# DEVELOPMENT OF SCHOOL ALGEBRA – A COMPARISON BETWEEN THE 1980 AND 2011 SWEDISH MATHEMATICS CURRICULA

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#### ABSTRACT

In search of the reasons for Swedish students' low achievement in algebra in international evaluations, this study analyses and compares the algebraic content in the 1980 and 2011 Swedish curricula for the grades 1-9. In order to characterize the algebraic content, Blanton et al.'s (2015) five big ideas of algebra have been applied as an analytical tool. The results show that algebra is introduced earlier in 2011 compared to 1980 and that the big idea generalized arithmetic is weakly represented in both curricula. There is a high emphasis on practical and everyday mathematics in both curricula and there seems to be an increased emphasis on verbal abilities as opposed to computational skills over time.

### 1 Background

During the past decades, research on algebraic thinking at primary school level has gained increasing interest within the research field of mathematics education (for an overview, see Kieran, 2018). The assumption that young pupils are not capable to think algebraically has been challenged by several mathematics educators (e.g. Blanton, et al., 2015; Carraher, Schliemann, Brizuela & Earnest, 2006). Also, the idea that students' development of learning algebraic concepts would be reflected in the historical development of algebra has been questioned (Bråting & Pejlare, 2015; Schubring, 2011). In fact, recent studies reveal that it is not only possible, but also beneficial, to start working with algebraic ideas and generalizations in parallel with arithmetic from the very beginning (Mason, 2018).

This development is also visible in steering documents where several countries have revised their mathematics curricula in order to introduce students to algebraic thinking already in the early grades (NCTM, 2006). This is also the case in Sweden, which is the focus of the study reported in this paper. In Sweden, algebra has always been a part of mathematics that has caused school students major difficulties. In the international evaluation TIMSS (Trends in International Mathematics and Science Study), Swedish students' results in algebra have been below average ever since the 1960s (Hemmi et al., 2018). Even in the TIMSS test from 1995, where the overall result was the best ever for Sweden, the result in algebra was still below the international average. In the two TIMSS tests from 2007 and 2011, when the Swedish overall result decreased significantly, algebra was the topic that deteriorated the most (Yang Hansen et al., 2014). In the recent TIMSS evaluation from 2015, the Swedish overall result slightly increased, but the result in algebra remained poor (National Agency for Education, 2016).

Besides the revisions in the curriculum documents, there have been various attempts to improve school algebra teaching in Sweden through in-service training projects for teachers, and the "Algebra for All" movement ensuring that all school students are studying algebra before graduating high school. In connection with the latter, a specific textbook (see Bergsten, Häggström, & Lindberg, 1997) was compiled and used in teacher education at several universities in Sweden. However, it is not possible to discern a general positive effect of these efforts on Swedish students' learning in algebra, at least not if we consider the results in the TIMSS evaluations.

The study reported in this paper is part of an ongoing research project aiming at characterizing Swedish school algebra (Hemmi, et al., 2018). Both diachronic and synchronic studies are being conducted focusing on both formulation and realization arenas (Lindensjö & Lundgren, 2000) in order to identify the specific teaching tradition developed in Swedish school algebra (Bråting, 2015). The formulation arenas refer to steering documents and curriculum materials, and the realization arenas to schools and teachers who develop and maintain their own more or less tacit traditions. The overall purpose of the project is to find reasons for the failure to raise the quality of algebra teaching in Sweden, but also to find possible ways to improve the situation.

The study in this paper focuses on the formulation arena as it investigates Swedish curriculum documents in a historical perspective. The aim is to analyze and compare the algebraic content in the 1980 and 2011 Swedish mathematics curricula by using Blanton et al.'s (2015) so called big ideas of algebra as an analytical tool. In a longer perspective, the intention is to analyze the algebraic content in all Swedish mathematics curricula from the implementation of primary school ("grundskolan") in 1962 until today. In total, there have been five curriculum reforms in Sweden during this time period; in 1962, 1969, 1980, 1994 and 2011. An investigation of all these curricula provides valuable knowledge of how school algebra has traditionally been treated in Sweden with respect to the formulation arena. The study reported in this paper is a first step towards that goal.

