DESCRIPTION OF OLD NEPALI MATHEMATICS BOOKS AND THEIR POTENTIAL IN IMPROVING CURRENT DAY TEACHING AND LEARNING

Deepak BASYAL

University of Wisconsin-Milwaukee, College of General Studies Department of Mathematics and Natural Sciences, West Bend, Wisconsin, USA basyal@uwm.edu

ABSTRACT

This paper is a result of a comparative literature study of four Nepali early mathematics books from the period 1880-1935. The books used in this study are the first mathematics book written in Nepali language *Vyaktachandrika* (Arithmetic) (1884) by Gopal Pandey; the first mathematics book written in Nepali poetry, *Ankendushekhar* (Pinnacle of numbers) (1900) by Pahalman Simh Swar; *Ganitchandrachandrika* (A treatise of mathematics) (1922) by Raghunath Pant and *Shishubodha Tarangini part II* (Series of lessons for children) (1933) by Tikaram Dhananjaya. A content analysis and comparison of these four books based on seven categories recognized a few historical features that can be made useful in the teaching and learning of mathematics today.

1 Introduction

The teaching and learning of mathematics in Nepal is not a very new phenomenon. However, the formal teaching of mathematics in schools in Nepal started quite recently at the end of the nineteenth century. The start of the first formal school in 1885 for the public (Sharma, 1990), and a slow-paced increase in the number of schools thereafter motivated the production of teaching and learning materials of mathematics. A handful of mathematics books came into existence during the next 70 years, some of these books found their way to antiquarian sections of local libraries, and some of them were lost. A historical account of mathematics teaching and learning situations in Nepal can be found in these articles (Lardner, 1967; Tuladhar & Jha, 2002; Jha, Adhikari & Pant, 2006; Maskey, 2013). Readers interested to learn history of Nepal are recommended to see (Whelpton, 2005). A historical narrative of education situation in Nepal can be found in (Vaidya, Manandhar & Joshi, 1993, pp. 269-329).

Several studies support the use of old mathematics in teaching and learning of current mathematics contents and concepts. Research suggests that using history to teach mathematics generates positive attitude among students for learning mathematics (Marshall, 2000; Tözlüyurt, 2008; Goktepe & Ozdemir, 2013), motivates abstract concepts (Laubenbacher, Pengelley & Siddoway, 1994), creates interest in learning of mathematics (Tözlüyurt, 2008), and engages student in metalevel analysis (Barnett, Lodder & Pengelley, 2014). The use of original sources in the mathematics classroom has been proven beneficial to students in many aspects (Van Mannen, 1997; Jahnke et al., 2002). In spring 2011, I had an opportunity to take a graduate number theory course. This class, taught by professor David Pengelley, at New Mexico State University was based on Sophie Germain's translated manuscript. Over the last few years, I have used primary historical source projects (Otero, 2017; Klyve, 2017) produced by TRIUMPHS (TRI, 2015) in my classrooms. These experiences made me realize the value and importance of primary historical sources in teaching and learning of mathematics. In the beginning of the

twentieth century, Barwell (1913) suggested to use history of mathematics in teaching and learning of mathematics in the west, whereas Nepali mathematicians were just starting to produce mathematics books in Nepali language at the moment. In the west, the use of history of mathematics in teaching and learning was happening in the 1960s and 1970s (Fried, 2001), while Nepal was still working on its first mathematics curriculum. Nowadays, many countries in the world are advocating the use of history of mathematics in teaching (Fauvel & Maanen, 2000); on the other hand, the modern curriculums of Nepal (MoE, 2014; MoE, 2009; MoE, 2007) and school sector development plans (MoE, 2016) does not even foresee the history of mathematics in teaching and learning as a possibility.

In 2017, a newspaper article (THT, 2017) reported that traditional teaching and learning techniques currently employed in the majority of Nepali institutions is a catalyst for the rapid decline in students' interest in mathematics. Moreover, it is also observed that current Nepali mathematics curriculum is formal, abstract, decontextualized and disconnected from everyday life (Luitel & Taylor, 2007; Wagle et al., 2008). The historical sources considered in this study are rich in cultural and contextual problems. These culturally based mathematics curriculum has a potential to increase student's understanding of mathematical concepts (Lipka & Adams, 2004), and possibly remove barriers to motivation and engagement (Miller & Roehrig, 2016). The growing interest in history of mathematics education research in Nepal can be realized from these two recent PhD dissertations (Acharya, 2015; Subedi, 2017), and this paper seeks to initiate a more formal discussion on the use of old Nepali primary sources in teaching and learning of mathematics hence in this study content analysis of Nepali mathematics books from the period 1880-1935 is carried out in this study in order to identify useful features that can be made beneficial in teaching and learning of mathematics today.

