MATHEMATICS TEACHING IN GYMNASIA AND REAL SCHOOLS IN POLAND IN THE YEARS 1795–1918

Schools with Polish and German as the language of instruction – comparison

Karolina KARPIŃSKA
L. & A. Birkenmajer Institute for the History of Science, Polish Academy of Sciences,
Nowy Świat 72, room A09, 00-330 Warsaw, Poland
karolinakarpinska001@gmail.com

ABSTRACT

This article is dedicated to the comparison of ways mathematics was taught in secondary schools with Polish and German as languages of instruction on the territory of Poland from 1795 to 1918. Particular attention will be paid to gymnasia (Pol. *gimnazjum*, Ger. *Gymnasium*) and real schools (Pol. *szkoła realna*, Ger. *Realschule*), which constituted the best types of secondary schools at that time. Mathematics teachers education, mathematics curricula and school leaving examinations, called also Matura examinations, will be compared. But firstly, there will be explained why there were schools with different languages of instruction on the territory of Poland from the end of the 18th to the beginning of the 20th century.

Keywords: mathematics, mathematics teaching in Poland, secondary schools, gymnasia, real schools, partitions of Poland.

1 Introduction

In the early 18th century, Poland was a prosperous country with its own mathematical culture and mathematical traditions. In 1773, the Polish government established the Commission of National Education. This Commission reorganized the system of Polish education, prepared new school curricula, new school textbooks, organized a system of teachers' education and school supervision system. Regulations prepared by the Polish Commission of National Education (*Ustawy Kommissyi Edukacyi Narodowej dla Stanu Akademickiego i na szkoły w kraiach Rzeczypospolitey przepisane*, 1783) were based partly on the rules of Jesuits and Piarist schools but they also took into consideration the pedagogical concepts the age of Enlightenment. New school curricula paid attention to mathematical and natural sciences, especially on: arithmetic, geometry, trigonometry and algebra. Those regulations also described the best method of teaching: professors should abandon the lecture method in favour of experiential teaching in which students will understand what they are learning (*Ustawy Kommissyi Edukacyi Narodowej dla Stanu Akademickiego i na szkoły w kraiach Rzeczypospolitey przepisane*, 1783, 36).

The Polish Commission of National Education was the first Ministry of Education in Europe, it was a secular organisation, independent of the Church. It was the first European attempt to transform chaotic teaching into an organized system of education – from elementary schools to universities. 10 years later everything was ready and implemented on the territory of Poland.

One year before establishing the Commission of National Education, so in 1772, the First Partition of Poland took place. Three empires, Prussia, Austria and Russia, occupied some territories of Poland as a result of the unstable political situation in Poland. In 1793 there was the Second Partition of Poland, in 1795 – the Third Partition of Poland. From

the Third Partition of Poland to 1918 all territories of Poland were occupied[1] (Fig. 1.1). For 123 years Poland disappeared from the map of Europe.

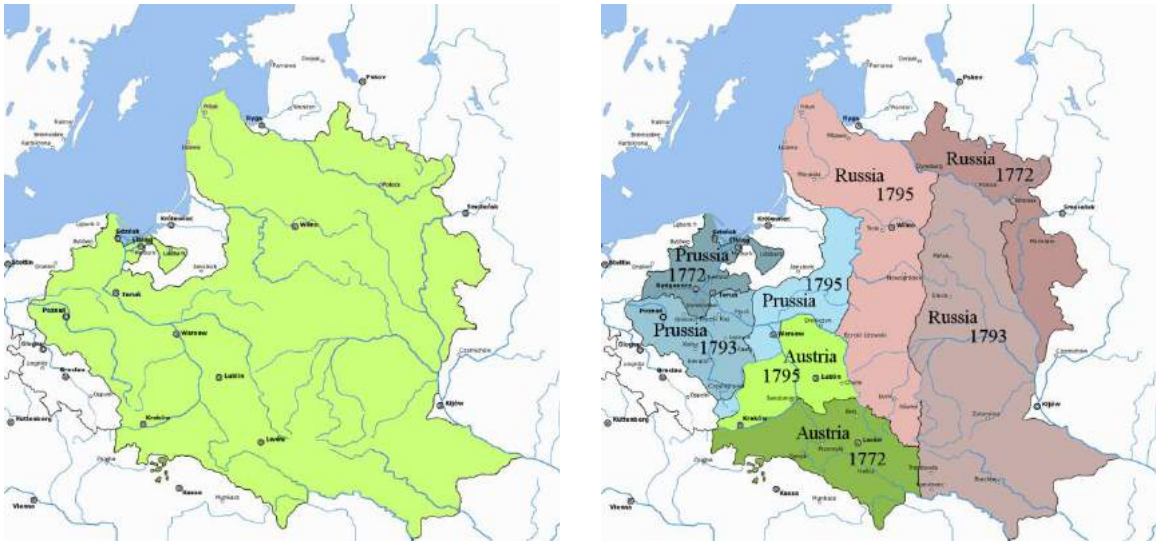


Figure 1.1: Partitions of Poland[2]

In 1795 Prussia, Austria and Russia began the process of germanization and russification of the territories of Poland.

Prussia and Austria almost immediately introduced German as a language of instruction to Polish schools, Polish teachers were replaced by those from Prussia and Austria, Prussian and Austrian curricula and textbooks were also introduced. In 1868 the territories of the Austrian Partition gained autonomy[3] in education and there was established the Polish Ministry of Education: National School Council (Pol. *Rada Szkolna Krajowa*), Polish was the language of instruction, there were Polish teachers, Polish textbooks and Polish curricula. In 1875 the freedom of Poles was limited and all decisions of the Polish Ministry of Education had to be approved by the Ministry of Education and Religious Affairs in Vienna (Świeboda, 1984, 111–112). In the Prussian Partition from 1795 to 1918 there were schools with German as a language of instruction and Prussian rules of education only.

In the Russian Partition from 1795 to 1831[4] there were Polish schools with Polish rules of education and Polish teachers. In 1831 freedom of Poles was limited and all decisions related to Polish schools had to be approved by the Department of Religious Affairs and Public Education of the Polish Provisional Government, which was under the supervision of Russian Emperor (Manteuffel, 1929, 26). The year 1839 is considered the beginning of the process of combining Polish education with the school organization of the Russian

[1] The Polish lands were occupied by Prussia, Austria and Russia almost for the entire time from 1795 to 1918. The exception are the years 1807–1815 when part of the Polish lands under the name Duchy of Warsaw were under the authority of the French emperor, Napoleon Bonaparte. After the fall of Napoleon, the Duchy of Warsaw was enlarged at the Congress of Vienna in 1815 and transformed into a Kingdom under the rule of the Tsar of Russia. Education in the Duchy of Warsaw remained in the hands of the Poles.

[2] Source of maps: https://pl.wikipedia.org/wiki/Rozbiory_Polski#/media/File:Partitions_of_Poland.png.

[3] Obtaining autonomy by the territories of the Austrian Partition was a process lasting several years: 1860–1873 (Majorek, 1980, XIII–XVI). It should be emphasized here that from the 1850s, there were schools on the territories of the Austrian Partition with Polish language of instruction, however there were only a few of them (*Sprawozdanie…*, 1885, 26–27).

[4] In 1831 November Uprising took place.

Empire (Królikowski, 2008, 285). Changes were introduced gradually – Russian teachers were successively brought to Polish lands and regulations concerning the functioning of Polish schools were issued, e.g. in 1851 another school reorganization was approved, under which Polish philology gymnasia were made similar to this type of schools in Russia (Królikowski, 2008, 287). After the January Uprising, taking place in 1863, Polish schools were revived for a short time thanks to Aleksander Wielkopolski, but already in 1867 they were thoroughly Russified (along with the introduction of the Russian language of instruction to all schools) (Manteuffel, 1929, 37-49). Until 1867, many Polish names appeared in the composition of the central educational authorities for the Polish territories of the Russian Partition (Manteuffel, 1929, 51-69). The managerial positions (such as the director, curator, minister, etc.) were usually filled by the Russians, however, also the Poles had a great influence on the form of teaching in the Polish lands.

This article will compare the teaching of mathematics in schools with Polish and German language of instruction functioning in Polish territories in all partitions. Particular attention will be paid to gymnasia and real schools, in which, in the 19th century, Matura exams were already carried out. Gymnasia were focused on teaching humanities subjects, and real schools – mathematics and natural science subjects. These were schools for the chosen, not the majority. These schools educated the intellectual elite of the country.

Young people starting gymnasia and real schools were specially selected. During the entrance exams to these schools, on the one hand, students had to demonstrate knowledge of the material obligatory in elementary schools, while on the other hand, they had to demonstrate high general abilities, whose determinants were intelligence, creative thinking, good memory and imagination. Only students with these skills could successfully continue their education in gymnasia and real schools and pass the 19th century Matura exam. Among students, especially in real schools, there were also people with special abilities in mathematics (see also: Karp, 2009). It allowed to implement the widest possible curricula, also in the field of mathematics.

Comparison of the method of mathematics education in gymnasia and real schools in the Polish territories during the partitions is therefore a comparison of the mathematical education of the intellectual elite in Poland with the way of its education in Prussia and Austria. It is a comparison of the mathematics education in the occupied country, seeking independence, and the way it is taught in the occupying countries.

Stefan Banach said that only countries that cherish mathematics can be strong and powerful. Mathematical education has often been an instrument of political reform (see, e.g., Karp & Funinghetti, 2016; Karp & Funinghetti, 2018). A lot has been written about this in the context of Prussia or France. Here we will focus on Poland. We will discuss the teaching of mathematics in a country that wants to free itself from slavery. We will try to answer the following questions: What kind of mathematics was taught to the Polish intellectual elites, which as a result, in 1918 raised Poland from the fall? Did Poles model themselves on the reaching systems implemented in Prussia and Austria or did they develop their own teaching system? Was the national element visible in teaching – was teaching adapted to the needs of the Polish society? Was mathematics aimed at practical application taught, or maybe the 19th century tendency was followed: "learning mathematics was intended to contribute to the development of one's mental capacities. Practical value or applicability of mathematics was typically much less important" (Smid, 580)? How were mathematics teachers educated?