### 2 Research on curriculum documents and school algebra in Sweden

During recent years, there has been an increased interest of research on mathematics curriculum and policy documents in Sweden with respect to the formulation arena (c.f. Boesen et al., 2014; Prytz, 2015; Bergqvist & Bergqvist, 2017). For instance, Prytz (2015) has given an overview of the Swedish mathematics curricula between the years 1850 to 2014 regarding structural aspects such as length (number of pages and words), the amount of time allocated to mathematics, and variation of mathematical topics through the years. The results show that the three curricula from 1980, 1994 and 2011 contain 50% more words compared to the curricula from 1962 and 1969. One reason is that the number of topics included in Swedish school mathematics has increased over the years and the descriptions of what it means to know that something has become more versatile. Moreover, Bråting and Österman (2017) illuminate a development from numerical and computational skills toward an increased emphasis on verbalizations and practical uses of mathematics in the Swedish school mathematics through the years.

Another study on the formulation arena is Boesen et al.'s (2014) investigation regarding mathematics teachers' response to the implementation of mathematical competency goals (see NCTM, 2000; Niss & Jensen, 2002) in the Swedish mathematics curriculum documents. The results reveal that the teachers are positive to the competences but it is difficult for the teachers to identify the meaning of the competence message by using national curriculum documents and national tests. Even though the competences were introduced fifteen years ago, Boesen et al. (2014) argue that the implementation should still be viewed as an ongoing reform in curriculum documents as well as in

textbooks, assessment and teaching. Drawing on Boesen et al's (2014) study, Bergqvist and Bergqvist (2017) have investigated to what extent and how clearly the national policy documents convey the competence message. The results show that the message is present to a large extent in the policy documents, but that it is vague and formulated with complex wording.

Although there is research on the formulation arena in connection to Swedish mathematics curricula, this research has not been conducted specifically on school algebra, the focus of the study in this paper. An exception is Bråting, Madej and Hemmi's (2019) investigation of the algebraic content in current Swedish textbooks in mathematics for grades 1-6, which is included in the same research project as the present study. In conformity with the study in this paper, Bråting et al.'s (2019) study applies Blanton et al.'s (2015) big ideas as a base for an analytical tool. However, in Bråting et al.'s study the focus is limited to the grades 1-6 and the five big ideas are merged into three. In this study, we use all five big ideas and we take all grades from 1 to 9 into consideration. The results of Bråting et al.'s (2019) study show that the big ideas "functional thinking", and "inequalities, expressions, and equations" are well represented both in the Swedish textbooks for grades 1-6. Meanwhile, the big idea "generalized arithmetic" is poorly developed in the textbooks. Apparently, this result is significant and helpful for the study in this paper.

Furthermore, Jakobsson-Åhl (2008) has conducted a historical study regarding the development of algebraic content in Swedish textbooks for upper secondary school between the years 1960–2000. Jakobsson-Åhl (2008) states that over the years the algebraic content has become more integrated with other school subjects, the level of complexity of algebraic expressions in textbook exercises has decreased, and algebra has more often been considered as a tool for solving practical and everyday problems.

There are some additional Swedish research studies focusing on school algebra within the realisation arena. For instance, Häggström (2008) compares how algebra is taught in Sweden and China, focusing on the treatment of systems of linear equation in grade 8 mathematics classrooms from the perspective of variation theory. The result reveals that the tasks used in the Chinese mathematics textbooks showed extensive variation in many relevant aspects, while the Swedish textbooks contained very similar tasks that did not open many dimensions of variation. Furthermore, a case study by Kilhamn (2014) shows that two Swedish grade 6 teachers using the same textbooks introduced variables in very different ways. The differences mainly depended on the two teachers' different views of the meaning of the variable concept, but also the meaning of algebra. A longitudinal study on students' algebraic understanding at upper secondary school is conducted by Persson (2010), who followed the same class of students for three years. Persson (2010) identified five main factors for success in algebra learning: pre-knowledge, concept development, instruction, time for learning, and interest, attitudes and feelings.

