ON THE MAIN MILESTONES IN DEVELOPING MATHEMATICS IN POLAND PRIOR TO THE XIX CENTURY THROUGH THE LENS OF MATHEMATICS EDUCATION

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ABSTRACT

This paper presents a short overview of the history of mathematics education in Poland prior to the nineteenth century, discussing a number of examples of old mathematical works and problems, and pointing out the main milestones in historical development of mathematics and educational transformations accompanying them.

1 Introduction

The main aim of this article is to present a short overview of mathematics education in Poland prior to the nineteenth century, in the context of educational transformations in that times. The knowledge about mathematics in the past, which I will discuss, was taken from the old mathematical works – their original or reconstructed forms available today, by considering them from two main perspectives: as the result of learning and, on the other side, as the subject to learn. These considerations are immersed in the context of the history of the ways of acquiring an education at different levels, which were practiced by people in the past.

2 Elementary Education in Poland in the Middle Ages – From the Perspective of Mathematical Knowledge

It can be assumed that the history of mathematics education in Poland dates back to the tenth century, the origins of the Polish statehood, connected with an acceptance of Christianity. However, we are able to identify even earlier origins of Polish mathematics, the first mathematical issues, on the grounds of prehistoric or early medieval artefacts, mainly paintings, patterns, which present the sense of symmetry and reveal an interest of their authors in astronomy, astrology or geometry. Unfortunately, other testimonies of mathematical knowledge have not survived to the present day. It is certain however that people living in the territory, later belonging to Poland as a state, possessed elementary knowledge of quantitative and geometric relationships, connected with everyday life, such as counting and measuring. Even if today there are no artefacts confirming an awareness of those relationships, it is possible to recognize some traces of this knowledge when exploring etymology of the Polish language – as words, phrases used to describe numbers, measurement, payment etc.

The adoption of Christianity in the tenth century in Poland resulted in the development of feudal culture and facilitated the permeation of cultural heritage from Western Europe (Høyrup, 2014). The first schools in Poland were founded in the eleventh century at the cathedrals. Their aim was to educate young boys to become clergymen. Latin was the language of the church, feudal administration and science, thus it was also a teaching language at school. The range of knowledge that could be obtained in the cathedral school was very limited. Teaching methods were based on memory training. Even the skill of writing was founded on memorizing the shapes of letters. The content of knowledge available for students depended on how well the teacher – a clergyman – who led the school – was educated himself.

In the later period of Middle Ages, when parish administration was developed in the territory of Poland, parish schools were founded, in addition to cathedral schools. In both those types of schools it was possible to obtain education, typical in Europe of that time: at the first degree – usually to parish schools – Trivium of ‘liberal sciences’, that is: grammar, dialectic (in sense of logic) and rhetoric (the art of arranging talks and letters), and at the second degree – usually in the cathedral or collegiate schools – Quadrivium, that is: singing (music), calculus (arithmetic), geometry (including geography) and astronomy. Parish schools were rather popular and also available for young people - without becoming clergymen. Young people attending parish schools did not gain much general knowledge and almost no mathematics; however some teachers taught pupils elements of counting. Cathedral schools were much less popular than parish schools thus the number of young people who learned arithmetic and geometry in the frame of Quadrivium, was small. One of the main reasons for such a limited level of education was that the teaching sources of that time – manuscripts – were almost inaccessible because of a very high price of paper, so it was common use Latin manuscripts belonging to churches as ‘school textbooks’. For example, the Cracow cathedral library at the beginning of the twelfth century did not contain any work with strictly mathematical content. Later, in the cathedral libraries, it was possible to read works – appreciated and known in Western Europe of that time – which contained mathematics and astronomical knowledge, such as the treatise of Isidorus of Sevilla ‘Libri etymologiarum origines’, the work of English monk Bedy, and writings of Boethius on arithmetic and Euclid’s Elements (Dianni & Wachułka, 1957, Gloger, 1900-1903, Wroczyński, 1996).

Although mathematical knowledge available in the frame of the medieval educational system in Poland was rather poor, in the thirteenth century, some young people, usually coming from rich families, after completing elementary schools and getting acquainted with first sources of mathematical knowledge at that level, felt the need to continue learning mathematics either under the care of individual teachers or at universities abroad – especially in France or Italy. That time can be considered as the beginning of a new period in the history of mathematics education of young Polish people: the smartest and wealthy enough students, after finishing elementary schools were pursuing their mathematics education in universities of Western Europe.

3 Mathematics Higher Education in the Middle Ages – Polish Students’ Experience in Studies Abroad

Before the foundation of the Cracow Academy in 1364, there was no other academic centre in Poland. Despite of it mathematical thought developed, as witnessed by the works of Polish scholars who had been educated in the major European academic centres.

