THROUGH A HISTORICAL DOCUMENT THE AUTHOR CAN GUIDE THE STUDENTS

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ABSTRACT

In the workshop, I described to the participants an episode of a classroom experiment in which historical documents was used with upper secondary school students. I asked the participants to write their remarks about both the work made by two students and some theoretical reflections. In the present article, I report and comment on these remarks. The purpose of the experiment and of the activity proposed in the workshop was to examine the relation that the students established with the author of the document who, although passive, can guide the students towards cultural considerations about mathematics and its evolution.

1 Introduction

A text is, for students, phenomenality of the author. If it is an author lived in the past, the historical distance can help students to highlight elements that connote the author, such as discussing in an unusual way or referring to the everyday life of the past. I consider these elements as traces of the author, that enrich phenomenality and "reveal" him/her as a human being. This contributes to creating the Student–Author relation. The students do not search for these elements which represent a possibility for their "unpredictable" encounter with the author (Demattè, 2019).

Based on these considerations, my research question is: Through the document, can the Other/Author summon students, so that they pose questions about him/her? In the present article, I discuss this question and I try to answer, also through the outputs of the workshop.

2 The experiment in the classroom

The document considered in this article is a problem taken from Filippo Calandri's *Aritmetica*, an Italian treatise of the 15th century, originally used to teach mathematics in the Abbacus schools whose main purpose was to prepare for mercantile, commercial, and artistic activities; see (Demattè & Furinghetti, 2022).

The problem was proposed as part of a competition between classes of ninth grade (14–15-year-old students), inspired by *Rallye Mathématique Transalpin* (http://www.rmt-sr.ch/). Each class had to tackle a number of problems, difficult to be solved in the allotted 50 minutes. So, the class had to distribute the problems among its members. The problem taken from Filippo Calandri was chosen considering that the students were facing algebra (polynomials and linear equations) and it was aimed at verifying whether students had internalized the use of letters for formalizing a general procedure. The competition took place in February, that is, shortly after the middle of the school year.

In Fig. 1, the English translation of the problem with a premise (common to each problem of the competition) is reported. For the students, the archaic Italian language has been modified to confine the difficulties of interpreting the text to mathematics, avoiding linguistic complications, but without changing spirit and meaning of the original text.

In addition to the text of the problem, the authors also report the resolution which, however, does not include all the steps, or in any case does not always explain the reason for the operations that are carried out and the results obtained. Write explanations to complete their reasonings.
Filippo Calandri (ca. 1467 –?) belonged to a family of Florentine abacists (the accountants and mathematics teachers of that time, too); his brother Pier Maria, his father and a great-grandfather also practiced the same profession. He is the author of one of the first printed mathematics texts: the *Aritmetica* (1491).

From Calandri's *Aritmetica*, Problem **LXXX**: **How to find the natural number your friend thought up**. If he would think 10, tell him to double that, and that's 20, then tell him to add 5, and that's 25, then multiply by 5, and that's 125, tell him to add 10, and that's 135, multiply by 10, and that's 1350. From this, subtract 350 and that's 1000 which is 10 hundreds for each of which you take 1. So, you say that the friend thought 10. And the same way you could do if he thought another number: every hundred is worth 1.

Figure 1. The problem delivered to students (material for participants, too).

In class, Calandri's document was assigned to the students Matilda and Benedetta. In the workshop, we focused on their work. Here it is a very short excerpt, sufficient to understand how they repeated the author's reasoning, using the number 9 instead of 10. "Ask your friend to pick a number, ex. 9. Tell him to multiply x 2, ex. 9·2 = 18 […] Now each hundred is worth 1 so the result will be 9".

About 20 days later, I interviewed Matilda and Benedetta. I asked them questions regarding Calandri's document. One of the questions was: "What questions (to ask an expert/teacher) did this problem raise you?". This question asked to the students was aimed precisely at investigating whether they had wondered *why*, whatever number the friend thinks, at the end of the game a value one hundred times greater than that number is obtained. I supposed their lack in mathematical tools and their consequent difficulties to answer in written form that *why*. They said: "Did I understand what the author is doing? Does the game change if I use other numbers and rules? Why did Filippo want to do that research on the number that his friend thought?"

