characteristics of the collaboration between the two populations and see how these characteristics are reflected in the construction and implementation of a joint learning unit in HoM.

1. Introduction

The basic assumption (not always acknowledged) guiding most studies on using HoM in mathematics teaching is that it is indeed feasible to integrate HoM into mathematics teaching and learning so that the main consideration is how to do it (Jankvist, 2007, 2009, Gulikers & Blom, 2001), however, it is indeed feasible to integrate HoM into mathematics teaching and learning. But this cannot be taken for granted. Fried (2001), for example, claims that along with difficulties such as lack of time and material supply constraints in the dense curriculum, the integration of the HoM contains within it a theoretical difficulty expressed in the tension between anachronism and relevance a dilemma in which the teacher is forced to choose between adopting an approach which sees the HoM as merely a "tool" (Jankvist, 2009) to be used in mathematics classes to teach mathematics relevant to the modern world and typical mathematical curricula and a historically sensitive approach that focuses on historical context, original sources, etc.

Since this dilemma is founded on the different demands made by the disciplines of mathematics and history and subsequently in the teaching of these disciplines, we need to examine the relationship between mathematics teachers and history teachers, the assumption being that teachers embrace the norms of their respective disciplines. In other words, this research sees the problem of incorporating HoM into mathematics education as a problem of interdisciplinary education or even transdisciplinary education (Clark et al. 2018; Thomaidis & Tzanakis, 2022; Barbin, 2022). Such studies take the nature of "collaboration" as something that is not self-evident.

Keeping in mind the framework of interdisciplinarity/transdisciplinarity, our study aims to bring history teachers and mathematics teachers together in order to see, first of all, what kind of considerations and presuppositions they have when they come to HoM, and second, whether they are able to work together to produce a chapter on the HoM for mathematics classes and perhaps for history classes. Our hypothesis is that integrating history of mathematics into mathematics education will benefit from the interaction between these communities of teachers, mathematics and history teachers. This has been done before to some extent (Moyon, 2013), but we have taken it up with the awareness that the attempt may involve a problem similar to that it is meant to solve, namely, a tension between the commitments and basic assumptions of history and mathematics teachers (Fried, 2001). That said, the awareness itself together with the particular choice of historical material may allow for the creation of genuine cooperative community, so that the solution to Fried's dilemma may be to think of HoM as fundamentally a trans-disciplinary subject that requires enlarging the community that deals with it.

Our overall aim is to investigate characteristics of the collaboration between mathematics and history teachers and how they are reflected in the construction and implementation of a joint learning unit in history of mathematics.

2. Research questions

Our overall research project is guided by three research questions: The first refers to the significant differences on considerations, presuppositions, and beliefs between teachers for mathematics and history regarding the integration of HoM into the classroom. The second refers to the impact of a course in the HoM on considerations and beliefs of the two teacher populations regarding

mathematics, history, and the HoM and what characterizes the learning experience of the two populations during their joint participation. The third refers to feasibility of an interdisciplinary collaboration between mathematics and history teachers, what characterizes this collaboration in transferring of a joint learning unit in HoM. In this paper, we focus mostly on the second and third research question.

3. Participants and Research Tools

In the present study, we focus on a group of mathematics and history teachers from Arab schools in Israel. In the first stage, a questionnaire distributed to 419 mathematics teachers and 138 history teachers to find out what are the considerations, presuppositions and beliefs of teachers for mathematics and history regarding the integration of HoM into the classroom. For the analysis of quantitative data, a descriptive analysis, paired samples t-test, correlation and regression analysis were used.

In the second stage, a group of 10 mathematics and history teachers (5 mathematics teachers and 5 history teachers) participated in a course-based on history of Islamic mathematics. The goal of the course, which included a joint activity, was to find out the teachers` views of collaboration, what are the characteristics of the collaboration between the two communities and how they are reflected in the construction and implementation of a joint learning unit in HoM.