In the thirteenth and fourteenth century young men who were interested in furthering their education at a higher level, travelled abroad to study at universities in France (Paris, Montpellier, Avignon), Italy (Padua, Rome, Bologna) or Bohemia (Prague). One of those medieval students interested in developing mathematical knowledge after completing elementary schools was Vitellon (1220?-1280?). After graduating from elementary schools, he studied at the universities in Paris, Padua and Rome.

On the basis of the knowledge about his works as a scholar, it is possible to discern the content of his studies in mathematics and physics (Vitellon, 1535, Birkenmajer, 1936, Trzynadlowski, 1979, Dianni & Wachułka, 1963).

As a student, he explored and mastered his knowledge of geometry, by getting acquainted with the most important works on the subject of that time, especially by exploring the fundamentals of ancient geometry. He presented the results of those studies in his own work titled: "De Elementis conclusionibus" – "On conclusions from Euclid's Elements", written in Latin. That work has not survived to our times. It seems that Vitellon understood geometry as a science that grew out of the needs of everyday life in connection with the phenomena of the surrounding world. Thus, as a scholar, he tried to develop that science, to be used as an aid to solve world problems. His second work, written in Latin, was devoted to optics and titled: "Vitellonis Mathematics Doctissimi Peri Optikis, id est de natura, ratione & proiectione radiourum uisus, luminum, colorum, atq[ue] formarum, quam uulgo, Perspectiuam uocant, Libri X [...]". the as: "On Optics: it is about the essence, the cause of the incidence of rays, sight, lights, colours and shapes that are commonly called the Perspective, ten books" (ca 1270). It seems that the work of the Arabic scholar Ibn al-Haytham (965-1040), titled: "The Book of Optics" (1011-1021) was the 'point of departure' for the Vitellon's work, which was known under its short name: "Perspectiva".

The work consists of ten books, the first of which is devoted to geometry and consists of 137 theorems with the proofs that are used in the next books on optics.

The book contains the author's knowledge about proportions, the theorems about parallel straight lines on the plane or in the three-dimensional space, the knowledge of angles and triangles, properties of a circle, a sphere and conical sections, and finally conclusions on the harmonic division of a segment. In order to show the author's style of work, two examples of propositions taken from this book are quoted below:

Proposition 53:

The arcs contained between the straight parallel lines on the plane of the circle are equal regardless of whether the straight lines are secant or tangent, or whether one is tangent and the other is secant.

Proposition 98:

There is no such a cross-section of a circular cone by a plane, that does not pass through its top, which is a triangle. (Vitellon, 1535, Dianni & Wachułka, 1963).

In order to prove these propositions Vitellon used his knowledge of ancient geometry, known from Arabic works, applying in the proof Arabic term of the notion of hyperbole (Vitellon, 1535, Dianni & Wachułka, 1963).

Among the theorems included in the book it is possible to find propositions which were originally formulated by Euclid and Apollonius and also some theorems which came from works of Arab and Greek scholars living after Apollonius.

It is worth noticing that European scholars began to study the works of Apollonius more thoroughly much later than Vitellon did it – only in the sixteenth century.

The work "On Optics [...]" has had a great impact on the development of mathematics and physics for the next generations of university students and scientists in Europe. The manuscript was repeatedly transcribed and then issued in print many times, especially in the sixteenth century (1535, 1551, 1572), so it must have served as an academic textbook for many generations of university students (Vitellon, 1572). Not only French, German and Italian scholars have referred to the scientific achievements of Vitellon: Nicoalus d'Oresmus (1320-1282), Regiomontanus (1346-1476), Luca Pacioli (1445-1517), Leonardo da Vinci (1452-1519) and Johann Kepler (1571-1630) – in his work titled "Ad Vitelloni paralipomena, quibus astronomiae pars optica traditur" (1604), but also Polish scholars such as Martinus Rex de Peremislia (1422-1460?) or Nicolaus Copernicus (1473-1543).

4 Development of Mathematics Education in Poland in the Fifteenth Century – the Cracow Academy as an Educational and Scientific Centre

In the second half of the fourteenth century, the need to establish an academic centre in Poland seemed to result from significant development in the area of social, political, economic and cultural transformations which took place in Poland of that time. The quality of education in the cathedral schools became higher, mainly because of better teachers' competencies, and the number of young people, who travelled abroad to study in foreign universities, after graduating from elementary education, was growing.

The first academic centre – the Cracow Academy – was founded in 1364. Early at the beginning of the fifteenth century it became an important European educational and scientific centre. Preliminary level studies at the Cracow Academy covered the programme of Trivium and Quadrivium which was also taught in the best cathedral schools. Students, after graduating from this level of studies, were able to continue their higher education studying such scientific disciplines as law or theology, but also astronomy and astrology, which were connected with mathematics. Although all these disciplines were called 'mathematical sciences', mathematical knowledge was considered as the basis necessary for investigations in astronomy (Gloger, 1901-1903, Karbowiak, 1923, Wroczyński, 1996).

