

# BOREL'S APPROACH TO MATHEMATICS, PROBABILITY AND CITIZENSHIP

**Alain BERNARD**

University of Paris Est Créteil (ESPE), rue Jean-Macé, 94380  
Bonneuil sur Marne, France and Centre A. Koyré (UMR8560)  
alain.bernard@u-pec.fr

## ABSTRACT

This paper proposes to reflect about contemporary issues on the place accorded to probability and statistics in the French mathematical curriculum, from an examination of the issues raised one century earlier by mathematicians like Emile Borel. For this purpose, we present and comment a series of selected quotes taken from two interesting papers, published by Borel in 1906 and 1908 in the *Revue du Mois*, a journal he had just founded with his wife. The first paper concerns the practical value of probability calculation, while the second bears on issues of its psychological and social acceptability. We show that the two papers are structured around the same kind of opposition between the mathematician's view on probability theory and its value, and the point of view of laymen opposing the latter's pretention to reduce decision making or social facts to calculation. We explain how Borel defines an original compromise between these two views, leading to a definition of the limits and strength of the application of probability to reality. We furthermore show that Borel's argumentation should be understood against the political and epistemological doctrines of his contemporaries. For this we comment on two papers published in the *Revue du Mois* of Alfred Croiset on the teaching of secular ethics, and Célestin Bouglé on the political doctrine of solidarism. Beyond showing the similarities between their argumentation and Borel's, we propose to explain this through the editorial background behind the publication of Croiset's paper. We conclude with some possible perspectives on present-day teaching concerns, that could be drawn from this historical interpretation: the first touches on the ambiguities of the probabilistic vocabulary; the second on the search for problems combining math teaching with the education to critical thinking; the last bears on the possibility of making interdisciplinary bridges between the political history and the rise of probability as a subject of public concern.

## 1 Mathematics, probability, citizenship: contemporary concerns and old issues

### 1.1 A contemporary dispute about probability and statistics in the French curricula...

One of the most significant changes in the French mathematical curriculum for secondary schools in the past two decades is the introduction of a new approach to statistics and probability. The latter combines a frequentist conception of probability and an experimental methodology to problems involving random processes, with the use of traditional models based on epistemic probabilities (Dutarte, 2011).

This change has been controversial from its very beginning, because it diminished the place of other important subjects in the mathematical curriculum, like geometry and geometrical thinking. This represented, therefore, a significant change in the underlying philosophy of mathematics: less emphasis on geometrical demonstration and calculus, more on model-making and experience in relation to the call to introduce and study of "real-life" problems. Among these problems, those calling for a critical reflection on the role of statistics and probability in society were given a significant role<sup>1</sup>.

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<sup>1</sup> See, for example, the following resources produced by the IREM of Paris Nord (Dutarte et al., 2007).

## 1.2 ... reminding old issues

What is striking in retrospect is that this mixture of epistemological and political debate on the very nature of mathematical thinking on the one side, with a reflection on the importance of statistics and probability for the modern notion of citizenship, on the other, is hardly a new one. Already at the beginning of the 20th century eminent mathematicians such as Henri Poincaré, Vito Volterra and Emile Borel had called the attention of mathematicians and of the educated elite, on the significant changes implied by the role of probability in science, industry and insurance or daily life in general (Durand & Mazliak, 2011).

Emile Borel and his later followers like Maurice Fréchet (1924), were especially influential in disseminating these ideas and pushing the idea that these questions should imply a reform of mathematics teaching, by the systematic inclusion of statistics and probability as a key subject (Courtebras, 2006, p. 94; 114). For them, it was not just an idea of augmenting the curricula with new subjects, but also reflecting on the value of mathematical knowledge as compared to scientific knowledge in general.

## 1.3 The principle of this paper: making out some of the reasons for Borel's interest on probability, through a series of quotes

The reasons why Borel, in particular, paid attention to the role of probability in contemporary science, industry and society around the years 1905-6 have been well studied in recent works (Durand & Mazliak, 2011; Mazliak, 2015; Mazliak & Sage, 2014). These studies show, in particular, that this interest is inseparable from Borel's efforts, in association with his wife Camille Marbo, to enlarge his intellectual relations and, by the same token, to promote his first political career through the foundation of a new journal, the *Revue du Mois*. In particular, two articles published by Borel in his own *Revue* (1906, 1908) are highly revealing of what were his key ideas and intentions concerning what he called the practical value of probability theory, on the one hand; and the ground reasons why probability should be made a subject of public interest and culture, and ultimately the subject of an institutionalized teaching, on the other.