The analysis carried out in this article is a contribution to the study of the history of mathematics teaching. It presents the means by which high effects of mathematical education were achieved in Poland in 1795-1918. It also allows to draw conclusions for modern educational practice.

Let us compare mathematics teaching in schools with Polish and German as languages of instruction in all partitions of Poland.

2 Mathematics teachers in schools with Polish and German as languages of instruction

2.1 Regulations related to teachers' education

2.1.1 Polish regulations

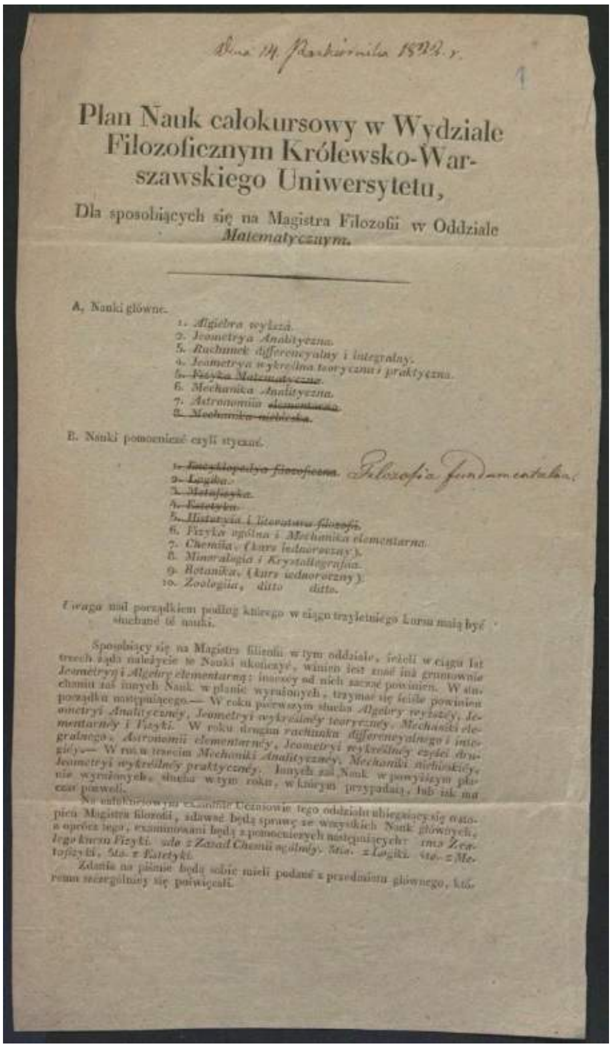
The first Polish regulations related to teachers' education were prepared by the Commission of National Education. It took place in the 1780s (see: Popławski, 1780; *Ustawy Kommissyi Edukacyi Narodowej dla Stanu Akademickiego i na szkoły w kraiach Rzeczypospolitey przepisane*, 1783, 15–18). The Commission operated until the Third Partition of Poland. However, its general regulations were in force for a dozen years after 1795. Moreover, its regulations related to teachers' education were kept for almost the entire first half of the nineteenth century in Polish schools[5].

The most important part of education, in the opinion of the Polish Commission of National Education, was teachers. According to the phrase, *the result of education will resemble the level of the teachers* (Popławski, 1780), the Commission of National Education has prepared a system for educating Polish teachers.

From 1780 those people who wanted to work in Polish secondary schools had to finish teachers seminars. The first Polish teachers seminar was opened in 1780 at the university in Cracow. Later, another seminars were opened – at universities in Vilnius and Warsaw[6]. Each teachers seminar lasted for 4 years and consisted of a one-year general pedagogical course and 3 years' university studies related to the main subject of the candidate's teachers course (Popławski, 1780).

Let's look at a schedule of 3 years' university studies in mathematics at Warsaw University in 1822 (Table 2.1). This is the framework of the 3-year part of the teachers seminar in mathematics.

[5] Certain regulations were also applied at the end of the 19th century and the beginning of the 20th century, an example here is delegating teachers to foreign scholarships in order to improve their skills.
[6] See: (Więsław, 2007, 110–114 and 256–276).



Plan Nauk całokursowy w Wydziale Filozoficznym Królewsko-Warszawskiego Uniwersytetu,

Dla sposobiących się na Magistra Filozofii w Oddziale *Matematycznym.*

A, Nauki główne.

1. *Algiebra wyższa.*
2. *Jeometrya Analityczna.*
3. *Rachunek differencyalny i integralny.*
4. *Jeometrya wykreślna teoryczna i praktyczna.*
5. ~~*Fizyka Matematyczna.*~~
6. *Mechanika Analityczna.*
7. *Astronomiia* ~~*elementarna.*~~
8. ~~*Mechanika niebieska.*~~

B. Nauki pomocnicze czyli styczne.

1. ~~*Encyklopedya filozoficzna.*~~ Filozofia fundamentalna.
2. ~~*Logika.*~~
3. ~~*Metafizyka.*~~
4. ~~*Estetyka.*~~
5. ~~*Historyia i literatura filozofii.*~~
6. *Fizyka ogólna i Mechanika elementarna.*
7. *Chemiia. (kurs jednoroczny).*
8. *Mineralogia i Krystallografiia.*
9. *Botanika. (kurs jednoroczny).*
10. *Zoologiia, ditto ditto.*

Uwaga nad porządkiem podług którego w ciągu trzyletniego Kursu mają być słuchane te nauki.

Main subjects[7]:

1. Higher algebra.
2. Analytic geometry.
3. Differential and integral calculus.
4. Descriptive geometry (theory and practice).
5. Mathematical Physics. (cancelled!)
6. Analytical Mechanics.
7. Astronomy.
8. Celestial mechanics. (cancelled!)

Associate subjects:

1. Philosophical encyclopedia. (cancelled!) Fundamental philosophy.
2. Logic. (cancelled!)
3. Metaphysics. (cancelled!)
4. Aesthetics. (cancelled!)
5. History and literature of philosophy. (cancelled!)
6. General physics and elementary mechanics.
7. Chemistry (one-year course).
8. Mineralogy and crystallography.
9. Botany (one-year course).
10. Zoology, *ditto ditto*[8].

Table 2.1: Teaching Plan at the Philosophy Department of the Royal Warsaw University for those who pursue a Master's degree in Philosophy at the Mathematical Department (1822)[9]

For example, let us discuss the program implemented during the lectures on differential and integral calculus. The then professor Adrian Krzyżanowski (1788–1852) during these lectures discussed the Taylor formula, power series, he determined the extrema of the functions of one variable and he introduced the basic theorems related to determination of integrals and their application. During the lectures he used Gottfried Leibniz's works (Więsław, 2007, 245). As historian says, mathematical studies at Warsaw University were at similar level of education to these at other European universities at that time (Duda, 2019, 115–117).

[7] On the whole-source Teaching Plan some subjects are cancelled, there is also one subject that was added in handwriting. This is probably a consequence of the fact that at that time, the teaching plans were prepared individually for each student (*Ustawy Kommissyi Edukacyi Narodowej dla Stanu Akademickiego i na szkoły w kraiach Rzeczypospolitey przepisane*, 1783, 16–17).

[8] *Ditto* is an Italian word that means "as above". *Ditto ditto* in this context should be understood as: (one-year course).

[9] State Archives in Łódź, sign. 39/ 592/0/2.1/734.

2.1.2 Prussian regulations

In Prussia teachers' education was finally settled at the beginning of the 19th century. In 1810 teachers examinations were introduced and they had to be passed after finishing university studies (Wiese, 1864, 545–547). Over the years, these ordinances have been improved, e.g. in 1831 the scope of material in mathematics which was applicable for the exam for mathematics and natural sciences teachers was determined: school arithmetic, geometry, plane trigonometry and algebra, higher mathematics, especially the application of mathematics to astronomy and physics (Kröger, 1837, 50–84).

2.1.3 Austrian regulations

In the Austrian empire in the first half of the 19th century the teachers of the highest grades of secondary schools, according to the regulations of 1786, were required to graduate from university studies, but in practice they were not often observed (Świeboda, 1984, 71–72). Teachers were employed on the basis of the so-called competition, which consisted of a written exam of subjects that the candidate was intended to teach. There were opinions that these exams often did not check the candidate's knowledge in a sufficient way, and did not check pedagogical knowledge and skills at all (Świeboda, 1984, 72). The first detailed regulations related to the teachers examinations in Austria were issued in the 1849. From then on, secondary school teachers had to graduate from university studies and pass a teacher's exam. These ordinances have been respected. In 1856, they were clarified (see: Seidl, Bonitz, Mozart, 1856, 673–686; Rakoczy-Pindor, 2012).

Since 1856 examinations could be passed in two subjects which the candidate intended to teach. Candidates had to prepare also a pedagogical, philosophical or didactical work at home on the topic given by the Examination Board (see: Rakoczy-Pindor, 2012, 164–167).

Requirements for mathematics teachers were as follows: higher algebra and number theory, elementary geometry, analytic geometry on the plane and in space, descriptive geometry, differential and integral calculus with applications, variational calculus, theory of functions (Rakoczy-Pindor, 2012, 165).

Austrian regulations were quite similar to Prussian. These regulations were also valid in Polish schools with the Polish language of instruction on the territory of the Austrian Partition.