The first one of these three questions can be interpreted as their way to get information about correctness of what they wrote as an explanation. The second can be an indication of their (partial) awareness of possibility to analyse the game in terms of variables and operations, that is, in the terms of the algebra they were facing at that time in class. I will discuss their third - unpredictable - question in the following pages.

3 *The workshop*

The audience consisted in teachers (in service or retired) from various school grades, researchers in education or history, and doctoral students.

After a 15–minute oral presentation in which the experiment in the classroom was described, I proposed to the participants to reflect individually, to discuss in small groups, and finally to write personal remarks about the materials reported in Fig. 1 and Fig. 2. The following premise introduced all the material delivered to them: "What would you say to Matilda and Benedetta? What is your opinion about the Theoretical reflections [see Fig. 2]? Suggestions and critical remarks are welcome!".

Theoretical reflections

In reading a mathematical text, students are called:

- to accept the *author*'s proposal, in its alterity (particularly evident in a historical document),
- to share the mathematical *content* (i.e., to get an agreement with the author through interpretation – hermeneutical process),
- to overflow *themselves* (through acceptance of a teaching and propensity to change their own foreknowledge).

Obedience to the request of the teacher who proposed reading the document is not enough to explain why that can happen. There must be a relation with the text and with the author, that excludes the teacher who has only created the conditions for this relation to take place. We may say that a main goal of mathematics education is to find ways so that students pass from their institutional tasks to their work with the Other; see (Demattè, 2021).

Even considering any mathematical document (not just historical) I think that not because it is manifestation of culture the students can be involved in analysing it, but because it belongs to the Other (I use the capital O considering that students have established a relation with that Other). Culture is a sort of surface.

According to Levinas (1991, p.191, endnote 10): "It is as possessed by a neighbor, as relics, and not as clothed with cultural attributes, that things first obsess. Beyond the "mineral" surface of things, contact is an obsession by the trace of a skin, the trace of an invisible face, which the things bear and which only reproduction fixes as an idol".

In conclusion, despite his passivity, the author can guide the students: in adherence to the mathematical reasoning he expounds, and in the questions he can raise about him and his work. Reminding Matilda and Benedetta, the fact that the author is a mathematician of the past gives them the indication that his work is inside the flux of the history of mathematics, so they consider it reasonable to pose questions about his "research" (of something new).

Figure 2. Theoretical reflections: material for participants.

4 Participants' remarks and questions

Each of the participants chose to focus on some aspects of the material delivered during the workshop, that is: the document, its educational role, the students' explanation, their interview, the theoretical reflections. No one has treated them all. Here, I use different letters to identify each individual participant or group who have written their remarks or communicated them orally.

Participants of group A discussed the choices made by the teacher. They considered the student–teacher–author triangle. They then asked a series of questions: With what criteria did the teacher choose the document? At what stage of class work did you introduce it to the students? For what purposes? Perhaps to introduce or to reinforce the use of algebra to generalize?

Participant B chose to work individually and proposed an observation regarding the student's difficulties in front of the document, establishing in some sense a continuity with the discourse introduced by group A: "There is a problem to translate the question in a formal language (algebraic) through the input of a variable. This is not trivial at that age, considering that the historical text doesn't use it".

Participant C worked in a group but produced his own contribution in which he formalizes Calandri's procedure with an equation and mentions historical and educational aspects: "$((2x+5)\cdot 5+5+10)\cdot 10=1350$. Find x. Linear equation. $1350-350=1000$ $1000:100=\boxed{10}$. Shifting from "rhetoric" to symbolic algebra? What do students need to know? The use of x (the "thing")?".