In a recent study on the contents and form of argumentation of these articles (Bernard, *forthcoming*), I found out that they had to be understood (among other contexts) against the background of contemporary debates about the foundation and teaching of secular ethics. This subject was in 1905 of great topical importance, due to the passing of the so-called law of separation of Church and State in France. This means, then, that part of the reasons for Borel's interest on probability theory, were related to political and philosophical reasons.

While there is no point at summarizing here the details of this complex story, implying many of Borel's contemporaries in this crucial period both his career and of French political history, I found interesting to render the flavour and style of these debates through an ordered series of quotes taken either from Borel's two papers mentioned above, or from other articles or conferences bearing on questions of secular ethics that were either published in Borel's *Revue* in the same period (1906-1908).

Among the reasons for this choice, is the simple fact that the articles mentioned here are beautifully written and thought-provoking, due to the special attention that the contributors to the *Revue* paid to remaining accessible to their readers (Ehrhardt & Gispert, 2018). Moreover, most of them are inaccessible in English, for even the major

book of Borel on Chance (1914) has received no translation. Finally, some of these quotes contain useful reflections that could inspire useful thought for teaching mathematics in a meaningful way<sup>2</sup>.

## **2 Borel's two papers on the practical value and acceptability of probability calculation**

The two papers in question (1906, 1908) were of high importance to Borel: this is revealed by the synoptic presentation that Borel made of their contents when he published in 1914 his first “grand public” synthesis on issues of probability. Introducing the third part of his book, he explains the following:

**Q1:** “We strove, in the first part of this work, to expose how one has succeeded in submitting the laws of chance to calculation. Then, in the second part, we have quickly explained the main practical and scientific applications of the methods thus created. We must now ask ourselves what the practical, scientific and philosophical value of these applications is; should one consider that their value is diminished by whatever mysterious connotation attached to the word “chance”, or should one consider that the comprehensive study of laws of chance will best tell us something about the value of any human knowledge?”

To deal with these essential questions, it seems necessary to me to reconsider the problem of chance *ab ovo*, so to speak, regarding the previous chapters as a mere introduction aimed at providing precise examples for the forthcoming reflections. In this spirit, we shall first examine (...) the value of probability in practical life and then look for how the individualist sensitivity is often an obstacle for the acceptance by many people of conclusions that nevertheless impose themselves to their reason. We will end up by brief indications on the role that the theory of chance could play in ethics based on solidarity and in the evaluation of the social value of individuals.” (Borel 1914, 213-4)

This introduction is directly followed by the quasi word for word reproduction of the two articles in question. We shall point out some key characteristic of the argument with selective quotes.

### **2.1 The practical value of probability (1906): a dialectical discussion, leading to an original position on probability theory**

Borel's 1906 paper begins with a dialectical confrontation, on which his whole argumentation is based. This is illustrated by the introduction:

**Q2** “*Probability calculus is the study of laws of chance.*”

One has already remarked that this definition does explain a contradiction by another contradiction. If one does not understand how one can speak about *calculation* in association with *probability*, one will even less understand that it can be question of *laws* in relation to *chance*. Is not chance precisely what remains outside any rule or law? And does not everyday experience teach us to be wary of laws, to which one pretends to submit fortuitous events? Does it not happen

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<sup>2</sup> For more details on my choice of quotes and its rationale, I forward the reader to my paper and the other studies mentioned above.

frequently bizarre coincidences, strange accidents that are apparently contrary to any probability?

‘Well, such events do happen actually, would say the mathematician, but less frequently than it appears, and their frequency is itself governed by the laws that their occurrence seems to contradict. One can furthermore provide excellent financial proofs for the rightness of probability calculation: a well-managed insurance company does always make profits; and no roulette game company has ever ruined its manager.’

‘But concerning the latter example you therefore hold for impossible, would answer the opponent, that the roulette company might be ruined? It might nevertheless be enough for me to be the only player and to play constantly the number that will appear. It might be very unlikely that I would have such flair, but this is obviously conceivable. What, then, of your principles?’” (Borel 1906, 424)

The originality of Borel’s argument lies in the fact that he takes this debate in earnest, insofar as it points out a ground dilemma on the very notion of probability. The first part of his paper takes up and legitimates to some extent the position of the „contradictor“; the second defends the rights of the mathematician to declare impossible in practice, certain events of very small probability<sup>3</sup>. We shall here only give an idea of the epistemological discussion contained in the first part. It is first introduced by small problems of the following kind:

**Q3.** “Peter exactly possesses one million [francs]; one proposes him to flip a coin for it, against one million (fair game) or even in exchange for one million and fifty francs (theoretically advantageous game). Unless very particular circumstances, it is clear that the interest of Pierre is to refuse.