The text used was "*On the Geometric Constructions Necessary for the Artisan*" by the Muslim mathematician Abu'l-Wafa Buzj'ani (940-998). This particular work was chosen because it contained genuine mathematical content, but the mathematics also had a clear social and cultural context. The treatise concerns the work of ordinary people, the artisans, and, therefore, reflects a distinctly Islamic emphasis on activities benefitting society. On the mathematical side, many of the examples in the book *were original*. The range of problems is very wide, from the simplest planar constructions (the division of a segment into equal parts) to polyhedrons inscribed in a given sphere (Affan et. al., 2019). Thus, the text should appeal to the needs of both groups of teachers. The course consist of five workshops of about 1.5-2 hours each session. In the first meeting, the researchers explained the rationale of the course and listened to the teachers' expectations, considerations and beliefs connect-

ed to it. In the second meeting, we presented Abu'l-Wafa's book including activities and original texts from the book; later, the participants were divided to pairs (each pair consisting of a history teacher and a mathematics teacher) focusing on an activity from the book. In the third meeting, we gave them three original historical texts from the Abbasid era (10th century) about the connection between the translation movement and the development of mathematics and sciences. In the fourth meeting, the teachers participated in further activities with original texts from Abu al-Wafa's book such as constructing polygons inscribed in a circle. In the last meeting, each pair was presented with examples of ornaments from Islamic art and were asked to construct the ornament on the basis of what they learned in the previous meetings and present the activity in the class.

 In the final stage of the research project, the teachers were divided into groups of two (each pair included a math teacher and a history teacher) and were asked to construct and implement a study unit on the history of Islamic mathematics inspired by the texts from Abu'l-Wafa 's book studied in the course given in the first stage. Three groups implemented their unit in the 10th grade and one is still in the construction process. The fifth group could not agree on the exact topic for the teaching unit and ended their collaboration.

   The data set of our analysis included questionnaires, observations and video recordings of the activities in the course and the joint study unit, and interview transcripts. In our analysis of the data, we were especially interested in teachers' views regarding collaboration, how collaboration developed during the in the course and in design of the study unit afterward.

4. *Initial Findings*

As mentioned previously, the questionnaire, which was designed principally to gain information regarding the first research question and to provide background for the course, contained 48 questions falling under four categories: considerations and attitudes concern mathematics; history and history thinking; history of mathematics; collaboration between history teachers and mathematics teacher. Since our main focus is the course itself, we will only summarize briefly the main conclusions from the questionnaire. Similar to previous surveys (Tzanakis & Arcavi, 2000), and others, mathematics teachers and history teachers were found to have positive attitudes about the integration of HoM. However, unlike mathematics teachers, history teachers think that inte-

grating HoM in classroom can contribute more to the understanding of the culture/ social/ human needs and less to mathematical thinking or to the understanding of mathematics concepts, which is the opposite to Mathematics teachers' views. Namely, the combination of the two subjects develops an awareness to human culture, and exposing the students to the contributions of mathematics to culture and humanity develops a historical thinking and critical arguments rather than a direct contribution to mathematical thinking or to mathematics itself. To answer the second and third research questions we analyzed the data using qualitative methods to determine categories describing the tensions, emotions and gestures among the teachers. The exact categories are still being revised. That said, from our observations and the videotapes recordings, we were able to discern four initial categories characterizing the degrees of cooperation or tension between the history and mathematics teachers.

4.1 Discomfort and Ease

At the beginning of the session, also during the first meetings of the course, most of the history teachers felt uncomfortable. Several facial expressions and gestures (such as combining hands, hands on the cheek or forehead, tense facial expressions…) indicated lack of ease or self-confidence with respect to the meeting. One of the teachers whispered to one of the researchers:" How did you convince me to come, I am a history teacher, what I suppose to do with mathematics". Another teacher wrote in a reflection after class, "As a student I was weak in mathematics and didn't like it, the thought that, after many years, I am going back to math again, gives me goosebumps….."
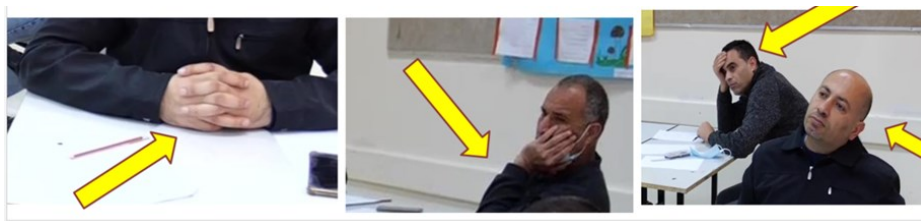

Figure 1: Discomfort and Ease expressed by facial expressions and body movements

4.2 Arrogance and Respect

In the activities and dialogues, we noticed a tendency for the mathematics teachers to be overbearing during the math activities and the history teachers during history lessons. As mentioned earlier, there was tensions between the

teachers of the two subjects and we noticed tension within some of the pairs (the mathematics teacher during the mathematical activity and the history teacher during the historical activity) who tried impress the other side, to show mastery on the material and sometimes trying to imprint there opinion. In contrast, in other pairs, we noticed a mutual respect among the participants and tried to help each other without hurting the other's feelings.

