POTENTIAL FOR COLLABORATION BETWEEN HISTORY AND MATHEMATICS TEACHERS

An investigation and framework based on a text by Abu'l-Wafaʾ Buzj'ani

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ABSTRACT

The main goal of our research study is to gain an understanding of mathematics and history teachers' beliefs and considerations towards the integration of history of mathematics, to investigate the potential for interaction and collaboration between them and what characterizes such a collaboration while they participate in a course of history of mathematics and in conducting and transferring a joint learning unit in class. To achieve our aims, a course on the history of Islamic mathematics based on Abū'l-Wafāʾ Buzj'ani book "On the Geometric Constructions Necessary for the Artisan" was designed for a group of history and mathematics teachers in the Arab sector of northern Israel. The research presented in this paper comes out of a project which is ongoing; therefore, only theoretical and methodological aspects of it will be discussed.

Keywords: History teachers, mathematics teachers, beliefs and considerations, Abū'l-Wafāʾ Buzj'ani

1 Introduction

The integration of history of mathematics into mathematics education has been a persistent interest in mathematics education. The importance of history of mathematics in schools' curriculum has been pointed out in documents of many professional councils. The HPM (History and Pedagogy of Mathematics) itself, of course, was founded on the basis of feasibility and potential of integrating history of mathematics into mathematics teaching and learning. This paper, which describes a current ongoing project with teachers, makes the further assumption that since history of mathematics is both mathematics and history, it is relevant both to mathematics teachers and history teachers. Indeed, we see teaching history of mathematics as a join effort on the part of these two communities. Before we get to this point, however, we shall consider some of the general issues against which our teaching project is set.

1.1 Justifications for Integrating History of Mathematics in Mathematics Education

For some researchers, history of mathematics should be taught and introduced into the curriculum hand in hand with mathematics since the history of a subject is an integral part of the subject (Sui & Tzanakis, 2004; Heiede, 1992), and, studying history of mathematics in conjunction with mathematics is a matter of general mathematical culture (Schubring, 2011).

In the last three decades, some scholars attempted to gather and list explicit reasons for integrating history of mathematics in mathematics' lessons. Fauvel (1991) listed some twenty reasons for integrating history of mathematics, and later, Tzanakis et al. (2000)

adduced 17 arguments for integrating history of mathematics into classroom.

Fried (2001) observed that arguments in favor of integrating HOM in mathematics education generally fall into five categories: 1) Humanizing subject matter; 2) Adding variety to teaching; 3) Showing alternative approaches to scientific ideas; 4) Analyzing students' understandings and misunderstandings; and 5) Deepening a sense of the nature of the discipline. Elsewhere, Fried (2014) categorized these justifications into three groups: 1) The Cultural theme; 2) The Curricular theme; 3) The Motivational theme.

For the project, the group of justifications most relevant are those under the cultural theme "recognizing the enterprise of mathematical inquiry as part of students' cultural heritage […] it is both a reflection on general culture as including mathematics and also on mathematics as being cultural" (Fried, 2014, P.680). These are also those must connected to history as such. Pursuing such a justification, on the other hand, is not always easy—not only from a practical point of view but from a theoretical point of view as well.

1.2 Difficulties in Integrating History of Mathematics: A Challenge for Teachers

Fried (2001) claimed that beyond the practical issues such as lack of finding time for fitting historical material into an already crowded curriculum or inaccessibility to correct and appropriate resources, the integration of history of mathematics hides a theoretic difficulty reflecting the tension between anachronism and relevance, and, between useful rational reconstruction and faithful historical analysis. He points out a dilemma whereby teachers must choose between taking a genuine approach to history – in which case, they end up spending time on things that are not part of the mathematics curricula—and trivializing history or taking an anachronistic (Whig) approach in order to use history as a tool for learning relevant mathematics.