2.2 Teachers characterization

In what way teachers on the territory of Poland under partitions can be characterized? Teachers in Polish schools from 1780, in schools under Prussian Partition from 1810 and in schools under Austrian Partition from 1849 can be characterized in a similar way. They published articles related to mathematics education, for example discussing arithmetic sequences of higher degrees[10], binomial theorem and properties of binomial coefficients,

[10] Definition of arithmetic sequences of the higher degrees (Koppe, 1869):
1. Sequence $(a_n^1)_{n\in N_+}$ is an arithmetic sequence of the first degree if it is a sequence of natural numbers.
2. Sequence $(a_n^2)_{n\in N_+}$ is an arithmetic sequence of the second degree if : $a_1^2 = 1$, $a_i^2 = a_i^1 + a_{i-1}^2$, $i \in N_{\geq 2}$.
3. Sequence $(a_n^3)_{n\in N_+}$ is an arithmetic sequence of the third degree if: $a_1^3 = 1$, $a_i^3 = a_i^2 + a_{i-1}^3$, $i \in N_{\geq 2}$.
4. Sequence $(a_n^k)_{n\in N_+}$ is an arithmetic sequence of the k^{th} ($k \in N_{\geq 2}$) degree if: $a_1^k = 1$, $a_i^k = a_i^k + a_{i-1}^k$, $i \in N_{\geq 2}$.

continued fractions and their applications (examples of teachers works related to Mathematics education: J. Słonimski, *O ułomkach ciągłych*, Kalisz, 1829; J. Piegsa, *Ein Beitrag zur Theorie der höheren arithmetischen Reihen*, Ostrowo, 1855). They published also school textbooks (examples of mathematical textbooks written by teachers: W. Karczewski, *Początki arytmetyki*, Kielce, 1822; E. Fassbender, *Anfangsgründe der beschreibenden Geometrie, der analytischen Geometrie, der Kegelschnitte und der einfachen Reihen*, Essen, 1860). Teachers very often conducted their own scientific work (examples of teachers scientific works: Z. Krygowski, *O pewnym zastosowaniu funkcji theta*, Przemyśl, 1890; K. Weierstrass, *Beitrag zur Theorie der Abel'schen Integrale*, Braunsberg, 1849). Sometimes they had Ph.D. degrees. In schools with Polish language of instruction in the second half of the 19th and at the beginning of the 20th century[11] it was about 7–8% of all teachers (Rakoczy-Pindor, 2012, 163). Teachers with Ph.D. degree in schools with German as a language of instruction can be roughly estimated at 30%[12]. Moreover, there were schools in which in some years the percentage of teachers with a doctorate was equal to 50%. For example this situation took place in Toruń Gymnasium in 1860/1861 school year – there were 20 teachers and a half of them had a Ph.D. degree, among them there were three teachers of mathematics: Eduard Fassbender (1816–1892), Rudolf Brohm (1807–after 1864), Hermann Rietze (1831–1862) (School reports, Thorn, 1861, 41–42).

There was one very important difference between Polish teachers and those who worked at schools with German as a language of instruction. The Polish ministries of education sent teachers abroad on scientific trips, for example to France, England and Prussia. During these trips teachers had to visit local schools and learn mathematical sciences at local universities[13].

It was a way to improve Polish teachers' education. After returning they were secondary school teachers during their whole educational career, for example[14]:

Ignacy Przybylski (1770–1838) – trip around Prussia; mathematics teacher in Płock and Kalisz.

Wincenty Wrześniowski (1800–1862) – trip around Europe; mathematics teacher in Radom and Warsaw.

August Bernhard (1804–1861) – trip around Europe; mathematics teacher in Piotrkowo and Warsaw.

Sometimes, after a few years in secondary schools, they were promoted to university teachers, for example[15]:

Franciszek Armiński (1789–1848) – visit to Paris; mathematics teacher in Warsaw; since 1816 astronomy professor at Warsaw University.

Adrian Krzyżanowski (1788–1852) – visit to Paris, mathematics teacher in Kielce and

[11] At that time, the most schools with Polish language of instruction were located in the Austrian Partition.

[12] In the Gymnasium in Trzemeszno in 1856–1862 people with doctoral degree accounted for approx. 27% of all teachers (School reports, Trzemeszno). In the Gymnasium in Braniewo in 187–1880, 1882–1912 and 1915, it was 27% (School reports, Braunsberg). In the Real Gymnasium in Bydgoszcz in 1888–1912 and 1915 the percentage of teachers with a doctorate was 34% (School reports, Realgymnasium zu Bromberg). In the Gymnasium in Bydgoszcz in 1856–1865 and 1869, 1871, 1873–1876, this mean amounted to 23% (School reports, Gymnasium zu Bromberg). In the Gymnasium and Real School (Real Gymnasium) in Toruń in 1860–1874, 1881, 1885, 1889–1892 and 1894–1900 it was about 41% (School reports, Thorn).

[13] See: (*Sprawozdanie…*, 1896, 6–7) and State Archives in Łódź, sign. 39/592/0/3.2/815.

[14] State Archives in Łódź, sign. 39/592/0/2.2/762, 327–328, 356–357, 148–149.

[15] Ibidem, 125–126, 275–278.

Warsaw; since 1821 mathematics professor at Warsaw University.

Kajetan Garbiński (1795–1847) – visit to Paris, since 1820 mathematics teacher at secondary schools in Warsaw and mathematics professor at Warsaw University.

At schools with German as a language of instruction mathematics teachers were also promoted to universities. I would like to mention three widely known names: Karl Weierstrass (1815–1897), Wilhelm Killing (1847–1923), who were teachers in Gymnasium in Braniewo and Martin Ohm (1792–1872) – teacher in Gymnasium in Toruń.

2.3 Conclusions

Teachers working in secondary schools preparing for Matura exams in the 19th and early 20th century had a very good professional education – university studies in the subject they taught. By publishing articles on didactics and preparing textbooks, the school tried to improve the level of teaching in secondary schools. On the other hand, they also took care of their scientific development. Often these were people with a PhD. In the Polish-language schools in the second half of the 19th century and at the beginning of the 20th century, people with a PhD constituted about 7-8% of all teachers, in German-language schools this percentage was greater – approx. 30%. Polish educational authorities, wanting to improve the education of teachers, decided on a solution that was not used in German-language schools – sending teachers to foreign internships. Teachers' scientific activity was often so advanced that they were appointed as academic teachers, where they also achieved significant successes. Mathematics teachers in schools on Polish lands during the partitions can be described as people who have a passion for mathematics. According to the statement *you have to burn in order to shine*, they had all predispositions to also light the passion for this subject in students. They were people with a predisposition to educate intellectual elites.

3 Mathematics teaching in real schools with Polish and German as languages of instruction

Now we are going to compare mathematics teaching in real schools on the territory of Poland under partitions. Real schools were those oriented on mathematics and natural sciences.

3.1 First real schools in Prussian, Russian and Austrian Partitions. First Polish real school

The first real schools were founded in Prussia in the 18th century. The so-called real school established by Christoph Semler in Halle in 1709 was considered to be the first school with a mathematical and natural profile (Heinen, 1863, 13).

The first Prussian ordinances regarding the organization of real schools, which were also valid for real schools opened in the Polish territories under the Prussian Partition, were issued on March 8, 1832 (Wiese, 1864, 27), but they have not yet been rigorously adhered to. After 1832, there are real schools in which the curricula were slightly different. This was the case, e.g., in the Real School in Krotoszyn (School reports, Krotoschin) and St. Peter School in Gdańsk (School reports, Danzig). Much influence on the teaching of mathematics was exerted by the Ordinance of October 6, 1859 (Der

Minister…, 1859) introducing the Matura exams in the selected types of real schools.

In 1864, there were 65 real schools operating in Prussia (Wiese, 1864, 46–48), including 10 in the Polish lands of the Prussian Partition (Wiese, 1864, 46–48), among them there were: St. John School in Gdańsk (Ger. *Johannisschule zu Danzig*), St. Peter School in Gdańsk (Ger. *Petrischule zu Danzig*) and real school in Toruń, Elbląg, Poznań, Międzyrzecz, Wschowa, Rawicz, Bydgoszcz and Grudziądz. Of these, only the real school in Grudziądz did not have the right to conduct Matura exams.

The first real school on the territory of Poland with Polish language of instruction was founded in 1840 under Russian Partition (ВЬДОМСТВО ПРОСВЬЩЕНІЯ…, 1868, 287). It was the Real Gymnasium in Warsaw. Polish students attended this school and the Polish language of instruction was used there. However, it cannot be called a Polish school. Both Poles and Russians influenced the curriculum in this school.

A special curriculum was prepared for the Real Gymnasium in Warsaw. It was not imitative in relation to curricula in other European countries. It was adapted to the local, Warsaw, industrial, commercial and agricultural needs (*Sprawozdanie urzędowe Dyrektora Gimnazjum Realnego Karola Frankowskiego z upłynnionych trzech lat szkolnych, czytane na akcie publicznym dnia 26 czerwca 1844 roku*, 391). In 1840, arithmetics (1st grade – 6 hours a week, 2nd grade – 6 hours, 3rd grade – 6 hours), elementary geometry applied to practice (3rd grade – 5 hours, 4th grade – 5 hours), algebra (4th grade – 4 hours, 5th grade – 3 hours), trigonometry (5th grade – 3 hours), descriptive geometry (5th grade – 5 hours, 6th grade – 4 hours) and conics (6th grade – 2 hours) was lectured in the Real Gymnasium in Warsaw. In the seventh grade, there was no mathematical theory and only and applications of mathematics in chemistry, construction, painting and machine construction (ВЬДОМСТВО ПРОСВЬЩЕНІЯ…, 1868, 301).

In 1845, the Russian Ministry of Public Enlightenment in Petersburg issued regulations concerning curricula in two types of schools in the Polish lands under Russian rule was: real schools (four grades) and higher real schools (six grades). (See: ВЬДОМСТВО ПРОСВЬЩЕНІЯ…, 1868, 343–457).

In Austria, including the Polish territories of the Austrian Partition, real schools were founded since the beginning of the 19th century, e.g. in 1817 there were founded Real School in Lviv (with German language of instruction) (*Bericht der Handels-…*, 1859, 136). But the first general regulations about organisation of the real schools were issued there only in 1849, under the name *Entwurf der Organisation der Gymnasien und Realschulen in Oesterreich* (*Sprawozdanie…*, 1885, 57; Ministerium…, 1849).

The first completely Polish real school was founded on February 19, 1856 in Lviv (*Sprawozdanie…*, 1885, 59). Initially, only the Polish rules of education were in force there. Since 1875 the freedom of Poles under Austrian Partition was limited and the curricula of this school and other Polish real schools had to be approved by occupant (*Sprawozdanie…*, 1885, 61). In 1869, two real schools functioned in the Polish territories under the Austrian annexation with a full seven-grade cycle of teaching: in Lviv and Krakow. There were also three real schools with a three-grade teaching cycle: in Tarnopol, Jarosław and Śniatyń (*Galizisches Provinzial-Handbuch für das jahr 1869*, 1869).