4.3 Compassion and Embarrassment

Some teachers expressed understanding for the other side and tried not to make them feel embarrassed. In one case, the participants were asked to draw a circle with a given diameter, a history teacher asked, "*what is a diameter ?*". Some of the history teachers laughed, and a part of mathematics teachers smiled, and he teacher looked very embarrassed (figure 2). On the other side, his "partner" (a mathematics teacher) explained to him patiently what a diameter is and how to draw a circle with a given diameter.

Figure 2. History teacher look embarrassed after he asked what a diameter is.

4.4 Dominance and Passivity

In the mathematical activities, each pair was asked to read a [geometric] text from Abu al-Wafa's book and apply the activity in the class. The mathematics teachers were more dominant when performing the activity while the history teachers were passive, watched them and followed their instructions. On the other hand, in the history activity of analyzing texts in history we noticed that history teachers took over the discussion most of the time, while mathematics teachers barely participated in the discussion.

Figure 3. Mathematics teachers more dominant in HoM activities

The aim of the joint work was to examine what characterizes the collaboration between the two communities during constructing and transferring a learning unit based on Abu al-Wafa's book. From our observations and the videotapes recordings, we were able to discern four initial categories characterizing the degrees of collaboration between the history and mathematics teachers. These types can be divided into four categories:

4.5 Complementing Pairs
In this type of collaboration, both teachers complemented each other. The history teacher explained to the mathematics teacher the interpretation of the historical text and discussed the significance of the historical events in it, in the other side, the mathematics teacher guided the history teacher in carrying out the mathematical activity.

4.6 Domineering Teacher
Mathematics teachers are more domineering in HoM activities (especially in mathematical activities), they were the first to hold and use the ruler and compass, draw the figures and explain to the history teacher what to do. On the other side, they swap roles in the historical activities.

4.7 Together But Each Separately
Both teacher sitting around the table, but everyone works alone and only at the end of the activity, they discuss what they do separately and compare the results. In the case of this pair, they did not succeed and failed in performing the activity.

4.8 Fully cooperating pair
Both teachers worked in harmony with full collaboration, motivating and encouraging one each other in order to produce a perfect product.
The following diagram (figure 4) shows the categories mentioned above about kinds of collaborating between mathematics and history teachers while participating a history of mathematics course.
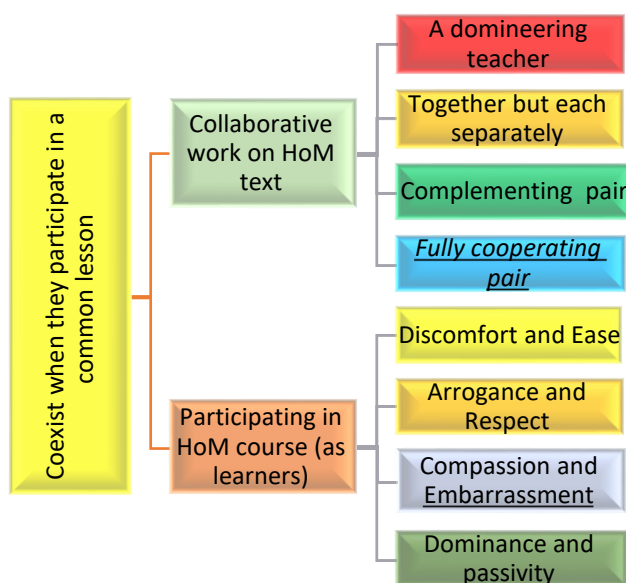
Figure 4. Kinds of collaborating Categories between mathematics and history teachers.

5. Conclusions

In this study, we observed that mathematics teachers and history teachers are two different communities according to their attitudes and considerations with respect to HoM in the classroom. Thus, a course in the history of Islamic mathematics to examine the feasibility of interdisciplinary collaboration between them was set up. Initial findings indicate that difficulties arose during the course and the joint unit study afterward. However, there was at least one fully collaborating pair. This shows that despite the difficutlies, it might, nevertheless, be feasible to enlarge the HoM community in order to promote collaboration between both communities. Further research is necessary to determine the exact mechanism for such successful collaboration.

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