2 Methods

The suggestions for analyzing the printed documents are followed from (Robson, 2002). Robson suggests a content analysis of printed documents, the textbooks in this case, with the following specific steps: start with a research question, decide on sampling strategy, define the recording unit, construct categories for analysis, test the coding on samples of text and assess reliability, and carry out the analysis. With a major research question on how to identify the useful features from these sources that can be beneficial in modern day teaching and learning of mathematics, a sampling strategy is initiated. Based on published newspapers and research articles, a list of books published in between 1880-1935 was formed. A total of 17 books were recorded. Robson further suggests reducing the task to a manageable dimension from population of interest. However, in this case, the options were limited. The major challenge was to locate a sample of these books from local libraries in Nepal. To locate a sample of books published in that period, the website of local libraries and their collections were searched. Google search was not helpful. The collection of Madan Puraskaar Pustakaalaya (MPP), Keisher Pustakaalaya, and Tribhuvan University Central Library were searched. These libraries are located in Kathmandu. MPP houses a collection of old Nepali mathematics books. Ten out of seventeen books from my list is found in MPP website. A total of six books were received. The library takes months to produce a digital copy. They charge cash amount to be paid in exact change. There is always a need for a person to go there on my behalf. A few friends came forward to help

me. All six books were received as a digital copy in different times. There might be other books in other libraries outside Kathmandu. I did not have easy access to those materials. Taking suggestions from Freid (2013), I confined the data collection to those sources, which I have an easy access to from my working location in the US. After receiving the sources, I worked on another crucial step of categories construction. Robson (2002) suggested creating mutually exclusive and exhaustive categories; however, I followed suggestions from Frejd (2013) in this regard too. As Frejd suggested, I considered the curriculum contents, which are purposed in a different level of current Nepali mathematics curricula (MoE, 2014; MoE, 2009; MoE, 2007), as well as pragmatic considerations, for example, the contents must be possible to compare (Freid, 2013). Two books out of six were not considered for this study for the following reasons. Bhasvati (1931) by Tikaram Dhananjaya is dropped because it has more astrology content than mathematics, and Ganitasagara (1921) by Gangaprasad Shrestha is not considered because it only lists questions and answers throughout the book. It was not easy to compare this book with other books based on the seven categories discussed below and shown in Table 3.1. This study attempts to answer the following specific questions: What does the preface say? What does the table of contents look like? What is the structure of the textbook? Are there any definitions/procedures explained? Are there any story problems? What is the presentation style of the book? How do the authors treat zero? Are there any answers/solutions provided?

3 Results and Discussion

A brief discussion about the books and their authors is provided in this section. This section also attempts to answer the questions considered in each category.

3.1. Books and Authors

The first book *Vyaktachandrika* (Arithmetic) was originally published in year of Vikram known as Vikram Samvat 1940 which is equivalent to 1883/1884 AD. A digital copy of second edition of *Vyaktachandrika* was received. Record shows that this book was in demand for about 30 years, and a several editions of the book were produced. The second edition was published in 1895, third edition in Hindi was published in 1907, and the fourth edition was published in 1914 (Maskey, 2013). The second edition considered for this study was published by Nirnayasangara Chhapakhana located in Mumbai, India. The front page of the book has a rubber stamp mark 'Kamalko Nepali Samgraha' indicating that the book was collected in the Nepali collections of Kamal. Late Kamal Mani Dixit was the founder of the Madan Puraskaar Pustakaalaya. It is not clear how this book landed in Kamal's collection.

Gopal Pandey (?1847-1921), the author of the book Vyaktachandrika, was a teacher at Sanskrit *pathasala* located in Kathmandu for more than four decades. He was taught by Bapudeva Shastri (1821-1900), a prominent mathematician and astronomer in India. Pandey has written several other books. Interested readers are suggested to see (Panta, 1980) for more information.