We will divide the analysis of mathematics teaching in real schools in Polish territories during the partitions of Poland into two periods: before 1868 and after 1868, that is before and after obtaining autonomy in teaching by the Polish territories under the Austrian Partition.

3.2 Mathematics teaching before 1868

Before 1868, most schools with Polish language of instruction were in the Russian Partition. The best real school in the Polish lands under the Russian Partition was then the Real Gymnasium in Warsaw. It was a school elite with an extremely wide curriculum. Higher real schools were schools of a more common nature. Thus, the analysis of the curricula in the real schools will allow to present a better, more popular image of mathematical education in the Polish territories under the Russian rule.

This part of the article will compare: ordinances regarding teaching mathematics in higher real schools in the Russian Partition (1845) (ВЬДОМСТВО ПРОСВЬЩЕНІЯ…, 1868, 349–457), ordinances regarding teaching mathematics in the Austrian Partition (1849) (Ministerium…, 1849) and mathematics curricula in two schools under Prussian rule – the Real School in Krotoszyn (School reports, Krotoschin) and St. Peter School in Gdańsk (School reports, Danzig). The analysis will cover the years 1849-1854. The starting data is conditioned by the issuance of Austrian ordinances regarding the functioning of real schools. The year 1854 was chosen as the ending date. It can be assumed that after 1854, the process of including Polish schools in the Russian education system was so advanced that there were no signs of Polishness in them anymore.

In 1849-1854, higher real schools in the Russian Partition, real schools in the Austrian Partition, the Real School in Krotoszyn and St. Peter School in Gdańsk were seven-grade schools. In Krotoszyn and Gdańsk, learning in some classes lasted for two years. The table below illustrates a weekly hour timetable for mathematics.

school		grade												number of hours in a week in the education cycle
		1st		**2nd**		**3rd**		**4th**		**5th**		**6th**		
Russian Partition		4		6		8		8		7		8		41
Austrian Partition		5		4		4		5		4		4		26
Prussian Partition	Krotoszyn	4		3		3		5		4	4	5	5	33
	Gdańsk	5		5		5		6	6	6		5		38

Table 3.1: Weekly hour timetable for mathematics in schools on the territory of Poland under partitions

Over half of the time reserved for mathematics classes in all these schools involved discussing arithmetic-algebraic issues. Most time to discuss these issues was devoted in the school in Krotoszyn – 82% of time, in Gdańsk – 74%, in higher real schools in Russia – 56%, and in real schools in Austria – 54%. In the three lowest classes, mainly four arithmetic operations were discussed on concrete and non-concrete numbers, on ordinary and decimal fractions, and the rule of three and its application. In three higher grades, the following was discussed: power, elements, logarithms, 1^{st}, 2^{nd}, 3^{rd} and 4^{th} degree equations, arithmetic and geometric progressions and their applications. In schools in the Russian and Austrian Partitions, as well as in St. Peter School in Gdańsk, additional exponential equations, combinations, Newton's binominal and probability theory and its application to calculating the expected life expectancy were discussed. In all schools, attention was paid to the use of arithmetic in everyday life – for calculating interest, discounts, pensions, retirement and calculations related to deposits and loans.

The remaining part of the curricula was filled with geometrical-trigonometric problems. The Austrian regulations from 1849 introduced geometry already in the lowest 1st grade in 2 hours a week. Discussing geometry started with the introduction of basic planimetric concepts. Stereometry appeared only in the highest 6th grade.

Regulations regarding higher real schools in the Polish territories under the Russian Partition introduced geometry a year later than in Austria – in the 2nd grade. The students first learned the basic planimetric concepts, stereometry was introduced in the 4th grade.

Geometry appeared at the latest in the curricula of schools under the Prussian rule. In Gdańsk, planimetry was usually introduced in the 3rd grade, the exception was year 1850/1851, in which education commenced in the 4th grade. Basic stereometric concepts appeared only in the 6th grade. In Krotoszyn, planimetry was always introduced in the 4th grade, the basic stereometric concepts were discussed a year later.

Geometry in the period under consideration was an important part of the curricula of schools under Austrian and Russian Partition. Almost half of all time devoted to teaching mathematics was devoted to discussing geometric issues. In schools under the Russian Partition, 18 hours week were dedicated to geometry in the education cycle, while at the school in Krotoszyn it was only 1/3 of that time.

When analysing the curricula included in school reports and relevant ministerial ordinances, it can be seen that all schools discussed similar planimetry issues, including, among others: equivalent conversion of figures, division of figures into a given number of parts with equal areas, similarity and congruence of figures, regular polygons, circles with theorems concerning chords and tangents, areas of figures, solving triangles. From the stereometric issues, the basic three dimensional figures were discussed with their areas and volumes were calculated. Conics and their properties were discussed in all schools.

Differences in the curricula were as follows:

1. In Krotoszyn and Gdańsk in 1850/1851 and 1852/1853 and in the Austrian Partition schools in 1849–1854 the spherical trigonometry was discussed. This issue was not present in the curricula of higher real schools under the Russian Partition. In addition, the applications of spherical trigonometry into the mathematical geography and astronomy were discussed in Krotoszyn and Gdańsk.
   In Krotoszyn in 1849–1851, gnomonics was additionally discussed – the construction of sundials and their setting. This issue was not found in other schools.
2. In Austria and Russia, the geometric constructions and their applications in geodesy was an important part of curricula – measuring instruments such as cords and measuring chains, a coal-casing and a measuring table were used and tasks, such as: find the distance between two places on the ground between which the river flows, were solved.
3. In the Russian Partition schools, 4 hours a week in the education cycle was exclusively devoted to the teaching of descriptive geometry. Among others, the tangent and normal to curved lines and curved surfaces were discussed. In 1853/1854, descriptive geometry appeared in the curriculum of the real school in Krotoszyn (5th grade). In the school report from Krotoszyn from this year, there was even an article of the teacher from this school about the need to introduce descriptive geometry to real schools and gymnasia. This is a preview of the upcoming changes in the curricula of Prussian schools – in 1859 descriptive geometry became an obligatory element of the teaching programs of real schools

conducting Matura exams (Der Minister der geistlichen, Unterrichts- und Medicinal-Angelegenheiten, 1859). In real schools in the Austrian Partition, descriptive geometry was not discussed.

After 1854, the russification of schools on the Polish territory under the Russian Partition was already very advanced. Schools were included in the Russian teaching system and the Polish language of instruction was removed. What was it like in the Prussian and Austrian Partitions? In the Prussian Partition, there were school with parallel grades with German and Polish as languages of instruction. Polish language of instruction was applicable only in the lower grades. Two highest grades were German-speaking. However, schools with two languages of instruction were rare. One of such schools was the Real School in Poznań. Comparing the 1857 curricula there from both types of grades, it can be observed that they were almost identical the Prussian teaching system was in force (School reports, Posen, 1857). Polish youth, even if it was taught in Polish, it was done according to the Prussian rules.

In 1856, the first completely Polish real school was created (*Sprawozdanie…*, 1885, 59). It was established in Lviv – in the Austrian Partition. After acquiring autonomy in teaching by the lands of the Austrian Partition (1868), other Polish schools joined it. In 1875, the freedom of Poles was limited and all decisions had to be approved by the invader.

3.3 Mathematics teaching after 1868

3.3.1 Austrian autonomy (1868–1875)

Only a few school reports have survived from the period of "Polishness" to the present day. It is hard to find materials from the 1868–1875 period regarding the teaching of mathematics in the three types of schools that we are interested in: Austrian Partition schools, Austrian schools under the Austrian Partition and Prussian Partition schools which would survive to this day. Nevertheless, it was possible to do so. Higher Real School in Lviv (Austrian Partition) from 1874 (School reports, Lwów, 1874), Higher Real School in Bielsko (Austrian Silesia) from 1874–1877 (School programs, Bielitz, 1875, 1877), Real School in Toruń from 1873–1874 (School reports, Thorn, 1873-1874) and Real School in Poznań (Prussian Partition) from 1874/1875 (School reports, Posen, 1875) were selected for further analysis. The table below presents the curricula implemented in these schools.

school	Higher Real School in Lviv		Higher Real School in Bielsko		Real School in Toruń		Real school in Poznań	
grade	curriculum	hours/week	curriculum	hours/week	curriculum	hours/week	curriculum	hours/week
1st	Arithmetic operations on integer numbers and decimal fractions. Divisibility and prime factorization. Greatest common divisor and least common multiple. Common fractions. Introduction to geometry: two- and three-dimensional figures, perspective, curves constructions.	8	Decimal system. Arithmetic operations on denominate and abstract numbers. Divisibility. Greatest common divisor and least common multiple. Common fractions and decimal fractions. Introduction to geometry: basic planimetric figures with their drawings – triangle, square, quadrangle, regular hexagon, circle and their combinations.	9	Arithmetic operations on denominate and abstract integers.	6	Arithmetic operations on denominate numbers and common fractions.	4
2nd	Ratios and proportions. The Rule of Three and its application. Percentages. Congruence and similarity of triangles. Curves: circle, ellipse, hiperbola, parabola, cykloid, and helical curves. Solids.	7	Ratios and proportions. Percentages and their applications. Calculating the area of plane figures. Drawings using the drafting tool, protractor and set square.	6	Arithmetic operations on denominate integers. Fractions. Counting in memory.	4	Arithmetic operations on common fractions. The Rule of Three and its application. Introduction to geometry: two- and three-dimensional figures, circle, sphere, cylinder, cone.	4
3rd	Percentages and their applications. Second powers and square roots. Third powers and cube roots. Congruence and similarity of	7	Percentages and their applications. Second powers and square roots. Third powers and cube roots. Arithmetic operations on algebraic expressions.	6	Arithmetic operations on common and decimal fractions. The Rule of Three.	3	Arithmetic operations on decimal fractions. The Rule of Three. Basic planimetric figures, triangles and	3