'International exchange' of students and scholars between the Cracow Academy and many European universities was popular: after graduating from the Cracow Academy scholars often moved to some foreign universities for further studies and then, after getting deeper scientific experience, they returned to their alma mater as professors. On the other hand, foreign students or scholars came to Poland for their studies or to develop scientific research, especially combining mathematical knowledge with the secrets of astronomy. International contacts made mathematics knowledge known to scholars, irrespective of their nationality.

In order to understand what the subject of mathematical studies at the end of the fourteenth century was, one should get acquainted with the content of manuscripts which were used as 'academic textbooks' at that time. Some of them were just direct notes written by students or scholars when studying in foreign universities or in the Cracow Academy, some were the scholars' own treatises. One of those manuscripts was the work titled: ‘Algorismus Anno 1397’, written in Latin by an unknown author. That manuscript contained arithmetic of the whole numbers, in particular: a knowledge about series of numbers, squaring, cubing and taking square roots of numbers. All that knowledge was necessary to consider problems in the area of astronomy. Except for arithmetic, Euclid’s plane geometry was also the basis of mathematics at that time (Dianni & Wachułka, 1957).

Among other manuscripts used for studying mathematics in the fifteenth century there are works of Martinus Rex de Peremislia (1422-1460?), one of the members of the Cracow Academy community. Martinus Rex studied at the Cracow Academy and after graduation he became a scholar at the Academy and began to develop his own scientific research. One of his most important works written in Latin: “Algorismus minutiarum” (ca.1445) was devoted to arithmetic of fractions (Dianni & Wachułka, 1963). It is worthwhile to consider the content of this work more carefully. In that extensive lecture on fractions, the author introduced modern notation of fractions using the fractional bar, the numerator and the denominator. Moreover, he discussed basic arithmetic operations on fractions, and illustrated with drawings his method of reasoning. Martinus Rex considered not only the ordinary fractions – ‘fractiones vulgares’, but also the sexagesimal positional fractions, which were characterized by a different notation system, similar to today’s decimal number system; he called them: ‘fractiones physicae’ – ‘physical fractions’. He investigated the relationship between these two types of fractions, but also explained the principles of taking the square root or the cubic root of a fraction, removing an immeasurability of the denominator of a fraction, multiplying and dividing the fractions. The style of that work was modern, in comparison to the old medieval style of presenting mathematical arguments. It differed in its methodology from the dogmatic lecture of canons supported by the authority of the ancient sages, and took into account the usefulness of mathematics. The content of this treatise became the subject of the main systematic university courses in the Cracow Academy.

After elaborating the course of arithmetic of fractions, Martinus Rex went abroad, maintaining a contact with his alma mater, and studied in Padua, where he became acquainted with the practical geometry of Prosdocimus de Beldomandis (?-1428). The second important treatise of Martinus Rex was devoted to geometry: ‘Geometria Regis’ (ca 1450) and –similarly to the previous one – became a basic ‘academic textbook’ for students and scholars (Birkenmajer, 1895, Dianni & Wachułka, 1957). That work begins with the statement: ‘Geometry has two main parts: the theory and the practice.’ It seems that Martinus Rex understood this statement according to his general philosophical attitude that ‘the theory’ – mathematics – and ‘the practice’ – nature – are in fundamental relation: ‘the theory’ should be convincingly argued by ‘the practice’. The ‘Geometria Regis’ is considered to be the first work on so called practical geometry in Poland. In this work, the author provides solutions to practical issues, practical measurements, which are an introduction to theoretical considerations and research. Issues discussed in the treatise dealt with the following mathematical concepts: proportions, similarity of triangles, measurement on the circle, the sphere, cylinders, cones, barrels – here the author used the approximate method of ‘smoothing the cross-sections’, provided the rules for calculating the area of plane figures and the volume of geometrical solids and illustrated them with examples. The lists of measures used for length measurements as well as the relationship between them are included in the treatise. The work ‘Geometria Regis’ became the basis for further studies and investigations in geometry in Poland (Birkenmajer, 1895, Dianni & Wachułka, 1957).

When we analyse the area of Martinus Rex's mathematical research we can easily deduce that he also dealt with astronomy. He noticed the shortcomings of the Ptolemaic system and tried to make corrections to the calculations of quantities in the Toledan Tables. Martinus Rex, in his astronomical considerations, investigated trigonometric relationships in a modern form –he used in his calculations the half-chords (sinus) instead of the whole chords (chord), which were the basis of Greek calculus. His research on the Toledan Tables, which he initiated, was continued by consecutive scholars of the Cracow Academy, including Wojciech (Adalbert) from Brudzewo (1446-1495). Adalbert from Brudzewo noticed the defects of the Toledan Tables and the need to improve them, but it was Nicolaus Copernicus who posited the hypothesis that the ambiguities in the calculations might have come from the faulty construction of the entire system, not from the incorrect construction of the Toledan Tables.