The second book *Ankendushekhara* (Pinnacle of numbers) was written in the Falgun month of Vikram Samvat 1956 on Tuesday, which is equivalent to the beginning of 1900 AD. The date was given in page six in the form of shloka. The copy from MPP has a

handwritten name Prof. SR Pant, who is a former department head of the Central Department of Mathematics at Tribhuvan University. It appears that the book was in his possession at some point. The second edition of the book was published in 1953 AD by Sarba Hitaishi Company located in Banaras, India. This book was quite popular in the first decade of 20th century Nepal and India (Subba, Sinha, Nepal & Nepal, 2009).

Pahalman Simh Swar (1879-1933) is considered the first drama writer in the Nepali language. He worked for the government of Nepal for some time and moved to India. He had written several books on drama, poetry, story and spiritual writings. More on his life and work can be found in this biographical publication (Swar, 1982).



Pandey (1884)

Swar (1900)

Pant (1922)

Dhananjaya (1933)

Figure 3.1 : The cover pages of the books

The third book *Ganitachandrachandrika* (A treatise of mathematics) was published in 1922 AD according to the MPP website. The digital copy received is the second edition of part I, which was published by Bhagawati Press, Kathmandu, Nepal in 1933 AD. The book was written by Raghunath Pant.

It was not possible for me to find any relevant sources to write biographical information about Raghunath Pant, the author of the book *Ganitachandrachandrika*.

The fourth book *Shishubodha Tarangini Part II* (Series of lessons for children) was believed to be published in 1991 Vikram Samvat which is equivalent to 1933/34 AD. There is a handwritten date 1991 in the cover page of the book. There is no other printed publication date provided for this book. The purported author of the book is Chandrakala Dhananjaya, who did not receive any formal education in her lifetime. Later, when she was interviewed by Modnath Prashrit she revealed that she did not write the book (Prashrit, 2002). She also confirmed that the book was written by her husband Tikaram Dhananjaya. I found this book in MPP, and it is probably the one out of three-surviving copies of the book. Part I seems to be lost. Although, the part III was in the plan for writing, it is possible that the part III was never written because of sudden death of Tikaram at the age of 26. Interested readers can find more on this in Basyal (2015).

Tikaram Dhananjaya (1909-1936) was a poet, mathematician, translator, astrologer, commentator, writer, grammarian and an inventor of a sign language. In his short life of about 26 years he produced a good number of books on different area of knowledge. Tikaram earned his high school degree from Banaras, India. After returning back from Banaras, he worked as a mathematics and astrology teacher in a local school.

3.2. Categories

In this section, the categories being considered are discussed and commented on what is observed in each category. Table 3.1 provides a summary of the results. Now, each category will be discussed in detail with examples and explanations.

Books	Arithmetic	Pinnacle of Numbers	A treatise of mathematics	Series of lessons for children
Categories	(1884) by Gopal Pandey	(1900) by Pahalman Swar	(1922) by Raghunath Pant	(1933) by Tikaram Dhananjaya
Preface	1	1	×	×
Table of contents	1	×	1	1
Definitions/Procedures	1	1	1	1
Story problems	1	1	1	1
Presentation style	Normal (Sanskrit)	(Nepali)	Normal	(Nepali)
Treatment of zero	Eight operations with zero	Placeholder	Placeholder	Eight operations with zero
Answer/Solutions	Few detailed worked out examples	Few worked out examples	A long list of problems with answers	Few worked out examples

Table 3.1:	Summary	of findings
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What does the preface say?

Only two books provide the preface. In *Arithmetic*, it is stated that the first edition was well received by the readers and all 500 copies were sold soon after it was published. It does not state the number of copies published in the second edition.

My translation of the preface in Gopal Pandey's book is given here:

The calculations we make with one, two, three and more such numbers, is called Arithmetic. The European learned Arithmetic from Arabian, and Arabian learned from Indians and they still call it 'Hindu-mathematics'....