# 3 Methodology

We have analysed and compared the algebraic content in the 1980 and 2011 Swedish national curricula in mathematics for grades 1-9. In this section, a brief characterization of

the two curricula will be given<sup>1</sup>, followed by a description of the analytical tool and the procedure of the data analysis.

# 3.1 The two curricula – Lgr80 and Lgr11

The 1980 Swedish mathematics curriculum for compulsory school, included in "*Läroplan för grundskolan*" (*Lgr80*), consists of the two sections: 1) Goals, and 2) The main content. In the first section, the following two main goals are prescribed:

- 1. The teaching in mathematics should be based on the students' experiences and needs, and prepare them for the role of adult citizens. Students should therefore, in the first place, acquire an ability to solve such mathematical problems that usually occur in everyday life. This means that the students, by means of the teaching, should acquire
  - numerical abilities with and without technical resources,
  - skills in mental arithmetic and estimate calculations,
  - knowledge primarily in percentage calculations, practical geometry, units and unit transformations, and descriptive statistics.
- 2. By means of the school activities the pupils will also acquire mathematical knowledge and skills usable for studying other subjects, to further studies after primary school, at leisure and in working life. This requires, in addition to the above, that the students acquire knowledge about
  - the real numbers,
  - geometric relationships,
  - algebra and basic knowledge about functions,
  - statistics and probability, and
  - the usage of computers and computer knowledge (Lgr80, 1980, p. 98).

Section 2, the main content, is divided into nine topics where each topic is described for the grade levels 1-3, 4-6 and 7-9. These nine topics consists of problem solving, arithmetic, real numbers, percentages, measurements and units, geometry, algebra and basic functions, descriptive statistics and probability, and computer knowledge. The topic Algebra and basic functions is, besides computer knowledge, the topic with the least space in this section. In the study reported in this paper, all three sections of the curriculum are considered but only section 2, the main content, is included in the investigation.

The 2011 Swedish mathematics curriculum for compulsory school, included in *"Läroplan för grundskolan, förskoleklassen och fritidshemmet"* (*Lgr11*) consists of the three sections: 1) Introduction to the subject, 2) Central content, and 3) Knowledge criteria. Section 1 is the same for all grades 1-9 and includes a historical background to the subject and a description of the aim of school mathematics, which is summarized in terms of five competencies in the following way:

The teaching in mathematics will give students the opportunity to develop their ability to

- formulate and solve problems by using mathematics as well as evaluate selected strategies and methods,
- apply and analyse mathematical concepts as well as relations between concepts,
- select and use appropriate mathematical methods to make calculations and solve routine tasks,
- conduct and follow mathematical reasoning, and

<sup>&</sup>lt;sup>1</sup> For a more detailed summary regarding the overall structure of the Swedish curricula between 1850-2014, see (Prytz, 2015).

• use mathematical expressions to discuss, argue and account for issues, calculations and conclusions (Lgr11, 2011, p. 2).

Furthermore, sections 2 and 3 are split between the grade levels 1-3, 4-6 and 7-9. Section 2, the central content, is divided into six mathematical topics; Number sense and the usage of numbers, Algebra, Geometry, Probability and statistics, Relationships and change, and Problem solving. Section 3, the knowledge demands, is based on the five mathematical competencies in section 1, mentioned above.

In the present study, all three sections are considered but only section 2, the central content, is included in the investigation.

# 3.2 The analytical tool and data analysis

In order to characterize the algebraic content in the material, Blanton et al.'s (2015) so called *big ideas* has been used as a base for an analytical tool. These are the areas that should be developed through the grades as students develop their algebraic thinking. Next, we give a brief description of each big idea and how these have been interpreted in this study.