To summarize, Martinus Rex's works served as the basis of studies and research for subsequent generations of students and scholars of the Cracow Academy in the Renaissance period (Birkenmajer, 1895).

5 Mathematics Education in Renaissance Era in Poland

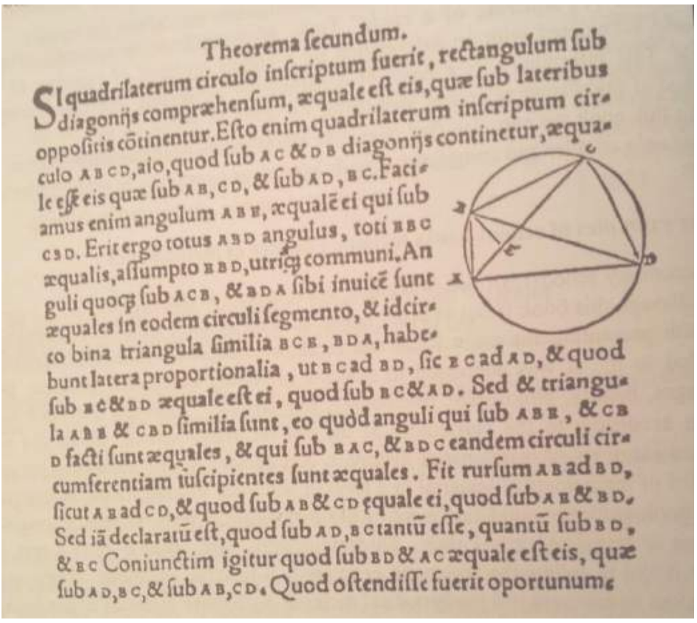
The end of the fifteenth century as well as the sixteenth century is recognized as 'the golden age' of Renaissance era in Poland. In the field of mathematics, the wide spectrum of sophistication levels in mathematical investigations can be observed: on the one side – further development of scientific research, especially connected with astronomy and its necessary mathematical tool, trigonometry, and on the other – new tendency to make elementary mathematics more available for wider groups of people, by writing textbooks in the Polish language, which was connected with creating mathematical terminology in the Polish language.

The most important representative of Renaissance science was Nicolaus Copernicus (1473-1543), who studied at the Cracow Academy from1491-1494, probably as a student of Adalbert from Brudzewo. He was especially interested in astronomy – its phenomena, and their mathematical tables. That knowledge and the strong confidence that the Ptolemaic system could not be correct, led him continuing his research, on the scientific verification of his hypothesis on the Earth motion as profoundly as possible. During his lifelong research, hecreated the scientific justification of his hypothesis on the construction of the world, and based hisarguments on mathematical considerations (Baranowski, 1854).

The most important mathematical basis for his astronomical arguments he gathered into a separate part of his treatise titled 'De Revolutionibus Orbium Coelestium' (1943, printed edition1543), (Birkenmajer (1920, 2004). That is the First Book: 'Trigonometry', devoted to considerations regarding the solution of triangles.

Chapters 12, 13 and 14 of that book were acknowledged as a kind of complementary part to Euclid's Elements. They were focused on spherical triangles and linear triangles and gave rules for calculating chords and halves of chords for given angles. Among theorems presented in those chapters there are two – the Second Theorem and the Third Theorem – which were used by Copernicus to elaborate the table of chords.

The Second Theorem: “In a quadrangle inscribed in a circle, a rectangle consisting of two diagonals [of the quadrangle] is equal to the sum of rectangles consisting of opposite sides [of the quadrangle].” (Fig. 5.1)