Jacques is isolated, with no contacts, in a remote country; he is fairly rich and should receive tomorrow a large sum of money; but he has a major interest in taking a boat that leaves within an hour and he has only 300 francs with him, although the boat crossing, to pay paid in advance, costs 400 francs. One proposes to toss a coin for his 300 francs, in exchange of 200 (theoretically disadvantageous game); his interest is obviously to accept.” (Borel 1906, 426)

These and other examples of the same kind, lead Borel to argue that expressions like “mathematical expectation” (*espérance mathématique*) are essentially misleading, because the calculation in itself does not express what one should really *expect* from a game. The calculation might be exact; it still has no practical value, because decision making has other incentives than a mere numerical result, like one’s character or economic condition:

**Q4.** “[one should] renounce to speak about what one calls *mathematical expectation*, or at least to understand this expression has only designating a quantity that it is often advantageous to introduce in probability calculations. But it should not be taken at face value; one gives to Peter a ticket in a lottery in which the unique prize is 100 000 francs within one million tickets; the mathematical expectancy is ... 10 cents. But, if he pessimistic and starving, he will prefer by far to be given two

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<sup>3</sup> This second part of the argument leads to arguments akin to Borel’s famous evaluation of improbable events by the estimation of the probability that „typewriting monkeys” would reproduce the books of the French national library within one year.

pennies<sup>4</sup> to buy his bread; whereas if he is imaginative, he will be for one day as happy as if one had given him the 100 000 francs. By evaluating his mathematical expectation, one has perhaps made an exact calculation, but with no practical value.” (Borel 1906, 427)

Borel develops further this idea with other, slightly more sophisticated examples ultimately leading to the idea that probability calculation yields no certitude, but has only what he calls a relative value. It only serves to simplify a complex problem into simpler data, and nothing more; it does not provide infallible criteria for decision, but only constitutes one component of it. This is what he summarizes through the following, interesting sentence:

**Q5.** “The practical value of probability calculation thus appears to us as relative; the practical problem to be solved is simplified in its terms, but not modified in its essence: it keeps being a problem of probability.” (Borel 1906, 430)

By this Borel does apparently not mean a problem in which mathematical probabilities should be calculated, but a problem of decision making through the pondering of various factors which do not depend only on calculation.

Then Borel generalizes these thoughts on any kind of application of mathematical calculation to practical decision and raises one of his main points, about the very possibility of discussing numbers:

**Q6** “The intervention of calculation in the decision of practical life gives often way to two of the following, extreme judgments; for some people, it is absurd to mix up calculation to any decision which implies elements that cannot be expressed in figures; for others, figures have a magical power that make infallible anyone who used them according to the rules.

These two opposite tendencies actually correspond in fact to the same state of mind; this is only because figures do appear to them to have an absolute value eliminating any kind of discussion, that certain minds fear about their invention and prefer to do without their help rather than subjecting themselves to their yoke.” (Borel 1906, 431)

Beyond any matter of psychology, Borel immediately points out that the origin of this illusion is related to the „magic of calculation“ itself, meaning that:

**Q7.** “... one can only do calculation on precise figures; at least, it is quite long and difficult to look for the diverse solution of a problem, that correspond to the diverse values of imprecise data, and one shrinks from this task. One is then led to adopt determinate values for each element of the calculation, even though this element is not known accurately.

On these precise data, one can effectuate precise calculations that lead to precise results. And the longer the calculation is, the more the result might be inexact and the more, nevertheless, we have time to forget that the data were imprecise and to let oneself trust in the exactitude inspired by correctly done arithmetical operations.

The same illusion frequently occurs in statistics; one reads every year in the journals that the total production of wheat in France is evaluated, for example, to 115 200 000 quintals<sup>5</sup>. This figure was obtained by the addition of many particular

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<sup>4</sup> That is, ten cents in Borel’s time (*deux sous*).

<sup>5</sup> A quintal is a hundred kilograms.

indications, each one of which was inexact; the mass, though, inspires a certain trust and one is ready to make economical deductions from it.

Let us be wary then, about trusting too much in figures; this would be the best way to avoid the contrary excess, which is to deprive ourselves of their help. (...) probability calculation then appears as justified as any other calculation; its practical value is just the same.” (Borel 1906, 431-2)

As mentioned above, the end of Borel’s paper insist on the fact that, in practice again, very tiny probabilities should be taken as equivalent of strict impossibilities. The ground argument is very similar: if we had to refuse this in daily life, or even in scientific conclusions, then life would become impossible because any event, however improbable, would have to be taken in consideration.