[...] People in various countries have written arithmetic in their own languages to teach their children. People might think that Gorkhali does not know mathematics because we still have no arithmetic written in Parvatiya language. Therefore, I am writing this book with my limited knowledge to prevent the situation. (Pandey, 1985, p. 1)

In the quote above *Gorkhali* and *Parvatiya* both refers to Nepali. In the preface, Pandey further goes on to indicate that the most contents for his books are taken from Bhaskaracharya's Lilavati. He also cites his teacher Bapudeva Shastri's work on page 43 for the method 'casting out by eleven'. As promised in the preface, Pandey tried to simplify the rules and provided thoughtful and useful examples. He also hoped that the book will be much easier for learners to follow as the book is written in local language.

The preface in Pahalman Swar's book roughly translates to:

Dear gentleman, for a long time, I had a desire to write mathematical shlokas in Nepali vernacular. I have realized that it would make a special contribution to write mathematical shlokas. Here, I present a few mathematics in shloka for gentleman. As my readers are aware of the fact, writing anything new is not an easy task, so writing mathematics is not easy either. With my limited knowledge, I am taking this daunting task. (Swar, 1953, pp. 2-3)

After reading these prefaces one can conclude that these books were written to fulfill the lack of mathematical resources in Nepali language. Note that Pandey managed to credit his sources; however Swar did not mention anything about his sources.

What does the table of contents look like?

Three books provide a table of contents: The content in *Arithmetic* is given in three parts, the third part being geometry. The *Pinnacle of Numbers* does not provide a table of contents; however, a table of contents is created by observing the content given in book. The order of contents is written as it was presented in the book. The *Series of Lessons for Children* provides 63 titles in mathematics and astrology in the table of contents. However, only mathematics titles are translated here.

A closer look at Table 3.2 suggests that these books were written to teach the basic rules and word problems related to arithmetic. The addition, subtraction, multiplication and division of whole numbers are presented in every book. The rule of three, five, seven etcetera, compound interest, mixtures, barter and series were also common topics of the time. These basic rules of arithmetic were used for illustrating how some real-life problems can be solved using these arithmetic tools. The work-rate problems, interest calculation, and conversion of local money and land units are also considered. These topics given in Table 3.2 can be identified in current Nepali curriculums (MoE, 2014; MoE, 2009; MoE, 2007), so there is an opportunity to create guided classroom worksheets and projects and use them in the classroom to test the usability of these contents.

Table 3.2: Table of Contents of the books

Arithmetic (1884)	Part I: Numeration and pronunciations of numbers, addition, subtraction, multiplication, division, exponentiation, taking roots, greatest common divisor, least common multiple, dividing by a same number, addition of fractions, [subtraction] of fractions, addition of fractions with its parts, multiplying fractions, compound fractions, addition and subtraction of a number with its parts, dividing by fraction, division of fractions, exponentiation of fractions, taking roots of fractions, eight rules with zero, rule of three, rule of five, seven and etcetera, barter, mixtures, working backwards, supposition, double false position, using sum and difference to find the two numbers, quadratic [Diophantine] problems Part II Method for checking if multiplication is correct, addition of decimals, repeating decimals, taking roots of decimals, taking roots of decimals, taking root of fractions, taking root [of fractions, ratio, adding whole in ratio, Part III Definitions, area and volume for simple shapes, triangles, rectangles, ratio with areas, circle, length measure.
Pinnacle of Numbers (1900)	Invocation, definitions, numeration, local values, addition, subtraction, multiplication, division, dividing in five, four, and seven equal parts, money exchange, rule of three, inverse rule of three, rule of five, seven, nine and eleven, compound interest, mixtures, shadow measures, series, miscellaneous problems
A treatise of mathematics (1922)	Definitions, number writing, pronunciations of numbers, local values, addition, addition of whole numbers, subtraction, subtraction of whole numbers, multiplication, multiplication of whole numbers, division, division of whole numbers, role of grouping symbols, several topics, concerning addition and subtraction, concerning multiplication and division, working backwards, concerning addition and subtraction, money, rate and miscellaneous, examples workout and various question, examples, answers
Series of lessons for children (1933)	Invocation, definitions, eight fundamental rules of arithmetic concerning whole numbers, greatest common divisor, least common multiple, eight fundamental rules of arithmetic concerning fractional numbers, multiplying fractions, adding or subtracting a fraction and a whole number, Increasing and decreasing a fraction by a fractional amount, rule for addition and subtraction of fractions, eight fundamental rules of arithmetic concerning zero, eight fundamental rules of arithmetic concerning decimals, working backwards, method for checking if multiplication is correct, method of supposition, using the sum and the difference of two numbers to find the numbers, the rule of three, simple interest, Investigation of mixtures, series

Are there any definitions/ procedures explained?