	polygons. Plane and three-dimensional geometrical constructions.		Basics of stereometry. Technical drawings.		Counting in memory.		parallelograms, construction tasks.	
4th	Arithmetic operations on algebraic expressions. Greatest common divisor and least common multiple. Common fractions. Equations of the first degree with one and two unknowns. Calculating the area of figures and volume of solids (with practical application). Converting a given polygon into another one in an equivalent way. Curves constructions. Geometry application to geodesy, measuring the area of the ground using measuring instruments. Orthogonal projection.	6	Arithmetic operations on algebraic expressions. Greatest common divisor and least common multiple. Common fractions. Equations of the first degree with one and two unknowns. Application of arithmetic operations on algebraic expressions to solving planimetric and stereometric tasks. Curves. Introduction to descriptive geometry.	7	Introduction to planimetry up to parallelogram and trapezium. Compound Rule of Three. Percentages. Decimal fractions. Tradesmen calculations.	6	Percentages and their applications. Powers. Arithmetic operations on algebraic expressions. Introduction of formulas: $(a+b)^2$, $(a-b)^2$, $(a+b)(a-b)$. Systems of equations. Parallelogram. Circle, chords and tangents. Tangent circles. Regular polygons. Equality of figures. Pythagorean theorem.	6
5th	Arithmetic operations. Divisibility of numbers. Fractions. Proportions and their application to tradesmen calculations. Powers. Roots. Logarithms.	8	Equations of the first degree with more than two unknowns. Diophantine equations. Number systems, especially the decimal system. Decimal fractions. Powers and roots. Complex	9	Geometry, especially theory of similarity and construction tasks. Arithmetic operations on	6	Powers. First degree equations. Everyday calculations. Similarity of figures. Calculating the area of plane figures. Area of the	6

	First and second degrees equations. Discussion of further planimetric problems. Descriptive geometry[16]: projections of a point, straight line and plane and their relations, projections of three-dimensional solids; sections of solids.		numbers and four arithmetic operations on complex numbers. Ratios and proportions. Quadratic equations with one and two unknowns. Construction tasks. Descriptive geometry: projections of a point, straight line and plane; projections of three-dimensional solids; sections of solids.		expressions using letters. First degree equations with one and two unknowns. Square and cube roots.		circe.	
6th	Equations of higher degrees, which can be reduced to the second degree equations. Continued fractions. Arithmetic and geometric progressions and their applications to pensions calculatings. Combinations, Newton's Theorem. Descriptive geometry: Curve lines, i.e. an ellipse, a hyperbola and a parabola; tangent planes to surfaces; surfaces and planes intersections.	8	Logarithms. Equations of higher degrees, which can be reduced to the second degree equations. Exponential equations. Arithmetic and geometric progressions and their applications to pensions calculatings. Series. Combinations, Newton's Theorem. Plane trygonometry. Calculating the area and volume of solids. Spherical trygonometry. Descriptive geometry: Curve lines; tangent planes to surfaces; surfaces and planes intersections;	8	Discussion of further planimetric problems. Trigonometry. Quadratic equations. Powers, roots and logarithms. Arithmetic and geometric progressions, calculation of interest and pensions.	6	Second degree equations. Roots. Medians and heights of triangles and their properties. Figures inscribed in a circle. Introduction to stereometry.	5

[16] Descriptive geometry in 5th and 6th grades was taught as a separate subject within 3 hours a week.

			learning about shadows.					
7th	Polynomial equations. Second and third degrees equations. Probability theory. Series. Spherical trigonometry and its applications to astronomy and stereometry. Analytic geometry. Geometric drawings: intersections of curve surfaces; tangent planes to curve surfaces.	8	Probability theory with application to calculate the expected life time. Continued fractions. Arithmetic sequences of the higher degrees. Curve lines, i.e. ellipse, hyperbola and parabola. Tangent planes to surfaces. Surfaces and planes intersections. Conics. Descriptive geometry: Perspective. Repetition of the material.	8	Descriptive geometry. Analytic geometry. Arithmetic sequences of higher degrees. Equations of higher degrees. Permutations, combinations, variations and binomial theorem. Exponential and logarithmic series.	6	Roots and powers with fractional exponents. Logarithms. First and second degree equations with one or more unknowns. Arithmetic and geometric progressions. Interest and pensions calculations. Calculating the area of plane figures. Area of the circe. Spherical trygonometry.	5
8th							Equations of the second degree with one or more unknowns. Calculation of interest. Permutations, combinations, variations. Figural numbers. Arithmetic sequences of higher degrees. Third degree equations. Plane and spherical trigonometry. Tasks in mathematical geography and astronomy.	5

Table 3.2: Mathematics curricula in Higher Real School in Lviv, Higher Real School in Bielsko, Real School in Toruń and Real School in Poznań

The comparative analysis of the above curricula allows to see the following differences:

1. Certain issues in different schools were introduced at various stages of education. For example, the logarithms in Lviv were discussed in the 5th grade, in Bielsko and Toruń in the 6th grade, in Poznań – in the 7th grade; Combinations and Newton's binominal in Bielsko and in Lviv were discussed in the 6th grade, in Toruń in the 7th grade, and in Poznań in the 8th grade.
2. One of the most important differences between the above curricula was the moment of introducing basic geometrical objects to school teaching. Basics of geometry in the Higher Real School in Lviv were already introduced in the lowest 1st grade and they included the simultaneous introducing planimetric and stereometric concepts. In Bielsko, geometric concepts were also introduced in the 1st class, with the fact that they only included planimetry. Stereometry in this school appeared only in the 3rd grade. In Poznań, introduction to geometry was planned for the 2nd grade and, just like in Lviv, the planimetric and stereometric concepts were introduced at the same time. In the Real School in Toruń, geometry was introduced in the 4th grade and only the basic planimetric concepts were discussed then. Stereometry has not been included in the above curriculum of the Real School in Toruń, but undoubtedly it was discussed there. Most probably, stereometry was discussed only in the highest grade of the Real School in Toruń (School reports, Thorn, 1873). In schools located in Prussia, also on the territories of Poland under Prussian control, the introduction of geometry was moved to the lowest grade of secondary schools at the beginning of the 20th century – as a result of the reform called Merano Programme (1905). Merano Programme managed the simultaneous introduction of planimetric and stereometric concepts.
3. There are also issues that are found in the curricula of some schools, and not in others. Usually, the lack of mentioning certain issues in curricula means that they were not taught at school. However, this is not always the case. Let us note here the applications of geometry in geodesy, spherical trigonometry and construction tasks.
   a) In the 5th grade in Lviv there are applications of geometry for solving measurement tasks. This issue is not in Bielsko, Toruń and Poznań.
      The application of geometry to geodesy in Higher Real School in Lviv was taught in such a way: teachers took students to forests or meadows and using measuring cords, goniometers and measuring tables, they solved tasks such as finding a distance between two places on the ground between which a mountain was. During solving tasks, they had to use theorems related to congruent and similar triangles. This issue wasn't mentioned in Bielsko, Toruń and Poznań curricula, but it can be supposed that geometry application to geodesy was taught there. This application was discussed in the majority of Prussian and Austrian schools, for example, it was taught by Karl Weierstrass in the Lower Gymnasium (Ger. *Progymnasium*) in Wałcz in the 1840s (School reports, Deutsch-Crone, 1843–1848; Grunert, 1851).
   b) Spherical trigonometry in Bielsko was introduced in the 6th grade, in Lviv and Poznań in the 7th grade. While spherical trigonometry is not found in the curriculum of the Real School in Toruń.

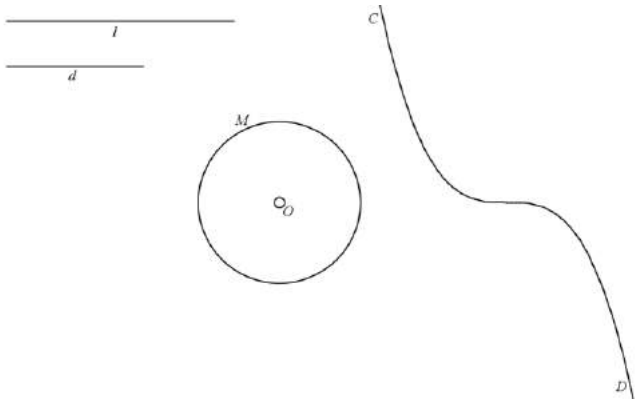
In 1874, Eduard Fassbender was the mathematics teacher of higher grades in the Real School in Toruń. Fassbender published the article *Die Kopernikanischen Sehnen- und Dreieckberechnungen* in 1872 (Fassbender, 1872). This article was a faithful representation of the three chapters of the first book of the *Nicolai Copernici Torinensis De Revolutionibus orbium coelestium, Libri VI,* which the astronomer devoted to the plane and spherical trigonometry. Fassbender, in his article, informed that students of the Toruń school were acquainted with the "world system" of Copernicus, and extensive fragments of the astronomer's work, after adapting to the knowledge and skills of students, were placed in school textbooks. Due to the fact that Copernican plane and spherical trigonometry constituted the basis for all the considerations regarding the structure of the universe, the Toruń school paid attention to them. Fassbender's article, constituting a compendium of Copernicus' geometry, was only a form of systematizing knowledge for students. Thus, although the 1874 curriculum did not mention spherical trigonometry, it was taught there. In the Real School in Toruń, spherical trigonometry was taught according to Copernicus, meaning solving the spherical triangles.

c) In Lviv, Bielsko and Poznań, great attention was paid to geometric constructions. The curriculum implemented in Toruń does not have a clear emphasis on this type of tasks. However, from an article *Beiträge für den Unterricht in der Geometrie* written in 1866 by one of the teachers from Toruń, Otto Reichel, it is known that great importance was attached to the construction there (Reichel, 1866). Reichel in his article placed a list of 27 construction tasks found in the school register. These tasks were not in school textbooks. They were independently prepared by teachers to improve the level of geometry teaching and to prepare young people better for university studies. Among them was the following task:

Exercise (Reichel, 1866, 14)[17]. A circle *M* and a curve *CD* are given. Construct a secant *ABP* of a given length l such that the length of the outer part of this secant is given and equal to *d* and its endpoint *P* is on the curve *CD*.