With D, I indicate two people who worked together. In his writing, one of them formalizes the problem with an identity: $(((((x\cdot 2)+5)\cdot 5)+10)\cdot 10)-350=100\cdot x$. Then he focuses on the three questions posed by Matilda and Benedetta. He points out that the first one concerns "the process", the second the "logic behind the game", the third "the why" and "it poses some interesting insights on the motivational problem behind mathematics, and the things students usually do in schools". "Regarding the bond with the author", he considers "more interesting" to "read the explanations that the author gives himself" instead "just solving an ancient problem". This is related with what his mate writes. She says about the "relation with the text and with the author" that "excludes the teacher"; "the main goal of mathematics education is to find ways so that students pass from their institutional tasks to their work with the Other". She continues by observing that the activity with students suggests an expansion through interdisciplinary work to introduce them to the "historical context of fifteenth-century Florence".

Participant E worked with another person, who, however, did not produced a contribution. He insists on "Theoretical reflections" and highlights his point of view oriented to a "Philosophical practice", also with reference to (Radford & Sabena, 2015, p.159). He expresses his disagreement with the following statement contained in the "Reflections": "relation with the text and with the author [...] *excludes* [italics added] the teacher". He feels that by saying this, one forgets that the teacher is a mediator. He suggests clarifying the meaning of the term "passivity" in the context in which it is used here. He then underlines that the text belongs to the Other, it contains the trace of the Other – mostly a missing person – and it is the only link between student and author.

Participant F also focuses on the “Theoretical reflections” thinking about links with the Bakhtinian perspective (Guillemette & Radford, 2022). He tries to think about a dialogue between Matilda and Benedetta, the teacher, and Calandri. This raised questions about autonomy of the students, role of the teachers who can accompany them, what to "give" them, and what they have to discover/encounter by themselves.

5 Responses to participants' remarks

Some questions posed by the participants are answered hereinafter, and others have already been answered elsewhere in this article.

Participant C wrote some questions that suggest part of the possible answers. His question "Shifting from rhetorical to symbolic algebra?" recalls Nesselman’s three historical phases of algebra: rhetorical, syncopated and symbolic. The questions: "What do students need to know?" and "The use of x (the "thing")?" – posed differently by A participants – could open a long discourse regarding epistemological and educational aspects on the use of the variable: for example, see (Bråting, 2019). Note that the students did not use variables in their explanation.

When in the "Theoretical reflections" I affirm that the "relation with the text and with the author [...] excludes the teacher" I want to refer to the relation that I define "ethics" in (Demattè, 2021) and that I consider a necessary condition for the student’s learning in using a written mathematical text. The teacher has the role of creating the conditions for this relation to be established. He is only momentarily excluded. The student will return to the teacher for other phases of classroom work in which the teacher will possibly assume the role of mediator.

I used the term "passivity" recalling Levinas when he focuses the ethical relation between two human beings; see (Levinas, 1989) and (Guillemette, 2018). That relation is established without either of the two performs acts aimed at conditioning the Other. Referring this relation to the case of the au-

thor of a document and the student, I consider that the author offers a content without having the power to force the reader. Thus, he exposes himself to the violence of a distorted use of the document: let's think about when the student uses it for a mnemonic study, and therefore without a substantial adherence to the author's reasoning. The author is a defenceless Other – passive – but, when the ethical relation with the student is established, it happens that the author involves the student in his own reasoning (even if the temporal distance and the different personal experience do not allow to speak of coincidence of reasoning); see (Guillemette & Demattè, 2022).
Matilda and Benedetta asked a question regarding author's choices, namely the motivation that led him to the game. The two students moved the discussion beyond the strictly cognitive aspects. This is a case of the ethical events that connotate mathematics classroom; more situations in (Radford, 2021).

