## THE MAGIC OF THE EAST - FROM THE ALHAMBRA TO THE SAMMEZZANO CASTLE

#### Symmetries in mathematics, nature and art

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#### ABSTRACT

The lack passion some students have for mathematics is known. Understanding that the basis for further mathematical skill development is determined in the primary schools, we chose a project involving 9 to 14 year olds from the Reggello school (Florence, Italy). The project is part of a multi-year course titled "The Pleasure of the Beautiful" with the idea of appreciating the beauty of mathematics and its relationship with other disciplines (art, history, geology, ...). In this project we chose to talk about symmetry as a key reading idea to understand art and architectural works, considering it is one of the central ideas of mathematics of the twentieth century. The opportunity was offered by the presence in our landscape of the Sammezzano Castle, a beautiful Moorish style building, built in 1605 by the Ximenes D'Aragona family and restored by Ferdinando Panciatici between 1853 and 1889 according to Orientalist fashion, then in vogue in Florence. The castle is rich in geometric decorations and tiles with reference to the Alhambra mosaics.

#### **1** Introduction

At the beginning of school year, I used to get my first-class pupils to write a composition "Maths and Me" asking them to express openly their opinions. Here are some of the things they said: Elisa (first class) "I don't like maths at all, when I was in primary school, I was one of the best pupils, the same in the first class of middle school. In the second class I began to have difficulties understanding the teacher's explanations.....I would find myself shaking every time the teacher gave back tests". Gabriele (first Class) "Maths and me – like a cat and dog". Giulia "Now I'm in the secondary shool, and I think that I must live another five years with maths, I feel really unhappy. One thing is for sure. When I finish school I will never have anything more to do with maths".

In the light of such comments, our task was not easy, how could we change the opinions of our students about maths and let them appreciate the beauty (the actual beauty) of a subject which people consider arid and lacking in poetry? For this reason we worked on a project, lasting four years, which involved some classes of Primary and Secondary school. This project is called "Il piacere del bello" (The pleasure of beauty). Why have we titled this project "The pleasure of beauty"? Perhaps because we live in Tuscany, where the Renaissance and the Medici have left testimonies of incomparable beauty, as the architect Renzo Piano wrote to the inauguration of Columbias University Art School. Beauty is not only that of consolidated art, it is emotion, research, knowledge, discovery; it is also political gesture, it gives strength to desires and is one of the human emotions capable of competing with those more dangerous: money, power and conquest.

The mathematician's patterns, like the patterns of poet or painter must be beautiful – the ideas like the colors or the words, must fit together in a harmonious way. Beauty is the first test, wrote G. H. Hardy in his book *A Mathematician's Apology*, 1940. It is impossible to be a mathematician without being a poet in soul. (Sofia Kovalevskaya, *Recollection of* 

*my chilhood*, 1885). Mathematics is perceived as a new relationship between the lightness of the ideas and the heaviness of the world. (Italo Calvino, *American lessons*, 1985). In our opinion, one reason (not the only one), for studying mathematics is to educate the eye and the ear of mind to be able to see it or to feel this beauty. So, why mathematics appears beautiful this reason is necessary, although it is not measured, in case it is dampened. Creative thinking needs time, space, serenity, curiosity and enthusiasm: it must be able to conduct researches without restrictions, excessive fears or anxiety.

## 2 Interdisciplinarty of the project

The continuity between different orders of school and interdisciplinarity is the key to unify the different disciplines. "The supremacy of a fragmented knowledge, according to the discipline, must be replaced with knowledge capable of grasping mutual relations and mutual influnces between parts and a whole complex world" (Edgar Morin, *Manifesto pour changer l'education*, 2004).

Our project with the recovery of manual dexterity, carried out through the laboratories, the involvement of the students, who themselves become teachers to the other students and promoted initiatives towards adults with the realization of the works carried out with the involvement of the territory and of the family, make students the protagonists of its own knowledge. With the laboratories, learning recuperates its formative value, we recover the place where we search, we experiment, we give meaning to the students to work in harmony with their curiosity.

### 3 Italian school structure

Education in Italy is compulsory from 6 - 16, and is divided into five stages:

Pre-School (scuola dell'infanzia / not compulsory / 3-5 years), Primary School (scuola primaria / 6-10 years), Lower Junior High School (scuola secondaria / 11-13 years), High School (scuola superiore / 13-16 years), University (università).

Italy has a both public and private education system. In Italy a state-born school system has existed since 1859, when the Legge Casati (Casati Act) was published.

Our school is for students from age 3 to 13 years. It includes three stages: Pre-School: 301 students, Primary School: 532 students; Lower Junior High School: 315 students

## 4 Methods and teaching strategies

The 8 key competences of European Union are: Communication in the mother tongue; Communication in foreign languages; Maths competence and basic competences in Science and Technology; Digital competence; Learning to learn; Social and civic competences; Cultural awareness and expression; Sense of initiative and entrepreneurship. In our school we have five labs: music, ceramic, physical education, science and maths.