Solution

Data:



[17] This exercise was not solved by Otto Reichel. Solution of this exercise was prepared by the author of this article.

Sketch of the construction:

1. Construct an arbitrary chord AB of the circle M with a length equal to $(l-d)$.
2. Construct a circle Q with the center O tangent to AB.
3. Extend the chord AB by the segment of the length equal to d. As a result we receive the segment AF.
4. Construct a circle Q' with the center O and the radius OF.
5. The point of intersection of Q' and CD is the end of secant that we are looking for.
6. Construct the secant. Secant should be tangent to the circle Q.

Conditions under which the construction is possible to made:

1. $r \geq \frac{l-d}{2}$,
2. $Q'(O, \sqrt{r^2+ld}\,) \cap CD \neq \emptyset$,

where r is the radius of the given circle M, $Q'(O, \sqrt{r^2+ld}\,)$ is the circle with the center O and the radius $\sqrt{r^2+ld}$ and CD is a given curve.

4. Descriptive geometry was one of the mathematical problems, on which very high emphasis was placed in some schools, and was omitted in others altogether. Descriptive geometry was a very important issue in Higher Real School in Bielsko and Higher Real School in Lviv. In Bielsko it was taught as a separate subject in three highest grades, within 3 hours a week. In Lviv – in two grades: 5th and 6th, within 3 hours a week too. The 1874 report of the school in Toruń included descriptive geometry only as a slogan in the curriculum for the 7th grade. There are no written descriptions of the descriptive geometry discussed during classes. As it was mentioned before, Eduard Fassbender was the teacher in Toruń at that time. Fassbender in 1857 wrote another article *Abriss einer Einleitung in die beschreibende Geometrie* (Fassbender, 1857), in which he mentioned that descriptive geometry was taught by him in Real School in Toruń since 1855 and the lecture in this subject contained all issues discussed in this article. This issues were: the slope of the line to projection planes, calculating the length of a straight line contained between two given points (in other words: calculating the length of the segment), the mutual position of two straight lines and angles between them, the mutual position of a straight line and a plane and the mutual position of two planes. We can see that the descriptive geometry lecture in Real School in Toruń at that time was quite basic. Probably the same issues were discussed in 1874. In Lviv there were taught additionally for example: projections of solids and tangents to curve surfaces.

   Descriptive geometry was an important part of teaching mathematics in Bielsko and Lviv, it was also taught in Toruń, while it was not included in the school curriculum in Poznań.
5. Other issues that are included in the curricula of some schools and were omitted in others include:
   a) In Bielsko, in the 5th grade, complex numbers and arithmetic operations on these numbers are discussed. This issue is not found in Lviv, Toruń and Poznań, but certainly complex numbers were taught there – in school teaching

in the 2nd half of the 19th century, equations were always solved in complex numbers.

b) The exponential equation was discussed in Bielsko in the 6th grade. This was not in Lviv, Toruń and Poznań.
c) The series were discussed in Bielsko in the 6th grade. In Lviv and Toruń, they were introduced in the 7th grade. They are absent in the curriculum implemented in Poznań.
d) Analytical and conical geometry are discussed in Lviv, Bielsko and Toruń. These issues are not in the curricula of the school in Poznań.

The above analysis allows to note discrepancies in the teaching of mathematics carried out in the Polish Higher Real School in Lviv, the Austrian Higher Real School in Bielsko, and schools from the Prussian Partition: Real School in Toruń and Real School in Poznań. The method of teaching mathematics in a Polish school was not imitative in relation to the teaching of this subject in Austrian schools[18] and Prussian schools. The curricula differed, but so did the methods and time of introducing of certain concepts, e.g. in Lviv greater attention was paid to teaching geometry, it was already introduced in the lowest grade, in a visual way – the analysis of stereometric objects enabled the introduction of planimetric objects. Such a solution was not observed in other schools. Although geometry was also introduced in Poznań in an illustrative manner, but it was done a year later – in the 2nd grade.

3.3.2 Mathematics teaching after 1875

After 1875, mathematics curricula in schools with the Polish language of instruction in the Austrian Partition did not change much. The hourly timetables and mathematics curricula in individual classes were almost identical. Comparing curricula from the Higher Real School in Lviv from 1874 (School reports, Lwów) to the curricula for teaching mathematics at the Higher Real School in Krakow in 1876 (School reports, Kraków, 1876), it can be observed that the only significant difference concerned the introduction of geometry. The Lviv illustrative introduction to geometry in the 1st grade was changed in Krakow to introduction of planimetry in the 1st grade, and stereometry only in the 2nd grade.

After 1875, differences between teaching mathematics in Polish-speaking schools under Austrian rule and Austrian German-speaking schools were already small and usually concerned slight shifting of certain issues to the curricula of earlier or higher grades. When analysing teaching mathematics in the Higher Real School in Krakow and Higher Real School in Bielsko to the beginning of the 20th century (School reports, Kraków; School reports, Bielitz), it can be observed that certain issues were added to curricula in the school in Krakow, e.g. in 1899 rational and irrational numbers and exponential equations were added to the curriculum of the 5th grade, and in 1904 the logarithmic equations. The school curriculum in Bielsko actually remained unchanged –

[18] Bielsko, in the period under consideration, was part of the Austrian Silesia. The curriculum in the Higher Real School in Bielitz listed in the table was based on the Austrian regulations of July 19, 1870, August 9, 1873 and Silesian ordinances of November 5, 1874 (these ordinances did not apply to the teaching of mathematics) (School reports, Bielitz, 1875). The curricula in the Real School in Lviv differed from the curricula implemented in Bielsko. Curricula in the Real School in Lviv were also not in accordance with the Austrian regulations of 1849.

no new issues were added to it. In 1899, in both schools the method of introduction to geometry was unified – geometry was introduced in the 1st grade in the demonstrative manner: from stereometry to planimetry.

In schools under Prussian rule, program changes were also small. In 1884/1885, analytical geometry, cones and series were added to the curricula of the Real School in Poznań (School reports, Posen). In Toruń, in 1899/1900 minima and maxima were added to the curriculum of the highest grade (School reports, Thorn). The remaining changes were cosmetic.

The way of introducing geometry to school education and the scope of its discussion was still the basic difference in curricula. In schools under Prussian rule, descriptive geometry was discussed symbolically or was not discussed at all, whereas in schools under Austrian rule, descriptive geometry was taught as a separate subject 3 hours a week.

3.4 Conclusions

In 1849-1854, higher real schools, in which Polish youth educated, were mainly in the Russian Partition. 41 hours of mathematics a week was planned in them in the whole cycle of education. It was the largest number of math hours out of all real schools at that time in the Polish lands. While in the Prussian Partition schools the vast majority of time was devoted to discussing arithmetic-algebraic issues, in schools with the Polish language of instruction under Russian Partition and schools in the Austrian Partition, arithmetic-algebraic and geometric-trigonometric issues were treated equally – almost the same amount of time was devoted to their discussion. In curricula of schools located in all partitions, a lot of attention was paid to the use of mathematics, especially to perform the so-called civic settlements, i.e. calculation of interest, profit, loss, pension, etc. One of the basic applications of geometry was to solve measurement tasks. In schools under the Prussian and Austrian rule, spherical trigonometry and its applications to mathematical geography and astronomy were discussed; in Krotoszyn, the construction of sundials was even performed – these issues were not in Polish-language schools under the Russian Partition. On the other hand, the Polish-speaking schools paid a lot of attention to the descriptive geometry needed by future architects and constructors. Let us recall that descriptive geometry was not in schools under Austrian rule at that time, it was also rate in schools under Prussian rule.

The second important period from the point of view of the Polish education in Polish territories during the partitions was in the years 1868-1875. At that time, Polish schools were under the Austrian rule. Here, the difference in the teaching method is clear. In Polish schools, the geometric concepts in the lowest 1st grade were introduced in an illustrative manner – the abstract planimetric concepts were introduced based on stereometric concepts known to students from everyday life[19]. A similar solution was used in Poznań, but it was not a systemic solution, but a unitary solution. In Austrian schools, demonstrative teaching of geometry was introduced in the 1890s. In Prussian schools – only at the beginning of the 20th century. Polish schools paid a lot of attention to mathematical theory. They discussed most of these issues provided for teaching in

[19] At the end of the 18th century, school textbooks on illustrative geometry were published. A very popular textbook in the Austrian Partition, used, e.g. in the Nowodworski School in Krakow, was the textbook: F. Močnik, *Geometria poglądowa dla klas niższych szkół średnich*, Lwów, 1896.

secondary schools in the 19th century, together with analytical geometry and cones, which in Poznań were introduced in 1884/1885. In 1868-1875, in schools of all partitions, the concern to show the applications of mathematics to civic settlements continued, as well as to solving measurement tasks, along with the discussion of spherical trigonometry with its applications. A lot of attention was paid to structural tasks. In Polish and Austrian schools, descriptive geometry was a very important part of the curriculum.

When analysing the teaching of mathematics in Polish territories during the partitions, one can assume that the success of Polish-language schools under Russian Partition, and then Polish schools under the Austrian Partition, paying a lot of attention to teaching geometry was a success (these schools were the leaders in teaching descriptive geometry). Looking in general terms on the 19th century, schools for Polish students paid more attention to teaching geometry than in the German-speaking schools of the countries occupying Poland, that is, in Austria and Prussia.

It is worth mentioning there that geometry was a field of mathematics, which has been particularly valued by didactics for centuries. Samuel Dickstein spoke about geometry as an indispensable part of the school and home educational system. In geometry he saw the science that develops the mind of a young man, forces him to look for connections between different geometric forms, and *abstract, seemingly dry and non-absorbing claims, with a skilful lecture, become alive, arouse curiosity and prepare the mind for self-use of own strength* (Dickstein, 1889, 67). According to this statement, the construction tasks and descriptive geometry taught young people independent and logical thinking. Learning of mathematics by action – the self-made constructions – also arouses interest in the subject and helps to increase motivation for learning.