Theorema ſecundum.
SI quadrilaterum circulo inſcriptum fuerit, rectangulum ſub
diagonijs compræhenſum, æquale eſt eis, quæ ſub lateribus
oppoſitis cõtinentur. Eſto enim quadrilaterum inſcriptum cir-
culo ABCD, aio, quod ſub AC & DB diagonijs continetur, æqua-
le eſſe eis quæ ſub AB, CD, & ſub AD, BC. Faci-
amus enim angulum ABE, æqualẽ ei qui ſub
CBD. Erit ergo totus ABD angulus, toti EBC
æqualis, aſſumpto EBD, utriq; communi. An-
guli quoq; ſub ACB, & BDA ſibi inuicẽ ſunt
æquales in eodem circuli ſegmento, & idcir-
co bina triangula ſimilia BCE, BDA, habe-
bunt latera proportionalia, ut BC ad BD, ſic EC ad AD, & quod
ſub EC & BD æquale eſt ei, quod ſub BC & AD. Sed & triangu-
la ABE & CBD ſimilia ſunt, eo quòd anguli qui ſub ABE, & CB
D facti ſunt æquales, & qui ſub BAC, & BDC eandem circuli cir-
cumferentiam ſuſcipientes ſunt æquales. Fit rurſum AB ad BD,
ſicut AE ad CD, & quod ſub AB & CD ęquale ei, quod ſub AE & BD.
Sed iã declaratũ eſt, quod ſub AD, BC tantũ eſſe, quantũ ſub BD,
& EC Coniunctim igitur quod ſub BD & AC æquale eſt eis, quæ
ſub AD, BC, & ſub AB, CD. Quod oſtendiſſe fuerit oportunum.

Figure 5.1: The Second Theorem

The Third Theorem: “If in a semicircle the chords of two unequal arcs are given, the chord of the difference of these arcs will also be known.”

The last theorem of the book: “The ratio of two arcs: larger and smaller – is greater than the ratio of chords” indicates that Copernicus had the knowledge of the fact that the chords are not directly proportional to the angles.

The First Book includes also theorems concerning the problems of solution of triangles (chapter 13) and spherical trigonometry – spherical triangles and their congruence (chapter 14), (Copernicus, 1543, Birkenmajer, 1920, Dianni & Wachułka, 1963). This book afterwards was a research inspiration for Francois Viete (1540-1603) and John Napier (1550-1617).

The main aim of Copernicus’s mathematical research was to find the appropriate models for astronomical phenomena. In the seventeenth century, even after 1616, the work of Copernicus was the main subject of scientific studies at the Cracow Academy.

The sixteenth century – the period of the Renaissance – is also characterized by the development of humanistic thought; Polish mathematical treatises were written not only in Latin, but also in mother tongues. The group of recipients of scientific knowledge was growing, the methods of teaching mathematics were changed, the scholars more often took up issues arising from everyday life and presented their solutions in a more accessible and clear way. The basis of Polish mathematical terminology was then created.
Among those works, which contain straightforward and visual methods used to solve problems as simply as possible, two textbooks can be distinguished.

The first textbook titled: ‘Algoritmus’ (1538), written in the Polish language by Tomasz Kłos, is addressed to wide group of people who must have used the science of arithmetic, merchant bills, and accounting in their daily work (Baraniecki, 1889, Wydra, 2015, Dianni & Wachułka, 1963). In order to make those calculations easy for readers, the author introduced the so-called ‘Calculus on lines’ – a specific abacus in the form of a board. Parallel lines drawn on the board indicated the places of units, tens, hundreds etc. The fields between the lines indicate the half of the higher order unit. Calculations were based on the appropriate placement and shifting of pebbles on the board. The operations of addition and subtraction, as well as the application of the 'rule of three' – 'regula detri' were presented and explained in the textbook. The same textbook included the presentation of calculations on fractions, and numerous solutions to specific problems, encountered in the reader's everyday life. The main aim of the author, as seen in this textbook, was to give to the reader the simple methods of obtaining a solution to a problem, but without justifying its correctness. The only reader's task was just to gain a mechanical skill in applying the given rule.

The author's intention for writing that book was also to address it to young people. For many years to come, despite the development in arithmetic later on in publications in Latin which used digital notation of numbers, his method of 'calculus on line' was widely known and practiced in people's everyday life.

Apart from the textbook about arithmetic and accounting, which was described above, the second important textbook written in Polish in the sixteenth century was devoted to geometry. The author of that textbook titled 'Geometry' (1566), Stanisław Grzepski (1526-1570), was a professor at the Cracow Academy (Grzepski, 1929, Dianni & Wachułka, 1957, 1963). He was interested in Euclidean geometry and its practical applications, especially in metrology. His textbook was addressed to a wide variety of people, especially to farmers, who needed to have an elementary knowledge of the plane geometry and its applications. The contents of the textbook include problems known from the first four books of the Euclid's Elements, as well as Books 5 and 6. Geometric concepts and their applications, presented in the textbook, were connected only with plane geometry. The solutions were illustrated by drawings. The author introduced not only Polish terminology for geometric concepts, but also as the first scientist in European geometry, gave the name for the concept of parallel lines: 'aequidistantes' (Grzepski, 1929, Dianni &Wachułka, 1957, 1963).