## 5 The territory of the project

Our territory is located in Italy, in the central part of the peninsula. The area is about one hundred kilometers from the Tirrenian sea. We lie in the shelter of the Appenines mountains. This mountain chain passes through the entire peninsula like a spinal column. We are in Tuscany, near Florence, in a district named Upper Valdarno because of the

location in the upper part of the Arno river valley.

In our valley the Arno river flows through its central part. The river goes then to Florence and on to the sea. Sammezzano Castle is located in this valley. On the east side we have the ridge (about one thousand and six hundred meters above the sea level); on the west side we can see the Chianti mountains (about nine hundred meters above the sea level). The lower part of the valley is about one hundred meters above the sea level. Here the valley is like a trough. This hilly area has a very peculiar aspect. It is very interesting for many aspects: first of all the geological one, but also naturalistic and environmental ones. The name of this area is "Balze park". "Balze" is the name of particular rock outcrops that stretches for many kilometers across Upper Valdarno. A possible translation could be cliffs.

# 6 History of the Sammezzano castle and Ferdinando Panciatichi Ximenes d'Aragona

The castle of Sammezzano got its current appearance thanks to the work of one man: Ferdinando Panciatichi Ximenes d'Aragona (Florence, 1813 - Sammezzano, 1897) who renovated and expanded the pre-existing building during the period between 1843 and 1889. The history of the place however is significantly older: it is possible to trace it back to the Roman period, as well as to the subsequent centuries. In the *History of Florence* written by the great historian Davidsohn, it is reported that in 780 Charlemagne may have passed by the place on his way back from Rome, where he went to have his son baptized by the Pope. Ferdinando was owner and builder at the same time; although he had no university degree, he had the abilities of an engineer, an architect and a geologist. This allowed him to design, plan and finance the castle by using local products and employing mostly native manpower. The cultural wave of "orientalism" spread around Europe at the beginning of the nineteenth century and Florence was one of the main centers. Influenced by this movement, Ferdinando started to modify the existing Sammezzano structure and realized new halls.

As an expert and passionate botanist Ferdinando planned a vast area surrounding the castle of approximately 65 hectares, the so-called historical park. Around the ancient "Ragnaia", which was characterized by a great forest of evergreen oaks, he recreated the habitat for numerous rare and exotic plants. His intent was to prepare step-by-step visitors and guests to the magnificence of the "Moorish" style of the castle.

In the late 90's, after several events, the property passed into the ownership of a group of English and Italian societies. The society plan was to perform an important renovation and re-open Sammezzano for tourism. However, no actions have been realized and the castle remained unused until today.

#### 7 Geology lab. Upper Valdarno geological evolution

The Castle location in the "Balze area" was the occasion for an interdisciplinary work comprising not only mathematics, art and history, but also geology. The students have discovered the geological history of the area using books, internet, magazines, the direct knowledge of the places where they live, and the observations of rock outcrops. For example, the students have drawn a section of their territory, starting from the bottom of the valley to the top of the mountain. They noted the presence of clay, silt, sand, gravel, pebbles and huge blocks of rock: deposits usually carried by the rivers. They understood these materials had been eroded, transported and deposited by torrents. Then they noticed that the clasts' size increases from the valley to the mountain. They answered: "This is because we are closer to the place where the rocks have been eroded." They noted that the Valdarno consists of a kind of erosion plateau, so they confirmed what they had already heard in their childhood: in this area there was an ancient lake. This lake was filled with debris coming from mountain erosion. Then the materials that filled the lake were in turn eroded and gave rise to the Balze.

Through research on the Internet they also found more detailed explanations that we can observe in some slides taken from a work of the University of Florence, Department of Earth Sciences, in which we see the evolution of the area. The students then carried out research on the fossil record. In fact, the bodies of many vertebrate mammals, that then became fossilized, were deposited in this lake during Pliocene and Pleistocene (Paleontological Museum of Florence and Montevarchi). The suggestion of the "Balze" has most probably inspired Leonardo in the landscape of the Gioconda painting. According to some scholars, this is not true. However, we have proof that Leonardo studied the Valdarno area from his writings. For example, in the Hammer Code, in which the scholar talks about the Valdarno and the Pratomagno.

#### 8 The project in the primary school

The Hand touches, the Brain says (Rita Levi Montalcini, Nobel Prise in Medicine, 1986).