We thus see the very special mixture of epistemological thought on the value of mathematical knowledge at large, and psychological considerations about its use for practical decision. All this is the seed of a more general reflection on why (probability) calculation is acceptable or not to public understanding. Let us now see how these arguments are retrieved and developed in Borel’s 1908 paper, in the direction of discussing the social acceptance of probability calculation.

## **2.2 Borel’s defense of probability calculation and “social mathematics”, against “individualist sensibility” (1908)**

Borel’s 1908 paper begins with a very similar argument than the previous one, in the sense that a dialectical opposition is built between the „golden chain“ of those famous mathematicians and philosophers who, from Pascal to Poincaré and Bertrand, have developed the theory of probability; and what Borel designates as the ‚sensibility‘ of people who, though they are ready to accept the conclusions of theories of probability on the authority of such luminaries, are, nevertheless, reluctant to do so:

**Q8.** “... [for] this is not their reason, but their sensibility that is shocked by the conclusions drawn from probability calculation; or at least, from the manner they understand and interpret it. (...)

While trying to understand in depth the reasons why probability calculation is antipathetic to many minds, I hope to demonstrate that this antipathy mostly relies on misunderstanding; it would be desirable that such misunderstanding would be dispelled; for the vulgarisation of the conclusions, if not of the methods, of this branch of science, would have a great social utility.” (Borel 1908, 642)

The notion of “individualist sensibility”, on which Borel relies here, is directly taken from the contemporary writer and philosopher George Palante, for which the antinomy of individuals vs society was a key notion of his worldview (Depenne, 2015). Borel quotes an excerpt of a spiritual article published the same year by Palante in the *Mercure de France* (1908), the quote containing an obvious charge against the kind of political opinion and party to which Borel adhered:

**Q9.** “Pascal said: ‘the more spirit one has, the more one finds how many original people there are. Laymen do not see any difference between men’. The social and gregarious sensibility (*sensibilité*) delights itself with the banality of traits; it likes that one is ‘like everyone else’. The Christian, humanitarian, solidarist and democratic sensibility would like to erase the differences between egos. Amiel sees in it, with some justice, the sign of a rough intellectuality.” (Borel 1908, 642,

quoting Palante)

The allusion to the so-called *solidarist* sensibility, as opposed to the individualist one that Palante advocated, goes to a political doctrine of the same name, developed in Borel's time (Hayward, 1961) and to which he adhered very soon, as we shall see later. Borel's point consists in generalizing Palante's remark, which originally applies only to elite minds, to anyone. We all care, says Borel, for our individuality and prefer to think ourselves different from the others – a tendency which is directly opposed to the notion that we might enter anonymously into a statistical account. But Borel immediately adds that this is not the same point of explanation, opposing statistics to probability:

**Q10.** “Probability calculation does not confound itself indeed with statistics, to which the aforesaid can also be applied; it does not contend itself to count past events, but also pretends, to some extent, to forecast future events: in this, it is a science. This pretention hurts from the outset the psychological feeling of human liberty (the metaphysical value of which is not at stakes here); one asserts that, if there is no exceptional event like war, earthquake etc., there will be for sure more than 1000 marriages in France during the next week. Does it not depend on the fiancés to deny the prediction, by postponing the celebration of their union by one week? The objection does not sustain examination, but it is often implicitly admitted without examination.” (Borel 1908, 644)

The point made here is very similar that the one which constituted the point of departure of the 1906 paper. The following discussion, although based on different and more dramatic examples than those taken in the previous paper (since they bear on the classical discussion of life duration and on the advantage of inoculation campaigns) concludes with a similar point that two years before, about the relative value of probability theory. Having explained indeed, that in certain cases augmenting the global life expectation at the expense of two many losses, although mathematically advantageous, would be unacceptable for common sense, he adds:

**Q11.** “... but common sense, no more than calculation, does ensure against misfortune; and this will always be little comfort for any individual to think that the probability of misfortune was small, if he is the one who suffers from it. Anyone starving does pay little attention to the rise of average fortune: one should not look into statistics or calculation any argument to comfort those who suffer from social inequalities; but this observation does not diminish in anything the proper value of statistics nor the calculation by which one interprets them.