The first part of the textbook presented to a reader the axioms of Euclidean geometry, the classification of polygons, the measure of inner angles of the triangle and the area of a triangle. The author presented also the calculation of the area of a circle by considering its approximation. The second part of the textbook includes defining units of measurement of the length and the area of plane figures and applying this knowledge to solving practical problems connected with agriculture and building construction. Although many examples in the textbook are taken from Euclid's Elements and from Archimedes's works, i.e. the problem of the height of the tower or the problem of the depth of the well, their solutions gave readers a chance to become acquainted with these fundamental geometric works in a simple and mathematically accessible way.

6 Mathematics in the Seventeenth-century Educational Centres in Poland

The seventeenth century brought further development in Polish mathematics and mathematics education – the achievements of Polish scientists were known in the European scientific centres not only because of leading their research at European universities but also through giving academic lectures and writing works in Latin.

One of the most important representatives of Polish mathematics of that time was Jan Brożek (1585-1652). After completing his studies at the Cracow Academy he was a lecturer of mathematics and astrology, then he studied in Padua and after some years he became a professor in the Cracow Academy. His work titled ‘Arithmetica integrorum’ (1620), written in Latin, is considered as the first modern – at that time – academic textbook in Poland, addressed directly to the university students (Franke, 1884, Dianni, 1956). The book presented the complete academic knowledge of arithmetic (at that time), that means: arithmetic operations on integers, especially multiplying numbers illustrated by the method called ‘the calculus on fingers’, the calculus using exponents, roots of numbers, introduction to arithmetic and geometric series. One of the most interesting parts of that textbook was discussing the calculus using the exponents of the power of number 2, in which multiplication and division of powers consisted in adding and subtracting exponents. In this way calculations on numbers came down to calculating on exponents. It seems that this idea was inspired by the concept of logarithm introduced in 1614 by John Napier.

Another important academic textbook, published in the seventeenth century, was the work titled ‘Arithmetica vulgaris’ (1640), written by Jan Toński (?-1664), who was also a student and later a professor of the Cracow Academy (Toński, 1640, Dianni & Wachułka, 1957). That book included calculations on integers and fractions as well as on decimal fractions. The author introduced the notation of decimal fractions with a decimal colon, used in Poland even today. The second part of the book was devoted to plane and spherical trigonometry. The rules of calculations were described by the author in a general manner, without using algebraic formulas. That part of the book could be compared with Copernicus’s treatise ‘Trigonometry’. Although the spherical trigonometry was considered in the book as a part of the theory of mathematics rather than merely an aid in astronomy, the last part of the book included examples of applications of trigonometry in metrology and astronomy.

It can be concluded that both academic courses indicated a tendency to develop mathematics as a scientific theory rather than a tool used for solving practical problems. Both books were very popular in the academic community and were included in the supplementary list of academic textbooks for many years, even at the end of the eighteenth century.

The seventeenth century also brought a modern textbook for geometry written in Polish by Stanisław Solski (1622-1701): ‘The Polish User of Geometry – learning to draw, divide, measure lines, angles, figures and solids’ (1683) (Fig. 6.1 and Fig. 6.2).

This work consists of 14 chapters called ‘plays’, in which the author presented descriptions of geometrical figures, their properties and applications of geometry for solving practical problems (Solski, 1683-1684). The last chapters included arithmetic and combinatorics. The names of mathematical concepts introduced in that work indicated great progress in the Polish mathematical terminology.

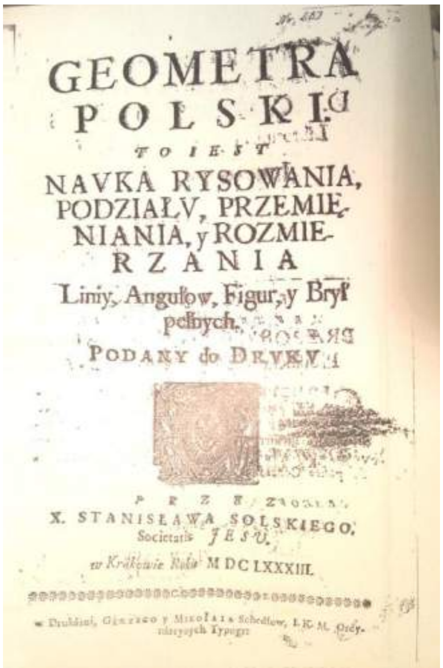

GEOMETRA
POLSKI
TO IEST
NAVKA RYSOWANIA,
PODZIAŁV, PRZEMIE-
NIANIA, y ROZMIE-
RZANIA
Liniy, Angułow, Figur, y Brył
pełnych.
PODANY do DRVKV
PRZEZ
X. STANISŁAWA SOLSKIEGO,
Societatis IESV.
w Krakowie Roku M DC LXXXIII.