- 1. Equivalence, expressions, equations, and inequalities (EEEI) include relational understanding of the equal sign, representing and reasoning with expressions and equations, and relationships between and among generalized quantities (Blanton et al., 2015, p. 43). An example of a task within this category is the solving of the open number sentence:  $8+5=\_+4$  and being able to reason based on the structural relationship in the equation. Number sentences such as  $8+5=\_$  have not been included in this category since this kind of tasks consider the ability to calculate.
- 2. Generalized arithmetic (GA) involves reasoning about structures of arithmetic expressions (rather than their computational value) as well as generalizations of arithmetical relationships, which includes fundamental properties of numbers and operations (e.g., the commutative property of addition) (Blanton et al., 2015; Kaput, 2008). In this study, this category also includes relations between operations, such as multiplication defined as repeated addition. Sometimes the term generalized arithmetic is referred to as the bridge between arithmetic and algebra (Fujii, 2003). A more detailed description of generalized arithmetic can be found in Bråting, Hemmi and Madej (2018).
- 3. *Proportional reasoning (PR)* refers to opportunities for reasoning algebraically about two generalized quantities that are related in such a way that the ratio of one quantity to the other is invariant (Blanton et al., 2015, p. 43). In this study, some specific applications of proportional reasoning, such as scaling and similarity are also included.
- 4. *Functional thinking (FT)* involves generalizations of relationships between covarying quantities, and representations and reasoning with relationships through natural language, algebraic (symbolic) notation, tables, and graphs (Blanton et al., 2015, p. 43). For instance, this can mean generating linear data and organizing it in a table, identifying recursive patterns and function rules and describing them in words and using variables, and using a function rule to predict far function values.
- 5. *Variable (VAR)* refers to "symbolic notation as a linguistic tool for representing mathematical ideas in succinct ways and includes the different roles variable plays in different mathematical contexts" (Blanton et al., 2015, p. 43). One typical

example within this category is the ability to use variables in order to represent arithmetic generalizations.

The data analysis was conducted in the following way. The two curricula were first analysed separately in the original language. The unit of analysis was a statement or part of a statement that addressed an issue connected to one of the big ideas. The results of this process were written down in five tables (one for each big idea) for each curriculum. Each table was divided into the three grade levels 1-3, 4-6 and 7-9 and consisted of all the statements connected to a specific big idea. For instance, the table representing the big idea FT for the 1980 curriculum was structured as in Table 3.1.

FT in Lgr80			
Grades 1-3	Grades 4-6	Grades 7-9	
Statement 1 Statement 2	Statement 1 Statement 2	Statement 1. Statement 2.	
	•••		

Table 3.1: FT-categorized content in the 1980 curriculum

In a few cases, it was not all clear whether a statement represented a certain big idea or not. For instance, some statements connected to the decimal system were excluded because they were considered to belong to the development of number sense rather than algebra. In these cases, the interpretation had to be reconsidered which led to minor corrections.

After this procedure, the statements from the two curricula were compared for each big idea. In order to do that, the five tables from the 1980 curriculum were merged with the five tables from the 2011 curriculum, as in Table 3.2 below.

FT in Lgr80 and Lgr11			
	Grades 1-3	Grades 4-6	Grades 7-9
Lgr80	Statement 1	Statement 1	Statement 1
	Statement 2	Statement 2	Statement 2
Lgr11	Statement 1	Statement 1	Statement 1
	Statement 2	Statement 2	Statement 2

Table 3.2: FT-categorized content in the 1980 and 2011 curricula

Within this process, specific features, gaps, similarities and differences between each curriculum were identified. The merged tables for each big idea are presented in the result section here below.

# 4 Results

The results of the analysis are presented separately for each big idea. Every section commences with a table displaying the categorization of the authentic expressions identified in the curriculum documents. This is followed by a comparison of the two curricula with respect to how they address the big idea in question at the different grade levels.

# 4.1 EEEI – Equivalence, Expressions, Equations and Inequalities

Table 4.1 shows the distribution of algebraic content categorized as EEEI with respect to the grade levels 1-3, 4-6 and 7-9.