This is only, indeed, from a particular point of view that the statistician or the mathematician study social phenomena; this study has a limited scope, but it does constitute an exact science, when one does not pretend to extend it beyond its natural limits. One should look in it neither for moral grounds nor for immediate reasons for action: but only, as in physical sciences, for one way to know accurately past events and to forecast with some approximation future events. When one predicts that more than 100 000 Parisians will take the „metro“ tomorrow, no one among them is obliged to choose such means of transportation; one only asserts a fact that is confirmed by experience.” (Borel 1908, 648)

Borel concludes this part of the argument by recalling the 1906 comparative arguments on the estimation of improbable events, however frightening they might be, when their

probability is much smaller than that of risks we take on a daily basis. Lacking the capability of making such comparisons is ironically qualified by Borel as „head in the sand policy“ (*politique de l’autruche*):

**Q12** “Ignorance might be convenient to those who practice this head-in-the-sand policy; it is never desirable for those who prefer to see clearly and do not let themselves be influenced by the more accurate knowledge of a possible danger, when its probability is much smaller and the unknown dangers to which the most fearful man does expose himself every day. One has nothing to fear from calculation, when one is decided not to align one’s conduct on its indications, without first estimating their proper value. To think that individual independence is augmented by ignorance is a peculiar illusion.” (Borel 1908, 649)

Borel does not stop his argument at that, but come back at the very end to Palante’s passionate defense of radical individualism, but an equally passionate defense of the notion of solidarity, to which mathematics contribute, *insofar as it can be educated*:

**Q13.** “Is there therefore no ground to the opposition we pointed out between probability calculation and individualism? There is, on the contrary, a very real one insofar as individualism is antisocial and probability calculation is the basis of what one may call *social mathematics*. Its study reminds us indeed that we are living within society and that the social phenomena have real existence and their proper interest. It reminds us that, while men are different from each other in many aspects, they are nevertheless similar in that they are all exposed to accidents, disease, death; and in that their various biological aspects (...) are distributed among regular laws about certain means; and in that, finally, one can establish *laws* that are checked by factual observation and in the statement of which they are considered constitutive elements of the whole to which the law is applied. (...). Such accounts are eminently proper to limit the excesses of the individualist mentality. Such or such a disease causes, on average, a certain number of victims; hail and floods make, on average, a certain amount of damage; one does not know why some are affected and others are spared, but society, as a whole, suffers from a more or less constant damage. The study of such facts can only contribute to develop the notion of solidarity, to recall to everyone that he should not consider himself as independent of the milieu in which he lives and that he should participate in repairing the casual damages that affected his neighbor and could have affected himself. Thus, the study of probability calculation does have a great educational value; one should hope that it would be within reach of all of those who want to take part in governing men and things.” (Borel 1908, 649-650)

This passionate defense of Condorcet’s ideals is highly revealing of Borel’s own political ideals, which were largely shared by the Republican and Radical-Socialist milieu to which he belonged. This now leads us to expose some contextual elements.

### **3 The political and philosophical background of Borel’s arguments**

We here give some elements of background that might help to understand better how Borel’s political and epistemological position, both as an author and as the chief editor of the *Revue du Mois*, might help to understand his argumentation and his final goal, which



was to promote the knowledge and teaching of probability calculus.

### **3.1 Alfred Croiset on the teaching of secular ethics (1906): the role of the State in education**

The second paper published in the first issue of the *Revue du Mois*, after Vito Volterra's translation of his inaugural lecture on the role of mathematics in biological and social sciences, is written by Alfred Croiset and touches on issues related to the teaching of secular ethics (*morale laïque*).

Alfred Croiset was an important figure in the Republican and Academic intelligentsia by Borel's time. Croiset was an eminent philologist and by then the Dean of the faculty of humanities in Sorbonne. He was also at the head of the "ethics division" (section morale) of the *Ecole des Hautes Etudes Sociales* founded in 1900 by Dick May (Prochasson, 1985). In this function, he organised with much talent several series of conferences, many of them having to do with the kind of "ethics" adapted to the scientific age. The conference, of which Croiset's paper in the *Revue du Mois* is a transcription, precisely introduces to one of these series of conferences. The subject was a great topical actuality in 1905, since it bore on the teaching of secular ethics. The latter had been instituted some 20 years before in the public school by one of the most famous laws passed under the third Republic (Loeffel, 2010).