All books show a procedure to write numbers. The mathematical rules and working procedures are explained either in prose or in verse in all books.

 (२) दुइ मैसीले एक खेतको घास पांच दिनमा र तीन गाइले सात दिनमा साझ सकतछन् भने एक मैसी र एक गाइले मिलेर खाया भने कति दिनमा सकलान् ? उत्तर ६३४दिन. (३) एक तलाउमा पानी आउन्या चारौटा धारा रहेलन् तिनमा एक लाइ छाडदा दुइ घडिमा, दोसरो लाइ छाडदा तीन घडिमा, तेसरो लाइ छाडदा तीन घडिमा, तेसरो लाइ छाडदा वा घडिमा, तो तलाउ मरीदो रहेल मने सपै लाइ एकहि पस्ट छाडदा कति वेरमा भरिवला ? उत्तर ४६७५भाम्म स्थे लाइ एकहि पस्ट छाडदा कति वेरमा भरिवला ? उत्तर ४६७५भाम्म स्थे लाइ एकहि पस्ट छाडदा कति वेरमा भरिवला ? उत्तर ४६७५भाम्म स्थे लाइ एकहि पस्ट छाडदा कति वेरमा भरिवला ? उत्तर ४६७५भाम्म स्थे लाइ एकहि पस्ट छाडदा कति वेरमा भरिवला ? उत्तर ४६७५भाम्म स्थे लाइ एकहि पस्ट छाडदा कति वेरमा भरिवला ? उत्तर ४६७५ माम खाल हुदो रहेल भने तुइ धारा र निकास लाइ एकहि पस्ट छाडदा कति वेरमा भरिवला ? उत्तर ५१५ पल. (७) एक पोलरीमा तीन ओटा पानी आउन्या कुला रहेलन् तिनमा पर्हिला र दोधरा लाइ छाडदा आठ घंटामा, अघ पहिला र तेखरा लाइ छाडदा भा देखा मा रेरदे रहेल्य स्थान स्थान श्रे प्रता ह छाडदा पर हुई घटमा, तो पोलरी भारते रहेल भने तीन आठा एकहि एक्ट छाडदा कति वेरमा भरिवला ! उत्तर ५१५ पल. 	 (३) मैले पउटा मानिसलाई घर किंज २४०१ रुपिझा र ४७४ रुपिझा दिएँ । अघि त्यस्ते घरखर्चलाई २४० रुपिझा र अभ् रुपिझा दिएँ । अघि त्यस्ते आफुने लिएका सब रुपिझा र त्य- केका थियो । हाल त्यस्ते आफुने लिएका सब रुपिझा र त्य- कोका थियो । हाल त्यस्ते आफुने लिएका सब रुपिझा र त्य- कोका थियो । हाल त्यस्ते आफुने लिएका सब रुपिझा र त्य- कोकि रुपिझा बुभिलिनुपर्ला ? (४) एउटा धर्नाका पहिलां कंकमा १२४००० रुपिझा, दोस्रो बंकमा पहिलांमा भन्दा ३४००० रुपिझा बढता, तेद्रो बंकमा दोस्रोमा भन्दा ४४४०० रुपिझा बढ्ता हालेका रहेछन् । त्यस्का साथमा ७२०७४ रुपिझा पनी रहेछ मने, त्यो धनी कति धनको मालिक रहेछ ? (४) १६३२ सालमा जन्मेका मानिस कुन् सालमा ७२ वर्षको होला? (ई) पउटा चैलीमा ४६६ कम्पनी, १२४० मोहर, ४६१ पैसा र ६४४ करपनांपैसा मिसिपकांछ भने, त्यस चैलीमा धान गन्ती कर्ति यान सिकाहुनुपर्वछ? (७) एउटा महाजनले ४०० गरीवलाई २ पैसाका दरले, १४४ गरीबलाई ३ पैसाका दरले र ६१६ गरीवलाई १ पैसाका दरले बिया भने जम्मा लाम्हर कर्जिय प्रले प्रताका ह र विसाका दरले
A few contextual examples in Pandey (1894)	ार्था मन जम्मा त्यस्का कात पता खर्च भएड्रन्? कति जना गरीबहरूले पाएड्रन्?
A lew contextual examples in Pandey (1884)	A lew contextual examples in Pant (1922)