> The main difficulty is that while one wants to see historical topics in the classroom or an historical approach in teaching, the commitment to teach the modern mathematics and modern mathematical techniques necessary in the pure and applied sciences forces one either to trivialize history or to distort it. In particular, this commitment forces one to adopt a "Whiggish" approach to the history of mathematics (Fried, 2001, p. 391)

The argument was that since teachers need to teach contemporary mathematics and its applications—for the power of contemporary mathematics is the predominant justification for its emphasis in school education— efforts to integrate the history of mathematics leads to a history that guided and measured by its relevance to the modern era. Thus, the teacher is compelled to adopt an unhistorical anachronistic history (Fried, 2001). The mathematics teacher's inner commitment to modern mathematics actually acts against history of mathematics into mathematical education, because the teacher has to filter out from history of mathematics the relevant and useful material, this the teacher is pushed to adopt a "whiggish" approach to history (Fried, 2001), that is, where the teacher measures what is important and what was the meaning of this thing in the past through the viewpoint of the present (Kragh, 1987). The term "Whig history" of course was a notion introduced into historiography by the British historian Herbert Butterfield (1931/1951), and to describe history in which the present is the measure of the past so that what is considered significant in history is precisely what leads to something deemed significant today, and, Butterfield's point was that that kind of history is not history at all.

1.3 Really a Matter of Two Communities

The difficulty described by Fried (2001) was conceived in terms of a hypothetical mathematics teacher who wants both to teach mathematics within the framework of a typical mathematics curriculum and also introduce history of mathematics into the lessons. In this way, he brought out a dilemma which the entire community of mathematics teachers interested in history must confront. In our opinion, this dilemma arises because the question is actually posed to one community only, namely, mathematics teachers. In fact, it can be thought of as referring to *two* communities:

- Mathematicians or mathematics teachers
- Historians or history teachers

The difficulty of the first community, that of the mathematics teachers, is been already stated. But, what about the history teachers? History teachers might do better than mathematics teachers in avoiding the trivialization of history if they took up the history of mathematics in their classrooms; on the other hand, history teachers cannot be expected to understand the spirit of mathematical thinking and may need to be convinced that history of mathematics is as relevant to history as it is to mathematics. So both communities have something to contribute and their difficulties to confront. They may not each by themselves be able reconcile the conflicts producing those difficulties. However, they may be able to do this working together. Therefore, we think of introducing the history of mathematics in mathematics teaching as a fundamentally multi-disciplinary effort that requires enlarging the community which deals with it. For since history of mathematics belongs both to history and to mathematics, it takes in both the communities of history teachers and mathematics teachers. Bringing together history teachers and mathematics teachers in the context of history of mathematics—even the specific context which we have chosen, namely, the work of Abu'l Wafa— we should point out has been attempted before by Marc Moyon and his colleagues (Moyon, 2013). However, our work takes up the problem in light of the theoretical issues described above.

2 Aims of the Study

Our study brings history teachers and mathematics teachers together in order to determine, first**,** the presuppositions of history and mathematics teachers, respectively, regarding the teaching of history of mathematics. We wish to see whether the presuppositions which are implicit in the disciplinary commitments of each community can be seen in the actual work of these teachers as they consider an appropriate historical topic. At a somewhat more basic level, before we ask mathematics or history teachers to integrate history of mathematics in mathematics education, we may need to stop for a moment and ask ourselves: what they think about this integration, who should teach history of mathematics, what kind of knowledge (along the lines of Shulman, 1986) mathematics teachers need to know about history, and what history teachers need to know about mathematics, and what they know about the nature of the two disciplines? Answering these questions is one aim of the study.

The second aim is to determine whether history teachers and mathematics teachers are genuinely able to work together to produce a classroom unit on the history of mathematics and what are the characteristics of this collaboration. This in effect asks the question whether the two communities truly can settle together the difficulties that cannot be settled

each alone.

We should point out here at the outset that the study is ongoing and we still do not have all of the results. Hence, this paper will focus mostly on theoretical and methodological issues.

3 Approach

Given the aims stated above, we need to investigate the following research questions.