Croiset, more precisely, summarizes the basis of the problem posed to teachers by this (by then) relatively new school subject. Delivered in the framework of a religiously neutral State, it has lost the dogmatic ground that was naturally attached to confessional ethics. Recalling then the contemporary tendency to provide a *scientific* ground to modern ethics, he opposes the following, which is a reflect of the kind of 'scientific uneasiness' expressed by contemporaries like H. Poincaré:

**Q14.** "For [teachers] have often heard that the most distinguished scholars that science was not an immutable and definitive work, that it was forever provisory in some sense, and it changed with all the rest. So then, in this flow of everything, what of the absolute character of duty, which seems to be the indispensable imperative of any ethics?" (Croiset 1906, 22)

This is contrasted with the urgency attached to action, thus defining the key problem he addresses to his audience:

**Q15.** "All these facts make it therefore particularly difficult, today, the task of organizing a secular teaching of ethics. But it also makes it more urgent as ever: for we must live, and since life is action, we need reasons to act. Reality does not wait for us to get out of our ignorance or doubts." (*ibid.*)

Croiset's next turn is to oppose any idea that the ethics in question could be qualified as "scientific", in the sense proposed by his contemporaries Durkheim and Levy-Bruhl, to whom he explicitly refers. The point is that the secular State cannot have any agenda of promoting a dogmatic ethical doctrine, even a positivist one. The problem, then, as defined by Croiset in full coherence with the doctrine of the Republican state ideology of that time, is to define the role of Secular State (*l'Etat laïque*) in the teaching of ethics:

**Q16.** "Can [the State] teach any ethics? To the name of what principles? And can it do this while keeping his proper field, without impeding on that of individual consciousness, without exposing itself to the reproach of tyranny, and without

meanwhile depriving its doctrine of any virtue and strength? Such is the problem on which I call your attention” (Croiset 1906, 24)

Croiset’s essential answer to this question is interesting and revealing of the underlying state ideology above mentioned. Having ensured that the secular State is perfectly capable of giving individual an ideal and a rule of conduct, he adds in grandiose terms:

**Q17.** “This end is *the development of social life*, which is for us the highest form of human life, for it constitutes the necessary framework of any individual life; it is, at the same time, the result of all past progresses and the condition for the future progresses of humanity. The State ... is the *real, concrete instrument of collective life*, without which there is no civilization. (Croiset 1906, 25, our emphasis)

Living out the details of Croiset’s own theory justifying this idea, based on his historical understanding of the grounding of common ethics on the sedimented past, let us turn to the underlying political agenda.

### **3.2 The political background: the solidarism of Bourgeois, Croiset, Bouglé.**

For this, let us turn to another cycle of conference organised three years earlier by Croiset together with Leon Bourgeois, an interesting and important political figure of the third Republic (Croiset & Bourgeois, 1902). Bourgeois had synthesised in a coherent doctrine, soon called *solidarism*, a social and political doctrine which defined, for proponents, a kind of mid term between individualism (attached to economic liberalism) and socialism (attached to Marxism). This influential doctrine soon became the „official doctrine“ of the Third Republic and the ground for important reform defining a welfare state „à la Française“ (Hayward, 1961).

Croiset, in his interesting and nuanced introduction, recalls the reason why people of his generation felt attracted to the notion of solidarity:

**Q18.** “If individuals are, so to speak, the cells of society, the word by which biologist express the interdependence of cells, is the one that should from now designate the interdependence of individuals. (...) Our modern generation, so thirsty of positive and objective science, needed such a word that would express the scientific character of moral law. The term “solidarity”, borrowed from biology, marvellously echoed this obscure and deep need.” (Croiset & Bourgeois 1902, x)

He then explains that the solidarist spirit that thus emerged from the progressive shift from a specialized term to a political one conveyed...

**Q19** “... a kind of scientific character that pleases the spirit of our time: we love this regularity of natural and social laws, that exclude the whim of individual wills and the incertitude of feelings” (*ibid* xii)

Croiset finally counterbalances these ideal views with the claims for respecting the rights of individuality:

**Q20.** “One should avoid going too far along this way. The best things have their dangers. During our talks, one has come out in favor of individualism understood in the most elevated sense, against the absorption of individuals in the whole. (...)

All intelligent solidarist people are perfectly convinced that the love of humans for each other, as well as the complete development of individuals in social harmony, are indispensable elements of true solidarity.” (*ibid.* xii-xiii)

We thus retrieve key ideas later developed in the 1905 conference published by Borel in 1906, that we commented above. These ideas are developed in another article published in April 1906 by Célestin Bouglé on the scientific grounds of solidarism (Bouglé, 1906). Bouglé was one of the first members of the early team of scholars who, around Durkheim, launched the *Année Sociologique*, which was the banner of the newly founded sociology (Vogt, 1979). Bouglé was also a militant scholar strongly advocating solidarism (Bouglé, 1907). In a much more sophisticated manner than Croiset, because of the highly learned references to contemporary thinkers he adds to the discussion, Bouglé develops the same ideas, grounded in the same paradox: necessity of grounding the solidarist ethics on a scientific and positivist ground, impossibility to do so without risking violating the right of individual freedom and consciousness.