	Grades 1-3	Grades 4-6	Grades 7-9
Lgr80 S e e	Solving simple equalities by trial and error.	Solving simple equations mainly by trial and error and on the basis of problems.	Setting up, simplifying and calculating algebraic expressions. Expressions with parentheses, factorization, and identities of the binomial squares are treated, with particular consideration to the students' maturity, interest and needs. First order equations,
			including unknowns on both sides of the equality sign and with parenthesis and fractional numbers.
			Problem solving with simple equations.
			Systems of linear equations and simple second order functions mainly in connection with problem solving and preferentially with graphical solution.
Lgr11	Mathematical equalities and the importance of the equal sign.	Methods for solving simple equations.	Algebraic expressions, formulas and equations in situations that are relevant
		Simple algebraic expressions and equations in situations that are	for the students. Methods for solving
		expressions and equations in situations that are relevant for the students.	Methods for se equations.

Table 4.1: The distribution of EEEI-categorized content in 1980 and 2011 curricula

In grades 1-3, the topic equalities are included in both curricula. In Lgr80, it is emphasized to solve simple equalities by trial and error, while in Lgr11 the term equality is mentioned together with the importance of the equal sign. In grades 4-6 equalities are still considered in both curricula, but the focus is directed to equations. While the Lgr80 document explicitly prescribe trial and error based on problems as a solution method, the Lgr11 document only mentions that methods of solving equations should be presented but does not pinpoint the character of the methods.

The topic algebraic expressions are considered already in grades 4-6 in Lgr11, while in

Lgr80 algebraic expressions first appear in grades 7-9. In Lgr11 it is pointed out that equations and algebraic expressions should be considered in situations that are relevant for the students. It is noticeable that the Lgr11 document mentions simple algebraic expression for grades 4-6, but leaves the decision of what to do with the expressions to the teachers.

In grades 7-9, the big idea EEEI consists of equations and algebraic expressions in both curricula. However, the Lgr80 document includes more detailed descriptions than Lgr11 and emphasizes abilities such as being able to *set up*, *simplify* and *calculate* algebraic expressions. It is also specified that expressions with parentheses, factorization, and identities of the binomial squares should be treated. Meanwhile, the Lgr11 document only refers to "algebraic expressions and formulas" (Table 4.1). Also in connection with equations, the content is more specified in Lgr80 compared to Lgr11. The latter refers to "methods of solving equations" and that the equations should be connected to situations that are relevant for the students. Meanwhile, the Lgr80 document prescribes that first and second order equations as well as systems of equations should be treated, followed by a specification for each kind of equation.

Both curricula highlight that the mathematical content should be relevant for the students and connected to the students' interests. In Lgr80, it is also stated that students' maturity and needs should be taken into account (Table 4.1).

### 4.2 GA – Generalized arithmetic

Table 4.2 shows the distribution of algebraic content categorized as GA with respect to the grade levels 1-3, 4-6 and 7-9.

	Grades 1-3	Grades 4-6	Grades 7-9
Lgr80			
Lgr11	Properties and relations of the four arithmetical operations and their usage in different situations.		

Table 4.2: The distribution of GA-categorized content in the 1980 and 2011 curricula

GA is the least represented big idea in both curricula. In fact, in Lgr80 we could not find any content connected to generalized arithmetic. One item could be found in Lgr11: in grades 1-3 the properties and relations of the four arithmetical operations are included. As within EEEI, the usage in different situations is pointed out.

#### 4.3 PR – Proportional reasoning

Table 4.3 shows the distribution of algebraic content categorized as PR with respect to the grade levels 1-3, 4-6 and 7-9.

In grades 1-3, simple examples of enlargements and reductions are PR-categorized content in both curricula. The term scale is introduced already in grades 1-3 in Lgr11, while in Lgr80 scale first appears in grades 4-6. In Lgr11, different proportional relationships, including doubling and halving, are also included in grades 1-3. It is noticeable that in Lgr80 the term proportion first appears in grade 7-9 in connection with linear functions (see Table 4.4 below).