3.1 Research Questions

1 What are the considerations, presuppositions, and beliefs of teachers for mathematics and history regarding the integration of history of mathematics into the classroom? and, is there any significant difference in these considerations and beliefs, in either population or both, according to years of experience, gender, education, grade level and school level (elementary, middle school, high school and teacher teachers)?

2 How and to what extent might a course in the history of mathematics impacts the considerations, and beliefs of the two teacher populations regarding mathematics, history, and the history of mathematics? and what are the characteristics of the learning experience (cognitive, interpersonal and mental emotions aspects) of the mathematics and history teachers during their joint participation in a course in the history of mathematics?

3 How feasible is an interdisciplinary collaboration between mathematics and history teachers? To the extent that it is feasible, what are the characteristics of this collaboration, and how it is reflected in the construction and transfer of a joint learning a unit in history of mathematics?

3.2 Participants and Research Tools

The participants in the first stage (Question1) in this research are **approximately 350 in-service mathematics and history teachers** from elementary, middle and high Arab schools in Israel. A questionnaire will be given to the teachers.

In order to answer the second question, we will examine how a course which is based on the history of Islamic mathematics impacts the consideration and beliefs of mathematics and history teachers, and how they act during participating in a common course and what are the characteristics of learning and thinking process. The researcher of this study is also the instructor of the course and the participants are **20 teachers** (10 mathematics teachers and 10 history teachers) from four schools (two elementary schools, one middle school, one high school) will participate in a course-based on history of Islamic mathematics.

The course has been designed to engage both communities at once. The specific historical material is based on texts published by the Islamic mathematician and astronomer, Abū'l-Wafāʾ Buzj'ani, who lived in the 10th century (940–998 CE) *"Kitāb fī mā yaḥtaj ilayh al-ṣāniʿ min al-aʿmāl al-handasiyya"* (Book on what is necessary from Geometric Construction for the Artisan). For this project we will use Istanbul manuscript [in Arabic] which was copied in the first half of the 15th century and Abū'l-Wafāʾ Buzj'ani book edited by Salih (1979). As mentioned above, this was also the text used by Marc

Moyon in his work, and, Moyon has discussed its importance in more than one paper (see Moyon, 2011, 2013). We were unaware of Moyon's work in this connection until recently and considered Abū'l-Wafāʾ as a central text independently: this only underlines the striking and suggestive character of that text.

In the third stage, about 10 mathematics and history teachers will work actively together from a pedagogical standpoint. The participants will be divided into pairs (**five couples**, each couple consist one mathematics teacher and one history teacher who took the course of Islamic Mathematics). Each pair will be asked to teach a short lesson along the lines of the material in the course, but not necessarily concerning Abū'l-Wafāʾ specifically. We chose this approach since we would like ultimately for the mathematics teachers and history teachers to work together in their own schools: this activity then allows them to experience this kind of cooperative effort and allows us to see the difficulties that may be involved. Accordingly, during the lesson, we will make observations and keep a log that will document events, interpretations, significant utterances, as well as personal thoughts and feelings. Our intention is also to incorporate video recording in the observations according to visual ethnography principles.

Abū'l-Wafāʾs treatise is a collection of 171 problems of geometry, divided into 11 chapters; it includes 150 problems of plane geometry and the rest on spheres and polyhedrons. The introduction in the book gives 13 chapters; two chapters are missing "On the division of scalene figures" and "On tangent circles".

Chap. I. Introduction

Chap. II. Basic constructions

Chap. III. Construction of polygons

Chap. IV. Inscription of polygons in the circle

Chap. V. Circumscription of the circle around polygons

Chap. VI. Inscription of the circle in polygons

Chap. VII. Inscription of polygons with each

Chap. VIII. Division of triangles

Chap. IX. Division of quadrilaterals

Chap. X. Division and composition of squares

Chap. XI. On dividing spheres

Abū'l-Wafāʾ Buzjani tells us in the introduction of his book, that he attended meetings between geometers and artisans in Baghdad, such meetings were a widespread phenomenon in the Islamic world. This book provides insights into how mathematicians and artisans collaborated in the Islamic culture.