The 4th class of the primary school in Cascia (near Reggello) composed of 16 9 yearold pupils, took part in the project. The central idea of our work is GEOMETRY that we have developed in various disciplines such as art, environmental education and history of our territory. The themes have been developed through manipulation and the construction with the use of hands. The manipulative aspects are in fact very important, in this phase of growth. Nowadays the children are "overwhelmed" by technology which, although important, has taken away from their childhood and much of their practical experience in play, movement, manual realization and the practical creativity that should distinguish this phase of age.

So our work has been based on reproduction of some geometric shapes of the Castle of Sammezzano and the Alahambra, using the square and the ruler, and on the construction of a mandala through the goniometer. The word "Mandala" means "Circle", and it is from this geometric shape that we created an infinite number of symmetrical and harmonious forms. Drawing, coloring and building a Mandala is also a satisfying and extremely relaxing activity, as well as being very effective to acquire the knowledge of angle and geometric shapes.

The steps of our program are: visit to castle of Sammezzano, the geometric transformations, reproduction of Sammezzano tiles, creation of our tiles using goniometer, ruler and especially our imagination. Finally the older pupils taught Mandala's constructions to the younger ones.

#### 9 The project in the secondary school

In the secondary school the same topics are gradually studied in more detail by developing the use of a more symbolic language, more specific to mathematics, but continuing to develop practical skills still using ruler, paper, pen, protractor, compass, glossy paper, mirrors, multimedia board and computer. Here we have a more systematic and analytical analysis of the same topics concerning the symmetries faced by primary school pupils. However, we tried to leave space for students' imagination and creativity without forcing them into too tight schemes. In this way the students have worked for a long time amusing themeselves.

We used the traditional blackboard and also the electronic dashboard. We created some examples from the notebooks concerning the rotations using the glossy paper. We treated the translations analytically using Cartesian coordinates, without forgetting the symmetries of the Castle of Sammezzano!

The students worked with mirrors in axial symmetry, succeeding in doing it thanks to their manual skills, to understand these geometric transformations in more depth. The application of symmetries and tiles was made by the guys using also the free Geogebra software.

The work concerns the tiles of the plane using transformations of central symmetry. The students have tried with triangles, quadrilaterals (convex and concave) and have always managed to fill the plan, deducing that it is possible because the sum of the internal angles is 360 degrees for the quadrilaterals and 180 degrees for the triangles. Then by joining the figures you can form a central angle without empty spaces, while they could not do it with the regular pentagon (internal angles 108°). With the regular pentagons is not possible to tile, empty spaces remain. We have an anecdote about a 8 year old primary school child: during a lesson on tiles, a teacher gave various geometric figures to the pupils (triangles, quadrilaterals, regular hexagons, regular pentagons). All the pupils succeeded except one: he had received the pentagon so he began to cry.

Here we have another example of the lesson on tiles given in the secondary school. This time the students worked in the three-dimensional space. During a lesson the students tried to make a dome using hexagons. They started by observing that with the regular hexagons it is possible to tile the plane. The materials used were: toothpicks, straws, cardboard, scissors and glue. They realized they couldn't get a dome using only the hexagons, that very well tiled the floor instead (they believed that they could apply the same laws to tile either three-dimensional space or the plan) ... the solution was at hand in front of everyone ... the football! Just adding the regular pentagon (maybe you could comfort the child who cried because he could not tessellate the plan with the pentagons!).

This experience was very useful also in the study of solids, in particular for the properties of the solid angles.

The students have applied the concepts learned from the symmetries and tiles to natural sciences that are an integral part of the scientific teaching in the secondary school.

They discovered the microscopic organisms (radiolarian, diatoms) that make up the marine plankton that have a siliceous skeleton that appears under the microscope in wonderful shapes. As the Aulonia exagona, which used hexagons and some pentagons to produce its skeleton in space. Then they again found symmetries in 3D space in the mineral world. They did various researches on this topic and some videos as well.

Again large-scale axial symmetries in the mid-ocean ridges, in the oceanic expansion, in particular the very interesting symmetry between normal and inverse magnetic polarization bands with respect to the medioceanic ridge, brought by scientists as a proof of the plate tectonic theory.

Also the music teacher with some students put the attention on the symmetry in music,

in particular very interesting the experience of the German Chladni who put into practice the saying of Pythagoras "Geometry is solidified music". Sand on a smooth metal plate that forms symmetrical and different interference figures according to the frequency of the sound.

The students participated in a ceramic workshop, with a reproduction of a bas-relief of the Castle of Sammezzano.

Finally we organized a trek in the Balze's area in collaboration with the local environmental association and the involvement of the family.

At the end we realized the conference and the exposition of the students' works to which the families, the town council, members of the committee, and local associations were invited. The students directly exhibited their works in front of a crowded audience in the town council auditorium.



Some phases of a lab about the translations.



Ceramic lab



Some phases of the construction of a dome



Trekking in the "Balze park"