A few contextual examples in Pant (1922)

Figure 3.2: A few contextual story problems

In Table 3.5, I provide a translation of mathematical procedure known as Rule of Three. The working rules with zero are provided in Table 3.4. In Table 3.7, the definition of Greatest Common Divisor and Least Common Multiple from both old book and modern book is provided. These procedures and definitions can be compared with the definition given in modern curriculums to gain some pedagogical insights.

Pandey (1884)	Pant (1922)
(2) Two buffalos can finish eating grass of a	(3) I lend 2501 rupiya and 475 rupiya to a
field in five days. Two cows can finish in	person to buy a house. He also received 250
seven days. How many days does it take for	rupiya for household expenses. Now he is
a cow and a buffalo to finish eating the	paying back all he received and 12 extra
grass?	rupiya for an interest. How much should I
(3) There were four taps connecting to a	get from him?
pond. First tap can fill the pond in two ghadi,	(4) One rich person has 125000 rupiya in the
second one in three ghadi, third one in four	first bank, 35000 more than that of in the
ghadi, fourth one in five ghadi. If all taps are	first bank is in the second, 45500 more than
open at a time, how long will it take to fill	that of second bank is in the third bank. If he
the pond?	have 72075 rupiya with him. How much
	money does this rich person have?

Table 3.3: Translation of contextual problems in Figure 3.2

Are there any story problems?

Yes. A few contextual story problems can be found in each book, the third one being the richest in the contextual problems. A few contextual story problems are given in Figure 3.2. A translation of the problems in Figure 3.2 is provided in Table 3.3. Studying how

these problems are similar and different mathematically in the current context could be a lesson for today's students. These problems are comparable with the problems given in modern textbooks. A sample of problems in modern textbooks is given in Table 3.6.

Table 3.3 provides a rough translation of first two problems from each book given in Figure 3.2. Pandey's problems are taken from (Pandey, 1895, pp. 34-35), and Pant's problems are taken from (Pant, 1932, p.22).

Looking at these representative problems from these books, one can get a sense of market products, rates, measurement and monetary units of that time. These problems where one talks about buffalos, *rupiya*, *ghadi* and the person with less than 1 million Nepali rupees would be considered a rich person are all foreign and old-fashioned to today's student. Similar problems can be seen in modern textbooks. Table 3.66 shows a few representative problems from textbooks used today in Nepali schools in various grades. These books are mostly used in private schools where the language of instruction is English. These books are also written in English.

What is the presentation style of the book?

Arithmetic is written in a mixture of Sanskrit shloka (verse) and Nepali prose, which consists of topics on arithmetic, algebra and geometry. The *Pinnacle of Numbers* and the *Series of Lessons for Children* are rich in Nepali shloka, whereas the *A Treatise in Mathematics* is rich in word problems. The *Series of Lessons for Children* book would be a great motivational and mnemonic resource as it lists 137 shlokas to explain mathematical rules, procedures and problems.

योगे खं क्षेपसमं वर्गादी खं खभाजितो राशिः	त्र्यरुत्ते गुणदा शुन्यै शून्यते गुणदापनि ।
खहरः स्यात खगुणः खं खगुणश्चिन्सश्च शेषविधी ॥	वर्गफेर्मुलमा शुन्यै खहा भौगर्दीदा त्रनि ॥ १ ॥
शून्ये गुणके जाते, खंहारश्वेत पुनस्तदा राशिः	योगान्तरमहां काम अरू वाकी हुँदापनी ।
अविकृत एव ज्ञेयस्तथैव खेनोनितश्व युतः ॥	राशित्रविकृतै जान शून्ये छेदुगुण् भयेपनी॥ २ ॥

Eight rules of arithmetic with zero in Pandey (1884)

Six rules of arithmetic with zero in Dhananjaya (1933)

Figure 3.3: Rules in Shlokas

How do the authors treat zero?