Figure 6.1: Cover of the book

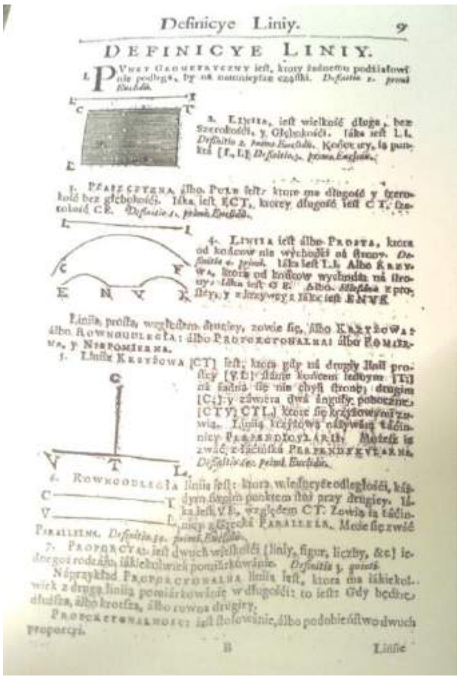

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Figure 6.2: Definitions of lines

The development of mathematics can also be inferred by studying certain scientific manuscripts written in the seventeenth century. One of the most interesting examples of mathematical investigations was the scientific achievements of Stanislaw Pudłowski (1597-1645). Many of his notices indicate his profound knowledge of mathematics, including some interesting proposals of solutions to problems of descriptive geometry, formulated later by Desargues and Monge (Dianni & Wachułka, 1963).

Adam Adamandus Kochański (1631-1700) was a student and later a professor of the Vilnius Academy. He was in correspondence with many outstanding European scientists of his time, such as Gottfried Leibniz (1646-1716) and Johannes Hevelius (1611-1687), thus he understood the need of sharing scientific experiences among scientists and its great impact on development of science. His works, mainly on mechanics and statics, as well as a very interesting and 'elegant' method of the approximate construction of rectification of the circle (1685), were published in thescientific journal 'Acta Eruditorium' edited in Leipzig (Barycz, 1935, Dianni & Wachułka,1963).

The Cracow Academy was not the only important scientific centre in Poland in the seventeenth century. There was also the Vilnius University founded in 1578. Apart from those universities there were academies, in which young people were educated: the Academy of Lubrański (1519-1780), the Academy of Zamoyski (1594-1784), and the Academy in Raków (1602-1638), where Jan Amos Komenski (1592-1670) was a professor (Gloger, 1900-1903). The education system included also the centres of – as we call it today –secondary education: there were colleges, especially Piarist colleges and schools (1642-) and Jesuit colleges (the period of the existence of Jesuit colleges in Poland: 1534-1773). Parish schools educated pupils at the lowest level; the teaching methods and educational programmes were rather similar to these applied in previous times. Young people belonging to nobility or rich townsmen families were educated individually by private teachers, often professors of academic centres, in order to be prepared to continue studies at the European universities (Gloger, 1900-1903, Suchodolski, 1972). The content of an education programme depended very much on the school staff. However, at the beginning of the seventeenth century, the parish schools introduced a common educational programme framework, elaborated by scholars of the Cracow Academy after analysis of educational programmes of the Italian, German and French elementary schools. The great impact which was felt with the development of teaching methods and contents was connected with the emergence of schools which were led by teachers belonging to protestant communities. In those schools it was important to educate pupils in developing humanistic thinking and learning based on understanding, gathering one's own experience rather than memorizing knowledge taken from a teacher. That style of teaching also helped students in learning school mathematics. Although Latin was still the teaching language in schools, mathematics textbooks written in the Polish language became popular among wide groups of people, especially those (i.e. merchants, farmers etc.) who had to apply mathematics to their professional activities (Gloger, 1900-1903, Suchodolski, 1972).

7 Mathematics Education in the Eighteenth Century – the Age of Modern Reforms of the Educational System in Poland

The first half of the eighteenth century was characterized by a gradual decrease in the level of Polish education. The need for reform in the whole education system as well in the political and social areas of the state was evident.

The education system of that time was dominated by the structure of parish schools and secondary schools – mainly Jesuit colleges. The old-fashioned school programmes were focused mainly on preparing students to lead philosophical disputes, thus rhetoric was still the subject of great importance and memorizing knowledge was the main teaching method. Latin was the teaching language. That style of teaching was not sufficient in the age of new European tendencies to develop humanistic thinking and gain the knowledge of natural sciences (Gloger, 1900-1903, Suchodolski, 1972, Wroczyński, 1996).

The first step towards reform was made by Piarist Stanislaw Konarski (1700-1773), who created an eight-year modern school – the Collegium Nobilium (1740). The programme of education included mathematics: algebraic expressions, proportions, algebraic fractions, roots, solving equations, geometry – plane figures, solids, and conics. Besides mathematics, economics, natural sciences, practical knowledge, foreign languages: French and German, and the Polish language as separate subjects were the components of the education programme (Gloger, 1900-1903, Suchodolski, 1972). That trend of modern reforms was expanding in a short time; other educational centers modernized their educational programmes, in which mathematics played an important role.