Through an epistemological reflection on what exactly should be understood under the name “scientific attitude”, Bouglé demonstrates that there is in fact no contradiction between the latter and the thirst for justice, fraternity, or social compensation. Having adopted this kind of compromise, he then argues that solidarism still retains something of its proximity to modern science, both on the levels of the means of action, as of its final ends. The key idea is that, although science cannot pretend to *create* ethical feeling (like that of justice, that pre-exist any social experience), it can still *modify* them. For this purpose, ethical education plays a crucial role:

**Q21.** “To shape the consciousness of the young in our schools, we actually apply the method advocated by solidarism: we fill them with science. By the history of inventions and institutions, we give them the feeling of what human progress is about and what it owes to universal cooperation. We thus inspire them the desire to fulfill their duties, to pay their share, to do their part (...) They are disposed to more altruistic effort through the very fact that they conceive themselves, as Condorcet would say, as ‘cooperating in eternal work’. The expansion of intelligence, obtained through the knowledge of facts, thus results in a dilation of the heart.” (Bouglé 1906, 450-451)

### **3.3 The editorial background**

As mentioned above, the various papers discussed above, by Borel, Croiset and Bouglé, were all published in the *Revue du Mois*, edited by Emile and Marguerite Borel. More than that, their contents - with the special emphasis on science in practical life and society - were in line with the declared purpose of the *Revue*, which was announced in the following way:

**Q22.** “The number and the importance of problems that can be treated adopting scientific methods grows every day. It seemed possible to us to imagine a journal which focused on these methods, not as a specialist publication but rather by aiming at the general development of ideas, and the exposition and critical appraisal of the advances in Knowledge and the resultant spread of ideas.

The *Revue du Mois* attempts to be this journal. It claims, above all, to be a journal containing free discussion, allowing the free unhampered expression of opinions based on science. The titles of the articles that follow this statement testify to the breadth of its scope; the names of the authors are an assurance of the seriousness with which its remit shall be fulfilled.” (prospectus translated in Durand and Mazliak, 2011, 314)

The issue of how science and scientific method could help develop general ideas and opinions was thus very much at the centre of the Revue's ambition. The notion of science was understood in a very wide and varied way going far beyond the tradition fields of mathematics of experimental science: it included indeed the nascent "social sciences" like sociology, psychology, economy or human geography.

The study of Borel's editorial correspondence around the publication of Croiset's paper shows that he intended it to become the point of departure of a debate on the teaching of secular ethics. He thus solicited two former friends of him, the physicist Bernard Brunhes and his brother Jean, a geographer, to find a Christian response to Croiset's paper. The Brunhes were indeed known to belong to the modernist movement, also related to "social Christianity". We cannot enter here into any detail on the detail of this exchange of letters; it is enough to say, that Borel was not only interested to have a Christian response, which in fact he never obtained because "modernist Christians" soon met with serious problems through the Pope's condemnation of their opinions (through the encyclical *Pascendi dominici gregis* issued in 1907).

But Borel was visibly convinced by the exchange to seek for another article than Croiset of a more elevated kind, of the level of Durkheim or Levy-Bruhl, and this was probably the reason why he ultimately asked Bouglé to contribute to the subject. Second, and more important, the correspondence also shows that he felt incited to contribute himself to the debate in his own way, and this might be one explanation for the particular tone and structure of his two papers of 1906 and 1908.

### **3.4 The epistemological and didactic background: Borel's ideas on mathematics and their teaching**

Another element of explanation for Borel's special positions on the value of probability calculation and of its teaching, surfaces through the papers analysed and quoted above. For we saw that he considered the reflection on probability and on the laws of chance, as part of a more general reflection of wider scope, on the value and applicability of mathematical knowledge in general. This appears, within his argument, through the comparison between the problems raised by the interpretation of probability calculation for real life, with the problems raised by any train of mathematical calculation based on approximate knowledge.

But this point was hardly secondary to Borel, as is shown through his very first public intervention on question of mathematics education, through his 1904 conference held in "Musée Pédagogique" on the role of practical exercises in the teaching of mathematics (Gispert, 2002). In it, Borel defended a kind of approach to teaching making a compromise between theory and practice. The end of the conference is worth quoting, because it shows again, that Borel had very early in mind, that this proposal had potentially both an epistemological and political signification:

**Q23.** "This new orientation towards the teaching of mathematics in our *lycées* and *collèges*, of which we just outlined, would exert the most beneficial influence on the philosophical ideas of the educated classes, these ideas governing in reality the evolution of the country. (...)