Interesting treatment of zero can be found in (Pandey, 1884) and (Dhananjaya, 1933). Figure 3.3 provides a screenshot of rules with zero given in Pandey and Dhananjaya. Pandey copied the Sanskrit shloka from Lilavati as it is. Dhananjaya gave a completely new shloka in Nepali to explain these rules. One change I noticed in Nepali shloka is that Dhananjaya removed the part where it talks about addition and subtraction with zero. Therefore, the rule reduced down to six rules from eight rules given in Sanskrit shloka. Dhananjaya provides examples in which addition and subtraction to and from zero is necessary. Table 3.4 provides a translation of the Sanskrit shloka taken from (Patwardhan, Naimpally & Singh, 2001, p. 47). My translation of Nepali shloka in Figure 3.3 is provided in the right side of Table 3.4.

It is fair to say that Dhananjaya was translating Sanskrit shloka into Nepali shloka without changing much information given in the original Sanskrit shlokas. Even the coherence of the presentation is kept intact. Out of 137 shlokas given in *Series of Lessons for Children*, a good number of shlokas seem to be a direct translation of Sanskrit shlokas

in Lilavati. Other two books considered in this study treat zero as a placeholder and do not mention the multiplication and division by zero at all.

Table 3.4: Translation of Shlokas in Figure 3.3

Pandey (1884)	Dhananjaya (1933)
If zero is added to a number, the result is the same number;	Zero multiplied by any
the square etc. (i.e., square, square-root, cube, cube-root) of	[number] is zero. [Any number]
zero is zero; any (non-zero) number divided by zero is	multiplied by zero is zero. The
khahara, i.e., infinite; the product of a number and zero is	square and square root of zero is
zero.	zero. [Any number] divided by
(If in some mathematical calculations, multiplication and	zero is infinity. If there is more
division by zero are likely to occur frequently then, though a	work to be done, the zero
number multiplied zero is zero,) one should maintain the form	divided by zero remains
of multiplicand and multiplier zero in rest of the operations	unchanged.
(until the final operation is reached). This is because if a	
number is multiplied by zero and divided by zero then the	
result is the (former) number. (Patwardhan, Naimpally &	
Singh, 2001, p. 47)	

ेश्लो०।ईइच्छा परमान् समान पनि हुन फल् हुन्छ सिन् पनी। पैल्हे लेखन प्रमार्या मध्य फलह इच्छा किनामां अमी ॥ इच्छाले फलगुन्न तेस विचमा भाग् ल वर्वायां ले जो झायो त्य लब्ध जनहो वयन त्तले ॥ १२ ॥

CIFAI पचाख SIGUI ? ৱতলাৰ गयो न्त्रासाक GI पश्चिस द्रव्य हिया मिल्छन भनिभन्य दशहरुये यदी हिडगा जिल्लन कली ती पनी ॥ १३ ॥

Rule of Three in Swar (1900)

Example related to Rule of Three in Swar (1900)

Figure 3.4: Rule of Three and Examples in Swar (1900)

Table 3.5 provides a translation of shlokas given in Figure 3.4. The rule of three on the left side of Figure 3.4 and the example related to rule of three given in right side of Figure 3.4 are both written in shloka (Swar, 1953, pp. 51-54).

The given words *Ichhya, praman, phala* and *labdha* translates to desire or the requisition, scale, fruit, and the desired result. The rule is essentially saying if we were to calculate a desired result given three things, we need to multiply desire by fruit and divide by the scale.

An equivalent rule from current grade 8 book reads:

In direct variation, the value of unit quantity is obtained by dividing the value of any quantity by that quantity and the value of any quantity is obtained by multiplying the value of unit quantity by that quantity. (Shrestha, Karki, & Bhandari, 2015d, p.60)

	_
Rule of Three in Swar	Example related to Rule of Three in Swar
Ichhya, praman are of	If five hundred pomegranates cost fifty rupiya,
similar type. Phala is the	how much does it cost for one hundred
smaller one. First we write	pomegranates?
the praman [on the left	If one thousand apples are sold in eighty-five
side], phala in the middle,	[rupiya], how much does it cost for four hundred
and <i>ichhya</i> on the edge.	[apples]?
Multiply phala by ichhya	Twenty-five <i>drabya</i> can buy eighty-five good
and divide by praman. The	pomegranates, then how many pomegranate does
result is a <i>labdha</i> .	ten <i>drabya</i> buy?