At the same time, in order to modernize the area of higher education, it involved Andrzej Załuski, the professor of the Cracow Academy, introducing on the basis of new foreign academic ideas, the three-year course titled 'Cursus mathematicus dogmativo-experimentalis' (1750). It consisted of three parts: the first part of the course was focused on studying arithmetic, theoretical geometry, plane and spherical trigonometry, and also algebra; the second part of the course was devoted to study mathematics applications, that is: lectures in mechanics, hydrostatics, aerometry, civil and military architectures. In the last part, at the third year of the course, students studied physics (Suchodolski, 1972, Dianni & Wachułka, 1963, Pawlikowska-Bożek, 1982).

In 1773 the Jesuit colleges were closed and that was the appropriate moment in Poland to introduce the institutional reforms concerning the system of education and the content of the teaching programmes. In the same year the Commission of National Education was established and officially took a care of all schools in Poland. The Commision of National Education was the first secular institution in Europe, similar to a modern ministry of public education. The group of modern scientists and educators, known as the "Society of Elementary Books', was responsible for drawing up modern programmes for school education and modern school textbooks. They decided to elaborate Polish translations of the best foreign textbooks of that time. These translations were published and used as the school textbooks in the Knight's School in Warsaw, founded in 1768 by Adam Czartoryski. In that school physics was taught on the basis of George Luis de Sage's and Christoph Pfleiderer's textbooks, and mathematics – on the basis of Simone L'Huillier's (1750-1840) textbook (L'Huillier, 1809). The translations of those textbooks into Polish as well as the Etienne Bezout (1730-1783) textbook's translation: 'Mathematics Education for the use of French Artillery' (1781), gave the basis for modern Polish terminology in mathematics and physics (Bezout, 1808). The textbook for mathematics included: arithmetic, geometry and algebra. It was addressed to the teachers, not to the learners, thus it included many methodical guidelines.

The Commission of National Education, working under the guidance of Hugo Kołłątaj (1750-1812), elaborated the complex and modern system of education in Poland, consisting of a three-level education. New curricula for primary and secondary schools were elaborated. The system assumed equal access to education for girls and boys. Although the idea of modern transformation of the education system was elaborated in detail, it did not gain the formal approval of the parliament. Despite that failure, the reform of higher education was carried out (1781-1788), the supervision of higher education institutions over secondary schools was introduced, and universities gained a modern organization. Also, vocational education was introduced (Gloger, 1900-1903, Suchodolski, 1972).

Although the period 1789-1794 brought a gradual loss of influence of those reforms, modern ideas in universities survived and gave the basis for developing modern branches of science, such as chemistry (i.e. works of Jędrzej Śniadecki, 1768-1838) and mathematics (i.e. works of Jan Śniadecki, 1765-1830) in the field of differential calculus, analytic geometry, and probability calculus) (Więsław, 2012-2013).

The reforms in the Polish education system and in mathematics and natural sciences that took place prior to the nineteenth century, gave the basis for development of modern mathematics and science in the nineteenth and twentieth century, becoming well known as the Polish School of Mathematics developed by the mathematicians atthe Lviv University and at the Warsaw University.

8 Final Remarks

Knowledge on the history of mathematics education provides many possibilities to apply it in today's school practice.

From the point of view of teachers the history of mathematics education is a rich source of information on the epistemology of mathematical concepts and possible epistemological obstacles which can be observed when teaching contemporary students. It is also an interesting area of mathematical problems which can be discussed with students and solved by them.

From the point of view of students some chosen old mathematical problems and their solutions can be interesting and may be helpful in understanding today's school mathematics.

The historical mathematical problems presented in original texts can also be helpful in studies of the language – Polish or Latin – for students who are interested in learning Latin or studying mother tongue, its grammar or etymology. For example, the textbook of Tomasz Kłos, Algorismus (1538), written in the Polish language, can be interesting for students because of the 'Calculus on lines' method of adding and subtracting numbers, presented and explained in the textbook. Another such example is the proof of the Second Theorem (Fig. 5.1) of the First Book of Copernicus treatise, which was included to the mathematics textbook addressed to students studying Latin (Lakoma, 2000).Also the textbook of Stanisław Solski (Fig. 6.1, Fig. 6.2), can serve as a source of old Polish geometrical terms which can be a good point of departure to study the history of the Polish language. Studying grammar and etymology of the Polish language can be also good motivation for students to learn mathematics and to find enjoyment in following historical ways of mathematical reasoning, comparing historical ones with contemporary ones, and searching for one's own solutions.

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