In grades 4-6, the main topics within PR in both curricula are percentage and scale. In both curricula, the relations between fractions, decimal numbers and percentages are emphasized. In Lgr11, the relation between proportionality and percentage is also considered. As already mentioned, in Lgr80 the term proportion first appears in grades 7-9.

In grades 7-9, the PR-categorized topics percentage and scale are still considered in both curricula. It is noticeable that the Lgr80 curriculum emphasizes calculations with percentages while in Lgr11 percentage is used to express change and change factor as well as calculations in everyday situations. Furthermore, in both curricula uniformity is introduced in grades 7-9. In Lgr80 the term congruence is also used. Both curricula point out the connection between the mathematical content and students' interests across all grades 1-9. In Lgr11 the expression "in situations relevant for the students" is used while in Lgr80 uses the terms "practical usage" and "everyday contexts". In Lgr80 the connection to other school subjects is also mentioned.

	Grades 1-3	Grades 4-6	Grades 7-9
Lgr80	Simple and practical examples of enlargements and reductions, for instance in connection with maps and handicraft objects.	Treatment of the percentage concept in connection with practical problems and other school subjects. Calculations with percentages. Relations between fractions, decimal numbers and percentages. Treatment of scale in everyday life.	Calculations with percentages, parts and the whole. Usage of the concept of scale mainly in practical contexts. Treatment of congruence and uniformity.
Lgr11	Different proportional relationships, including doubling and halving.	Proportionality and percentage and their relationship.	Percentage to express change and change factor as well as calculations with
	Students can use and give examples of simple proportional relations in situations relevant for the	Percentages and the connection to fractions and decimal numbers.	percentage in everyday situations and within other disciplines.
	students.	Graphs for expressing different types of	Scale with enlargement and reduction of two-
	Scale with simple enlargements and reductions.	proportional relations in simple investigations.	and three dimensional objects.
		Scale and the usage of scale in situations relevant for the students.	Uniformity in the plane.

Table 4.3: The distribution of PR-categorized content in the 1980 and 2011 curricula

# 4.4 FT – Functional thinking

Table 4.4 shows the distribution of algebraic content categorized as FT with respect to the grade levels 1-3, 4-6 and 7-9.

	Grades 1-3	Grades 4-6	Grades 7-9
Lgr80		The function concept is introduced through practical experiments. Interpretations of simple functions in the first quadrant in a coordinate system. Calculations of functions values by inserting them into formulas, connected	Interpretations and constructions of graphs in the whole coordinate system. Linear functions, especially those that indicate proportionality.
		school subjects.	
Lgr11	How simple patterns in number sequences and simple geometrical forms can be constructed, described and expressed.	How patterns in number sequences and geometrical patterns can be constructed, described and expressed. The coordinate system and strategies for scaling coordinate axes.	Functions and linear equations. How functions can be used to investigate change, rate of change and other relationships.
		Tables and graphs.	

Table 4.4: The distribution of FT-categorized content in the 1980 and 2011 curricula

In grades 1-3, the big idea FT is not represented at all in Lgr80. In the same grade level in Lgr11, FT-categorized content consists of simple patterns in number sequences and simple geometrical forms as well as how these can be constructed, described and expressed. This topic is also considered in grades 4-6 in Lgr11. The only difference is that in grades 4-6 the word "simple" (in connection with simple patterns) is removed.

Furthermore, in grades 4-6 the coordinate system and graphs are included in both curricula. In Lgr80, the term "function" appears in grades 4-6, while in Lgr11 the term "function" first appears in grades 7-9. In Lgr80, interpretations of simple functions in the first quadrant as well as calculations of function values by inserting them into formulas are emphasized in grades 4-6. In grades 7-9, not only interpretations of functions are emphasized but also constructions of functions in the whole coordinate system. Linear functions, especially those that indicate proportionality, are also mentioned in Lgr80. In Lgr11, the description of the content of functions are not as detailed as in Lgr80. However, the Lgr11 document prescribes the usage of functions in order to investigate change (Table 4.4).