Table 3.5: Translation of shlokas in Figure 3.4

Table 3.6: Problems in current Nepali mathematics textbooks

A few exercise problems in current mathematics books in various grades Deepak had Rs 2645. Sony gave him Rs 4675. How much money does he have altogether? (Shrestha, Karki, & Bhandari, 2015, p.67)

The monthly expenditure of Khadka family is Rs 12500 on food, Rs 2400 on fuel, Rs 6500 on education and Rs 9475 on miscellaneous. Find the total expenditure of the family in a month. (Shrestha, Karki, & Bhandari, 2015, p.67)

Hari earned Rs 85,250 in a year and spend Rs 78,375. How much money did he save in the year? (Shrestha, Karki, & Bhandari, 2015a, p.59)

If 6 men can finish a piece of work in 12 days, in how many days would 18 men finish the same work? (Shrestha, Karki, & Bhandari, 2015b, p.116)

In a barrack, 250 soldiers have food enough for 40 days. How long would the food enough for only one soldier? (Shrestha, Karki, & Bhandari, 2015c, p.102)

Tap A can fill a tank in 3 hours and Tap B can fill it in 6 hours. If they are open together, in how many hours the tank can be filled completely? (Shrestha, Karki, & Bhandari, 2015d, p.67)

Are there any answers/solutions provided?

Pandey and Pant provide a list of practice problems at the end of each chapter with answers, whereas, other two books provide mostly the worked-out problems.

Table 3.7: Definitions of GCD and LCM from old and modern books

A number which will divide two or more numbers evenly is called a common divisor [of the numbers]. The largest of such divisors is called the greatest common divisor. (Dhananjaya, 1933, p.9)

The greatest number that divides the given numbers without remainder is called H.C.F of those numbers. (Shrestha, Karki, & Bhandari, 2015b, p.42)

By reducing [dividing] all the numbers [by some numbers] until there is no remainder, keep the numbers separately. The product of these numbers with the numbers [which appear to be remainders] is called the least common multiple by mathematicians. (Dhananjaya, 1933, p.12)

Lowest Common Multiple (L.C.M) of two or more numbers is the smallest number that can be divided by those numbers without leaving any remainder. (Shrestha, Karki, & Bhandari, 2015b, p.45)

4 Concluding Remarks

These considered categories and identified historical features such as: the contextual problems, old fashioned units of measurements, presentation style of the book and problems and solutions may be a useful resource for current teaching and learning of mathematics. These may be directly used in the classroom, in teacher trainings, and in textbook writing. Avital's (1995) suggestions to replace current problems with old ones might improve instruction and learning of mathematics. Empirical research is needed to confirm the usefulness of these sources and features, which is clearly a future direction. However, as discussed in Jankvist (2009) these primary historical sources can be used both as 'history as a tool' and 'history as a goal'. As suggested in Freid (2013) these historical contexts can be used as a 'history as a tool' to assist students and teachers in the learning and teaching of mathematics, and 'history as a goal' to show that mathematics is a part of evolution of society and that mathematics has been developed and changed over time. As noted by (Schubring, 2011), these features can be helpful in teacher education to provide the meta-knowledge of mathematics so that it will eventually help increase the meta-knowledge in teacher and student. Glaz and Liang (2009) observed an improvement in student learning by using poetry as a pedagogical tool in a mathematics classroom, I do see an ample opportunity to use these shlokas in classrooms to enhance the teaching and learning of mathematics for both teachers and students alike. Moreover, these primary sources can be a resource for multidisciplinary study such as literature, linguistics, poetry, sociology, history, and Sanskrit study. Many topics observed in Table 3.2 can be identified in modern day primary schools' curriculums and textbooks (MoE, 2007; MoE, 2009; MoE, 2014; Shrestha, Karki, & Bhandari, 2015, 2015a, 2015b, 2015c, 2015d), and the topics such as series, compound interest, false position, double false position and topics in geometry are present in current school curriculum, so there is an opportunity of using these contents in different levels of school today. The creation of guided task based projects similar to TRIUMPHS projects, and use it in an empirical study to test the usability and usefulness of these resources is my future direction.

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