4 Mathematics teaching in gymnasia with Polish and German as languages of instruction

Now we are going to analyse mathematics teaching in gymnasia with Polish and German as languages of instruction.

The best way of checking both: the level of mathematics teaching and the most important topics in mathematics curricula, is to analyse the then secondary school-leaving examinations (Matura examinations) in mathematics. For almost the entire period which is took into consideration in this article Matura examinations tasks were prepared by the teachers of the schools in which these examinations were conducted[20]. It means that each school had an individually prepared set of examination tasks.

Regulations introducing the world's first examinations at the end of secondary schools enabling university studies were introduced in 1788 in Prussia. According to these regulations, mathematics wasn't an obligatory subject of Matura examinations (Domoradzki & Karpińska, 2017).

First Polish regulations introducing Matura examinations were published on February 17, 1812 (Potocki, 1812). A few months later, on June 25, 1812, the second Prussian Matura regulations were introduced. Much later, Matura exams were introduced in other European countries, e.g. in Austria these examinations were introduced in 1849, and in France in 1852 (Majchrowicz, 1869, 17).

[20] The sets of tasks had to be approved by the Chairman of the Examining Board, who was a member of a national body supervising schools.

4.1.1 Polish and Prussian Matura regulations from 1812

First Polish Matura regulations were introduced by the Directorate of National Education of the Duchy of Warsaw on February 17, 1812 (Potocki, 1812). From now on, only people with the Matura exam certificate could undertake studies at Polish universities. Polish regulations established mathematics as an obligatory subject of oral Matura examinations for all students. Moreover, mathematics was an obligatory subject of written Matura examinations in case of those students who wanted to study architecture. The introduction of Matura examinations required the unification of curricula in those schools in which Matura exams were carried. In the Polish Duchy of Warsaw, the framework curricula was set up in the 1812 regulations (Potocki, 1812).

Prussian Matura regulations from 25th of June 1812 established mathematics as an obligatory subject of every written and oral Matura examination for all students (Wiese, 1864, 484–485). These regulations set the requirements for secondary school graduates, but the curricula in individual classes were done a bit later as a consequence of the Humboldt reform.

It shows us that mathematics was as important a part of the Polish education as it was in Prussia.

Poland (Potocki, 1812)	Prussia (Wiese, 1864, 485)
• first, second and higher degrees equations, • logarithms, powers, roots, • planimetry and stereometry, • plane and spherical trigonometry, • application of mathematics to everyday life calculations, application of mathematics to physics and geography.	• first and second degree equations, • logarithms, powers, roots, • planimetry and stereometry, • plane trigonometry, math tables, • application of mathematics to everyday life calculations.

Table 4.1: 1812 requirements to the mathematics Matura examination

The scope of the material obligatory for the Matura exam in mathematics in Poland was broader than the scope obligatory in Prussia with the following issues: higher degrees equations, spherical trigonometry and application of mathematics to physics and geography. This allows to assume that Polish schools preparing for Matura exams have implemented wider curricula than in Prussian schools, including schools located in Polish territories under Prussian rule.

The quoted curriculum of the two highest grades of the Polish Warsaw Secondary School (Pol. *Liceum Warszawskie*) from 1812 (this year, the first Matura exam took place there) (School reports, Warszawa):

> 5th grade: The largest common divisor and the smallest common multiple, roots of the second and third degree, equations higher than the 2nd degree, stereometry, spherical trigonometry with the application to astronomy.
>
> 6th grade: Combinations, Newton's binominal, 3rd degree equations using to the division of the circle into 3, 5, 7 etc. equal parts, solving the 4th level equations with the discussion of the Bombelli and Euler rules, solving equations of any degrees by approximation, conics and their applications in optics, astronomy, civil and military architecture, etc., basics of descriptive geometry according to G. Monge.

The mathematics curriculum at the Warsaw Secondary School was very extensive, as for the school with a humanistic profile. The applications of 3rd degree equations to dividing the circle into 3, 5, 7 etc. equal parts or conics applications in optics, civil and military architecture, were not even discussed at the real schools established at a later time. Also descriptive geometry was rarely taught in humanistic schools. The establishment of real schools meant that in the humanistic schools, mathematics curricula and the scope of issues applicable at the Matura exams have been limited over the years.

After 1812, both Polish and Prussian, new Matura regulations were issued, which did not change the general organization of Matura exams, but refined, among others, examination subjects, the scope of the Matura material and the duration of exams. The regulations issued in Poland in 1819 were of particular importance, under which mathematics became a compulsory examination subject for all students (Potocki, 1867, 473–475). In Prussia in 1834, ordinances were issued according to which the written examination in mathematics was to last 4 hours and consisted of two arithmetic and two geometrical tasks (Wiese, 1864, 492–504). Further regulations were introduced, among others, in 1859, 1866, 1874.

4.1.2 Matura examinations under Austrian Partition

In Austria, the Matura exams were introduced in 1849. In the same year, they were introduced in schools located in the Polish lands under the Austrian Partition. According to the ordinances, mathematics was a compulsory subject of written and oral Matura examinations. These regulations were very generally related to the Matura requirements. In the case of mathematics, it was assumed that by taking part in the Matura exam, the student was to know all the mathematical theorems discussed at school along with their proofs and all the tasks that were solved during the classes. During the Matura exam, students were required to apply knowledge acquired at school to solve new problems or to prove theorems that were not discussed at school. During the written exam in mathematics, it was recommended to solve tasks in the field of planimetry and trigonometry, and in particular tasks that at the same time checked the geometric, arithmetic and trigonometric knowledge were recommended (Ministerium…, 1849, 195–196). In the 1850s and 1860s, a series of successive Austrian Matura ordinances was issued, which, among others, increased the number of subjects of Matura examinations and clarified the scope of the material used during the exams (Matauschek, 1864, 125–130). After obtaining the autonomy by the territories under the Austrian annexation, the Polish Ministry of Education modified the general regulations of the Matura examinations. At the beginning of the 80s, the number of subjects passed during the oral Matura examinations was limited in Polish schools to five: Latin, Greek, Polish, German and mathematics (*Sprawozdanie…*, 1885, 172).

4.2 Matura tasks in mathematics

In schools under the Austrian Partition, Matura exams were introduced in 1849, which is why further comparative analysis will cover the second half of the 19th century. When preparing this article, it was impossible to find mathematics Matura exams conducted in schools with the Polish language of instruction under the Russian rule. Thus, the comparative analysis will cover the Matura exams from the Austrian and Prussian Partitions.

We will begin the analysis with a discussion of written mathematics Matura exams conducted on the Polish territory under the Austrian Partition before it was granted autonomy.

In 1864, in the Gymnasium in Przemyśl, which was then a German-language school included in the Austrian education system, the written Matura exam was as follows[21]:

Exercise 1. Solve the equation: $3\sqrt{12-\sqrt{x-3}}=\sqrt{5x+21}$.

Exercise 2. Two forces P=37˙5 N and Q=89˙23 N are directed towards each other at an angle of $72°43'21\dot{.}7''$. What is their resultant force?

Exercise 3. A certain capital bears the 4% interest. How will it increase after 20 years of interest capitalization?

Examples of oral exercises were as follows[22]:

Exercise. We know the height and the bases of a truncated pyramid, calculate the volume of this solid.

Exercise. Calculate the surface of the circle.

It can be seen that apart from the mathematical theory, much attention was paid to the applications of mathematics in Przemyśl. Two Matura exam tasks are related to the applications: one task concerns the use of trigonometry in physics, and the second the applications of arithmetics to the accounts related to deposits. The purely theoretical task of the written exam is to solve the equation. It can be said that during the written exam, one of the basic ministerial orders has not been implemented here – paying attention to geometry. This postulate was implemented only at the oral exam.

In 1875, after 8 years of autonomy, the freedom of Poles under Austrian occupation was again limited, and curricula in schools with Polish language of instruction were similar to those implemented in Austrian schools (this was previously shown on the example of real schools). And what about mathematics Matura exams? It has been described above that in the 1880s, new Matura regulations concerning the Matura exams in the Polish schools under Austrian Partition were implemented. Did they introduce any change to the mathematics Matura exams?

In 1889, the Gymnasium in Przemyśl was already a Polish-language school. The mathematics Matura exam looked as follows (School reports, Przemyśl):

Exercise 1. Solve the equations:

$$\frac{\sqrt{3x}}{x+y}+\frac{\sqrt{x+y}}{3x}=2$$
$$xy-(x+y)=54$$

Exercise 2. Someone is entitled to a pension of PLN 800 annually for 20 years; however, he wants to convert it into another pension of PLN 1000 a year; how long will he collect this second pension if the rate is 4%.

Exercise 3. Calculate the volume of the scalene cone which shortest side $b=17^m$ is inclined to the base at an angle $\alpha=82°54'30''$ and the longest side a is inclined to the base at an angle $\beta=35°40'20''$.

[21] State Archives in Przemyśl, sign. 56/387/0/1.1/19.
[22] State Archives in Przemyśl, sign. 56/387/0/1.1/19.

Example of tasks from the oral exam[23]:

Exercise 1. Calculate the length of the string from the top of the major axis at an angle of 30° in the ellipsis.

Exercise 2. Solve the triangle if there is a given circumference and two angles.

As before 1864, there were three tasks in the written exam in the Gymnasium in Przemyśl. Two of them were arithmetic-algebraic, and one – geometric-trigonometric. During the written exam, more attention was paid to arithmetic-algebraic problems, while during the oral exams – to geometric-trigonometric problems. The tasks of mathematics applications in everyday life were still important.

The level of difficulty of tasks, in relations to those from 1864, definitely increased. In 1864, the Matura tasks were even at the elementary level, these were schematic tasks, which were certainly discussed during maths lessons at school. In 1886, the level of task difficulty is higher, and the tasks, especially from the oral exam, require logical, independent thinking and multi-stage reasoning.