### 4.5 VAR – Variables

Table 4.5 shows the distribution of algebraic content categorized as VAR with respect to the grade levels 1-3, 4-6 and 7-9.

	Grades 1-3	Grades 4-6	Grades 7-9
Lgr80			
Lgr11		Unknown numbers and their properties and also situations where there is a need to represent an unknown number by a symbol.	The meaning of the variable concept and its use in algebraic expressions, formulas and equations.

Table 4.5 7. The distribution of VAR-categorized content in the 1980 and 2011 curricula

In Lgr80, we could not find any content connected to the big idea VAR. In Lgr11, VAR-categorized content first appears in grades 4-6 and 7-9. Unknown numbers and their properties are included in the content in grades 4-6. It is also pointed out that situations where there is a need to represent an unknown number by a symbol should be treated. In grades 7-9, the focus is on the meaning of the variable concept as well as its use in algebraic expressions, formulas and equations.

# 5 Discussion

The result reveals both similarities and differences between the two curricula. Next, this will be discussed on the basis of the following issues: The earlier introduction of algebra over time, the representation of different big ideas, the movement from a focus on computational skills to verbal abilities between the years 1980 and 2011, and finally the high emphasis on practical and everyday mathematics in both curricula.

The result of the analysis reflects the recent international trend to integrate algebra in school mathematics already from primary school (Blanton et al., 2015; Carraher et al., 2006; NCTM, 2006). Several parts of the algebraic content are introduced earlier in the Lgr11 document compared to the Lgr80 document. For instance, in Lgr80 algebraic expressions and proportionality first appear in grades 7-9, while in 2011 algebraic expressions appear in grade 4-6 and proportionality already in grades 1-3 (Table 4.1 & 4.3). An exception in this study is the function concept which is introduced in grades 4-6 in Lgr80 and not until grades 7-9 in Lgr11. However, the Lgr11 document prescribes "patterns" already in grades 1-3 which might be viewed as a first step to understand the function concept (Blanton et al., 2015). That is, even though the function concept appears earlier in the Lgr80 document, the big idea 'functional thinking' (Blanton et al., 2015) is represented earlier in Lgr11 compared to Lgr80 (see Table 4.4).

Furthermore, the function concept is applied somewhat different in the two curricula. For instance, the Lgr80 document prescribes calculations of function values which is not mentioned in Lgr11. Instead, the Lgr11 document emphasizes the usage of functions in order to investigate rate of change and other relationships which is not included in Lgr80 (Table 4.4). One reason to this might be that 'Relationships and change' constitutes a new, separate category of mathematical content in the Lgr11 curriculum (see the Methodology section above, p. 5). In previous Swedish curricula, the content in 'Relationships and

change' was distributed among the different topics, especially algebra. The emphasis on 'Relationships and change' is probably an effect of a recent international trend where 'Relationships and change' has been identified as one of the four broad mathematical content categories in the PISA framework for school mathematics (OECD, 2010). This is also reflected in the current Swedish textbooks in mathematics, where the big idea 'functional thinking' is a dominating content with a clear progression throughout the grades 1-6 (Bråting et al., 2019). Meanwhile, as mentioned in the Methodology section above, in Lgr80 'Algebra and basic functions' is one of the smallest topics in the curriculum and it is also pointed out that individualization based on students' ability is necessary within this topic (Lgr80, p. 105).

'Generalized arithmetic' is the least represented big idea in both curricula. Apparently, it is not represented at all in the content description of the Lgr80 document and very little in Lgr11 (Table 4.2). As already mentioned, generalized arithmetic is seen as one of the most important parts of school algebra by several researchers (Blanton et al., 2015; Kaput, 2008). Sometimes generalized arithmetic is considered as a bridge between arithmetic and algebraic thinking (Fujii, 2003), that is, as a development of "algebra as generalized arithmetic" throughout compulsory school. However, we cannot find any notion of building a bridge between arithmetic and algebra in the two curricula. In fact, the terms 'generalize' and 'generalization' do not appear in neither of the two curricula. It can also be noted that the term variable (or unknown) is not mentioned at all in the Lgr80 document which probably is a reaction to the great focus on abstract mathematics in connection with "New math" during the 70s (c.f Prytz, 2015).