6 Further considerations regarding the "*Theoretical reflections*"

By the participants, there were no objections to the statement "Even considering any mathematical document (not just historical) I think that not because it is manifestation of culture the students can be involved in analysing it, but because it belongs to the Other [...]. Culture is a sort of surface". I expected that there could be a disagreement on the part of most of the participants. In fact, one of the reasons why the use of history in the teaching of mathematics is supported in the HPM group lies precisely in the fact that mathematics is included in the culture: mathematics "is the result of contributions from many different cultures" (Clark et alii, 2016). The approach I would like to propose, instead, is aimed at putting the relation between people in the foreground, in order to identify the origin of the possible motivations of a student facing a historical document. In this approach, the basic educational problem becomes helping students to see the author as an Other to be questioned and guided by. In the case of Matilda and Benedetta, the use of the term "research" ("ricerca", in the original report of the interview, in Italian) strikes me, and suggests some reflections and questions regarding the reasons for which they used it, even about a recreational mathematics problem. The term "research" indicates a process that has in itself something unfinished, not yet defined. It seems then that Matilda and Benedetta see exactly this in what Calandri illustrates - the first two questions formulated by the students testify that they do not fully master the situation of the game, which therefore they perceive as not defined. Research is a process that takes place over time and therefore it seems that the two students have chosen the term also considering Calandri and his work within the flow of the history of mathematics. The fact that the students got a biography of Calandri and the fact that they found an unusual and unsettling situation in the document gave them elements for their question about the reasons for the author's research. They thus expressed the desire to know something more about him. They made themselves open to be guided, first by retracing the author's reasoning – using another number – and then developing curiosities that go beyond what the document exposes.

Involvement of the students cannot derive from culture if there are no other previous conditions. Referring to mathematics education what Levinas affirms in the passage reported in paragraph 3, we can say that it is since it belongs to an Other that an element of culture – a mathematical text – involves ("obsessions"), not because it has a cultural garment ("clothed with cultural attributes"). Texts bear author's trace ("of an invisible face") and it is in this that the students find motivation to access mathematics in the document. History shows that mathematics is a product of culture; the complex of institutions, activities and spiritual manifestations that constitutes culture is produced by people. Thanks to the "obsessive" relations between people, culture grows. Mathematics is not an exception. What I have said about the historical document and the author's trace can also be referred to the use of any mathematical text, even if not taken from history. For a document of the history of mathematics, the attribution to an author is a salient fact. On the contrary, in the case of the school texts, it is a fact usually considered irrelevant; this deprives the students of one of the elements to establish the relation with the author.

7 Concluding remarks and didactical implications

We might wonder if the same question asked by Matilda and Benedetta about the research of Calandri would have been formulated by the students if the game had appeared in their textbook, having as author a person they do not know at all (from informal interviews with students of various classes, I have realised that they almost never know the name of the author of their textbook).

During the workshop, no one directly answered the question written as a first introduction of the material given to the participants, that is: "What would you say to Matilda and Benedetta?". One of the participants of the couple D highlighted the importance of motivation in learning mathematics. This seems a suggestion for teachers to give to students directions and information in sight of their possible "whys", similar to "Why did Filippo want to do that research on the number that his friend thought?" by Matilda and Benedetta.

A balance of the workshop allows to highlight the complexity of the activity proposed to the participants: from aspects regarding the specific historical document to theoretical reflections concerning the philosophy of mathematics education. According to their sensitivity and their interests, the participants oriented themselves towards some of those aspects. Also in this paper, only some aspects of the material delivered to the participants have been deepened. For example, the criteria on the basis of which the document was transcribed – modifying certain specific parts of the original, considered too difficult for the students – have not been examined. We reflected on the role of an author of a historical document and of an author of a modern text. We considered the student's relation with the author through the text in both cases. From this, we can remark that working with history can suggest aspects transferable to didactic situations that do not involve its use: educators can derive from history reflections on the pedagogy of mathematics in a broader sense, to improve mathematics education at all levels. Coming back to the students' questions in Section 2, we can find suggestions for projects of activities with the class. For example: a) use of letters for formalizing and generalizing (in different situations, for a long-term intervention), b) problems of recreational mathematics in the history, about which see (van Maanen, 2003) and (Moyon, 2019), c) historical context in which Calandri lived and worked (as a participant in the workshop suggested: see Section 4 of the present article).

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