Cracow St. Anna Gymnasium[24]	**Inowrocław Gymnasium** (School reports, Inowroclaw)
Exercise 1. Solve the equation: $\sqrt{\frac{3}{2}x-5}-\frac{1}{7}\sqrt{\frac{1}{5}x+45}=\frac{1}{4}\sqrt{10x+56}$ **Exercise 2.** The population of the city, in which there were 32500 inhabitants, rose after 24 years by 35566 souls. What is the annual percentage of growth? **Exercise 3.** Find an angle between the horizontal ground and the road leading up, if for a length of 10 fathoms (Pol. *sążeń*) on the road the height above the horizontal ground increases by one fathom.	**Exercise 1.** 6 numbers are given. The first 4 numbers of those 6 numbers creates a geometric progression and the last 4 numbers creates an arithmetic progression, such that the common difference of arithmetic progression is 12 times greater than the common ratio of the geometric progression. Moreover, the second last number is 15 times bigger than the second number. Find these numbers. **Exercise 2.** Construct a right triangle, if the sum of the hypotenuse and one of its legs is given and equal to a and the sum of the hypotenuse and the second leg of this triangle is given and equal to b. **Exercise 3.** Someone has a cash, which not exceed 350 thalers. When he distributes this cash into a sequence of 10 thalers, then he is left with 1 thaler. When he distributes this cash into a sequence of 15 thalers, then he lacks 4 thalers. Calculate how much money this person has, knowing that all cash can be completely distributed into 11 terms sequence. **Exercise 4.** Truncated cone with the radii of the bases equal to $r = 5'$ and $\rho = 3'$ and the side inclined to one of its bases at an angle $\varphi = 72°45'36''$, is given. On this cone a sphere is described. Calculate the area of the sphere bounded by the peripheries of the cone bases.

Table 4.2: Matura tasks in 1868

In 1868, after obtaining the autonomy, the first Polish Matura exams were carried out

[23] State Archives in Przemyśl, sygn. 56/387/0/2.1/91.
[24] National Archives in Cracow, sign. 29/482/191, 399, 402.

in the Austrian Partition lands. We already know that in the Polish lands before 1868 and after 1875 they were similar in form, only the level of difficulty of the Matura exams increased. Now we will check what the Polish Matura exams looked like from the period of the autonomy. We do this on the example of Cracow St. Anna Gymnasium conducted in 1868 (Table 4.2). This Matura exam will also be compared to the mathematics Matura exam conducted the same year in the Inowrocław Gymnasium under the Prussian Partition.

When analysing the tasks from the written Matura exam conducted in Cracow St. Anna Gymnasium in 1868, it can be noted that it has the same form as the examinations carried out in the territories of the Austrian Partition before obtaining the autonomy and after its limitation. The Poles took over the Austrian Matura exam rules for the mathematics exams.

There is a clear difference between the mathematics Matura in Cracow St. Anna Gymnasium and the Gymnasium in Inowrocław in 1868. Above all, Inowrocław paid more attention to geometric-trigonometric tasks. The Matura set included two tasks from this field.

Making a broader comparative analysis of mathematics Matura exams in Polish territories during the partitions, it can be stated that generally, from the mid-19th century, in schools with Polish as a language of instruction there were always three tasks in each written Matura examination in mathematics. The scopes of Matura tasks weren't established obligatory there, but usually the Matura exam sets had two arithmetic-algebraic tasks and one geometrical-trigonometric task. In schools with German as a language of instruction under Prussian Partition usually there were four tasks: one arithmetic or algebraic, one planimetric, one trigonometric and one stereometric task. The most popular tasks in both types of schools were those connected with: arithmetical and geometrical progressions, solving an equation or a system of equations, it was often an exponential, logarithmic or trigonometric equation, solving triangles, calculating the area and the volume of polyhedral or solids of resolution, everyday life calculations (calculating profits, losses, interest and the amount of pensions) and the application of mathematics to physics. There were two types of tasks which were only in schools with German as a language of instruction – construction tasks and those related to arithmetic sequences of the second degree.

4.3 Conclusions

The analysis of the form of Matura exams in mathematics conducted in classical gymnasium located in the Polish lands in the second half of the 19th century allows to conclude that schools with Polish language of instruction modelled the teaching of mathematics at the Austrian schools. While from 1875 it was imposed by the invader, in 1868-1875 it was a voluntary decision of the Poles.

Although the form of Matura exams in schools with Polish language of instruction under Austrian Partition and schools with German language of instruction in the Prussian Partition was slightly different, the detailed analysis of written and oral examinations allows to conclude that both types of schools discussed similar issues and equal weight was applied to both arithmetic and algebraic issues, as well as geometric and trigonometric ones. An important part of teaching in both cases were the applications of mathematics.

The most significant difference between the Matura exams and, hence, the mathematics curriculum, was the discussion, or lack thereof, of construction tasks. In Polish-language gymnasia, no importance was attached to this type of tasks. Certainly it was a shortcoming of the curriculum, because this type of tasks, due to their multi-stage aspect, shaped the logical and independent reasoning, and through their applications they could arouse students' interest of the subject.

In the second half of the 19th century, there is the already vivid activity of Polish teachers on the reform of teaching intended for Polish secondary schools. Kamil Kraft and Stanisław Ziobrowski from the 4th Gymnasium in Krakow along with seven other members of the Polish Society of Teachers of Universities in 1906 announced a reform project entitled "Polish secondary school – criticism of its foundations and necessity of reform" (Dropiowski, Kraft, Łopuszański, Nitsch, Sobiński, Stein, Tołłoczko, Wasung & Ziobrowski, 1906a, 1906b). The basis of the project was the removal of the Austrian educational system from Polish schools and the introduction of a Polish system adapted to the needs of the Polish society. The greatest demand in the Polish nation was for people educated in the field of mathematical and natural sciences – experts in their application in everyday life, in the economy and industry. Therefore, the reform project put a special emphasis on the teaching of mathematics. The Austrian teaching system of this subject has been accused of overloading the material, that the tasks are schematic, based on standard algorithms, and the whole science of arithmetics is based on obtaining mechanical skill in the calculations. In Polish schools, the plan was to limit the scope of the discussed material. It was believed that the basics should be taught, but in such a way that the students could understand them perfectly, not only solve schematic tasks, but also those that go beyond the patterns – requiring logical and creative thinking. Great emphasis was also placed on tasks related to the needs of everyday life. An illustrative study of mathematics and gradual introduction of the difficulty (the principle of accessibility) were recommended.

The above project has not been implemented. However, its main postulates were included in the reform introduced in Poland shortly after regaining independence (the reform introduced in 1919).

5 Summary

The comparison carried out in this article makes it possible to notice that Polish-language schools operating in Polish territories during the partitions in certain matters related to teaching of mathematics introduced pioneering solutions. One of these solutions was the issue of teacher training. Polish educational authorities have decided to improve the competence of secondary school teachers and send them on academic internships in foreign secondary schools and at universities. This solution was not used in schools under the authority of Prussia and Austria.

It was Poland, Duchy of Warsaw, that was the first country in which the first Matura rules were introduced establishing mathematics as an obligatory subject of examination. It took place on February 17, 1812. In Prussia, mathematics became an obligatory Matura exam subject a few months later on June 25, 1812. Polish ordinances established a broader scope of material for the Matura exam in mathematics than in Prussia. In Poland, knowledge of higher degrees, spherical trigonometry and the applications of mathematics in physics and geography was required. These ordinances were not purely theoretical.

Schools had to implement them. An example here is the Warsaw Secondary School, which met all the curriculum requirements. Polish mathematical curricula for humanistic schools were wider than those in the Prussian and Austrian schools of that period. Nevertheless, over the years, curricula implemented in humanistic Polish-speaking schools began to resemble those in German-language schools.

Mathematical education from the mid-19th century was the domain of real schools. In these schools, the mathematics curriculum was the widest and they show the greatest differences when it comes to schools with Polish and German languages of instruction. In the 40s, Polish-language schools under the Russian Partition and German-language schools under the Austrian Partition put a lot of emphasis on teaching geometry. About half of the time reserved for mathematics classes in these schools was devoted to geometric issues. Polish schools were the first in which systematic emphasis was placed on teaching descriptive geometry, especially due to its applications.

The period of autonomy of Polish schools, 1868-1875, brought about yet another systemic innovation of Polish schools – an illustrative introduction of stereometric and planimetric concepts in the lowest grade of real schools.

Looking in general terms at the 19th century, the most important difference in the teaching of mathematics in Polish and German-speaking schools in the Polish lands is the method of teaching geometry.

It is difficult to talk about the national characteristics of teaching mathematics in Polish-speaking schools. There are few sources to assess whether the special needs of Polish society have been taken into account in the teaching of mathematics. Clear information on this subject was found only in the case of the Real Gymnasium in Warsaw. The nationalist movement in the teaching of mathematics in the Polish lands can be seen only at the end of the 19th century thanks to the efforts of Kraft, Ziobrowski and others. The effects of their work were implemented in school education only after Poland regained its independence.

In all schools functioning on the Polish territory during the partitions, comparable importance was attached to the applications of mathematics (the exception was the descriptive geometry, which in schools under Prussian rule was treated negligibly). The applications of geometry were particularly important, e.g. in geodesy, architecture or astronomy. Geometry was taught in context. This is an indication for modern educational practice. The benefits of teaching geometry in this way are currently described by, among others, Bartolini Bussi & Boero (1998). Malkevitch (1998, 23) draws attention to the need to improve the quality of education in the United States, and among the councils indicates that it is necessary to show the increasingly powerful new applications of geometry in the world that surrounds the young man. Mammana &Villani (1998a, 4) say that in modern schools, the geometry curriculum should be prepared very carefully, which will show a balance between teaching contents, methods and motivations for learning. It is important to combine new and old ideas in new curricula (Mammana & Viliani, 1998b, 9–28). This article identifies issues from the history of teaching, which show the applications of geometry in the world that surrounds a modern man, e.g. gnomonics – sundials are an element of modern architecture. The history of teaching mathematics in the Polish lands provides new fields of experience, which can help develop students' activity, i.e. discussing descriptive geometry or measuring tasks.

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