The results reveal that the Lgr80 document emphasizes computational skills to a greater extent than Lgr11. Within the big idea EEEI, a typical example is the emphasis on *setting up, simplify* and *calculate* algebraic expressions in Lgr80 (Table 4.1). Furthermore, the Lgr80 document prescribes *solving* equations and specifies which kind of equations that should be treated. Instead, the Lgr11 document emphasizes *methods* for solving equations and *the meaning* of the equal sign (which is pointed out already in grades 1-3). Moreover, the Lgr11 document stresses abilities such as *expressing* and *describing*. For instance, within the big idea PR, percentage is connected to the ability to *express* change (Table 4.3) and within the big idea FT patterns should be *constructed*, *described*, and *expressed* (Table 4.4).

The emphasis on terms such as methods, expressing and describing in the Lgr11 reflects the implementation of the mathematical competency goals (NCTM, 2000; Niss & Jensen, 2002) in the Swedish mathematical curriculum. As already mentioned in the methodology section above (p. 5), the competency goals consist of abilities such as analysing mathematical concepts, evaluating selected strategies and methods as well as use mathematical expressions to discuss conclusions. These abilities cannot be found in the Lgr80 document where computational or operational aspects are in focus. Based on the results in this study, one could grasp a movement of focus from computational and operational abilities in Lgr80 to more verbal abilities in Lgr11 (c.f. Bråting & Österman, 2015). This is in accordance with the results of Jakobsson-Åhl's (2008) study which revealed that the level of complexity of algebraic expressions in Swedish textbook exercises had decreased over the years.

The implementation of the competency goals in the Swedish curriculum is an ongoing reform in Swedish school mathematics (Boesen et al., 2014) and according to Bergqvist

and Bergqvist (2017) it is problematic for teachers to convey the message which is vague and formulated with complex wording. The focus on the implementation of the competencies in Swedish school mathematics might be one reason why the traditional emphasis on computational skills has been disregarded during the past decades. However, we believe it is important to have a balance between the emphasis on verbal abilities on the one hand and computational abilities on the other.

Apparently, both curricula frequently point out the importance of practical and everyday mathematics within the algebraic content. However, the view of how these aspects should be acquired differs between the two curricula. In the Lgr80 document, practical skills and the acquirement of everyday mathematics are considered as a part of an overall ability, which should be acquired by means of learning computational, numerical and geometrical skills. Let us consider the same citation of one of the two main goals with mathematics from Lgr80 which was cited in the methodology section above:

Students should therefore, in the first place, acquire an ability to solve such mathematical problems that usually occur in everyday life. This means that the students, by means of the teaching, should acquire

- numerical abilities with and without technical resources,
- skills in mental arithmetic and estimate calculations,
- knowledge primarily in percentage calculations, practical geometry, units and unit transformations, and descriptive statistics (Lgr80, p. 98).

Here, it is clearly stated that the teaching must be based on the students' own experiences. However, the practical skills are closely linked to the specific mathematical content. This differs from the Lgr11 document where the specific mathematical content is more separated from the practical and verbal abilities within the descriptions of the goals (as in the citation of the competency goals on p. 5 in the methodology section above). Moreover, it is prescribed in Lgr11 that mathematics should be used as a tool to solve everyday and practical problems such as private economy, social life, and electronics. It seems that in Lgr11 the mathematical content is used to solve the practical and everyday problems, while in Lgr80 the practical and everyday problems are used as a platform to learn the specific mathematical content (see also Bråting & Österman, 2015). This is in accordance with Jakobsson-Åhl's (2008) study where the results revealed that algebra has more often been considered as a tool for solving practical and everyday problems through the years and that algebraic content has become more integrated with other school subjects.

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