A mathematical education, both theoretical and practical, as the one we tried to conceive, can exert the most beneficial influence on the training of the mind. We can thus hope to shape men having faith in reason and knowing that one should not try

to escape when facing a correct reasoning: one has just to accept it. They will have understood, on multiple examples, the determinism attached to natural phenomena and will be ready to understand the notion of physical law. But, on the other hand, they will be wary about any ungrounded reasoning (*raisonnement en l'air*) with no basis in reality, referring to badly defined words and to calculations effectuated on abstract numbers, the concrete signification of which is not made clear. They will seek to see the tangible object beyond the symbol.” (Borel 1904, quoted in Gispert 2002)

## 4 Conclusions

Having thus immersed ourselves into this literature aimed at the educated public in Borel’s time, we can then come back to the contemporary issues raised in the first place and ask what is and might be the interest of these reflections for nowadays teaching.

We propose three kinds of answers that are inspired by the work done in the research group, of which the present paper is an outcome.

### 4.1 Questioning the value of probability concepts for common understanding

We have seen how Borel’s paper are structured around the idea of taking in earnest an imaginary debate between competent mathematicians and „contradictors“, that is, laymen having their own intuitions, beliefs and reasoning on the day-to-day events and decisions, for which probability calculation pretends to provide help and insight. Transposed to the teacher and student situation, we retrieve a problem that is well known to middle school mathematics teachers, namely the conflict between the meaning of mathematical concepts such as „chance“, „probability“, „events“, „certain“ or „impossible“ events, „expectation“, and so on. This problem of making sense of the mathematical concepts, through a dialogue with the pupils own conceptions and understanding of the underlying situation, is much at the center of the French formal curriculum in probability.

What we find here, therefore, is an argument that gives room and legitimacy to the conflict between mathematical calculation, concepts, and models, on the one hand and, on the other, the matters of day-to-day life and decision making, the complexity of which cannot be reduced to the straightforward and passive interpretation of results. Many teachers are not at ease with these limits and with the questions of the complex relation of calculation with the reality it only helps to interpret. Getting into these arguments amounts to retrieving the historical fact that the application of probability theory to reality has been a matter of dispute for a long time, and, in fact, remains so.

### 4.2 Looking for “thought provoking” problems

As we have seen, part of the basic argumentative apparatus in Borel is made of small problems, that do not pose very deep difficulties in terms of calculation, but calls for reflecting on the results and their value. These problems were not Borel’s invention but the result of an already long tradition, well represented by the seminal treatise on probability of Joseph Bertrand (Bertrand, 1888). This treatise, which became in Poincaré and Borel’s time a basic reference, is entirely made of problems, instead of axioms or formal definitions as in Kolmogorov or other authors that took inspiration

from him. Moreover, Bertrand treatise is preceded by a long preface that introduces the reader to the epistemological problems raised by the application of probability to real and especially social events (Bru, 2006).

There is much emphasis, today, on the need for problems that conciliate the teaching of mathematics to the education to citizenship and especially to critical thinking. While many such problems are available, it is important to be conscious they are not new or the natural outcome of the contemporary concerns for the “overflow” of data around us. The historical tradition of probability writing constantly refers to famous dilemmas, like the problem of inoculation that constituted a famous controversy between Bernoulli and d’Alembert in the 18<sup>th</sup> century. Keeping alive this culture of controversy and problems, is certainly an important challenge.

### 4.3 Situating probabilistic thought in its political context

Finally, what Borel’s papers reveal, when understood against their political and philosophical background, is precisely the fact that their complete understanding depends on the latter. In France, the events surrounding the birth of the third Republic, the educational laws that were passed under this regime, and in particular the 1905 law of separation of Church and State to which Croiset directly refers, are of huge importance and part of a living tradition: thus, issues like those of *secularism* and the attached notion of *confessional neutrality*, are key concepts of our political culture.

What is less known<sup>6</sup> is the fact that the doctrine of the French state politics was inspired by such doctrines like *solidarism*, although it is at the ground basis of the idea of “social state”. There is much room, therefore, for interdisciplinary activities and training, showing how probabilistic thought is rooted in this political culture: the emphasis on the debate between the individual and society, inseparable from the rise of probability and statistics, as a subject of public concern.

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<sup>6</sup> Probably because the radical or radical socialist parties, that were so powerful and central with the third Republic, are no more central political forces.

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