> I was present at some meetings in which a group of geometers and artisans participated. They were asked about the construction of a square from three squares. A geometer easily constructed a line such that the square of it is equal to the three squares, but none of the artisans was satisfied with what he had done. The artisan wants to divide those squares into pieces from which one square can be assembled, as we have described for two squares and five squares.
>
> (Abū'l-Wafāʾ in Alpay, 2000, p. 174)

In his book, Abū'l-Wafāʾ understood the needs and problems of the artisans and was motivated by such meetings and by his efforts to advance Islamic art (Alpay, 2000), he displayed knowledge of pure geometry, familiarity with practical applications and skill in teaching theoretical subjects to practical-minded people.

3.3 Why this Book?

This particular work was chosen for three main reasons:

i) It contains significant mathematical content

Many of the examples in the book were original and some were borrowed from ancient Greek writings of Euclid, Archimedes, Theodosius of Berga, and Pappus. Abu'l-Waf ̄a' gave instructions on certain geometric constructions of two or three-dimensional ornamental patterns and also he gave advice on the application of geometry to architectural construction by using cut and paste methods as a didactical tool in teaching geometry to artisans (Alpay, 2000). Most of the constructions given in the book, however, are meant to be carried with compass and straightedge.

The range of problems is very wide, from the simplest planar constructions (the division of a segment into equal parts) to non-trivial problems concerning polyhedrons inscribed in a given sphere

ii) The mathematics had a clear social and cultural context,

Since the mathematics involved had clear, social, and cultural aspects, it touches upon Islamic history in addition to Islamic mathematics. Evidence to the second reason, we can see with the relationship between Islamic tradition/ religious and the Islamic sciences. The Prophetic tradition" May God protect us from useless knowledge" may shed a light on the treatment of the Islamic scholars with sciences. In this spirit, the philosopher and theologian Abu Hamid Al-Ghazali wrote:

> The problems importance of physics (*he was referring to aristotelian natural philosophy*) are of no importance for us in our religious affairs or our livelihood, therefore, we must leave them alone[…] Man has created only to know, but the knowledge man has been created to seek is that which brings him closer to his creator[…]this means, not only that religious knowledge is higher in rank and more worthy of pursuit than all other forms of knowledge, but also that all other forms of knowledge must be subordinated to it
>
> (Al-Ghazali in Sabra, 1987, p. 239).

iii) The treatise concerns the work of ordinary people, the artisans, and is therefore, directed towards a greater set of societal concerns.

In this work by Abū'l-Wafāʾ, there are problems concerning the division of a figure into parts that satisfy certain conditions, and problems on the transformation of squares (for example, the construction of a square whose area is equal to the sum of the areas of three given squares). In proposing his original and elegant constructions, Abū'l-Wafāʾ simultaneously proved the inaccuracy of some methods used by "artisans."

4 Significance of the study

Since the project described above is ongoing, we still do not have definite conclusions to report. In lieu of that, we cite three points that we think may be significant outcomes of the study when it is completed.

i. The integration of history of mathematics, according to many studies, significantly contributes to students. However, this subject is insufficiently integrated in schools. Assuming the teachers have central role in such integration, the current work might shed some light on the consideration of the teachers that lead to this situation and how those considerations may differ with respect to whether a teacher is a mathematics teacher or a history teacher (who indeed may never have considered history of mathematics as history at all!).

ii. Understanding what teachers should know about their profession when integrating the disciplines of mathematics and history, and what didactic skills are needed for them in order to generate an intellectual experience for their students. The research, in this regard, may open a new aspect of pedagogical content knowledge. Again whether that aspect of pedagogical content knowledge can take on a unified form across the disciplines of mathematics and history.

iii. Uniqueness of the study – to date, a study that examines the considerations mathematics teachers in comparison to those of history teachers regarding the integration of history of mathematics into classroom, in particular and the different practical and theoretical assumptions, and their openness to the interdisciplinary approach, in general, except a few isolated cases, such as the studies of Moyon, has not been done in intensive way.

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Appendix

Constructing a square from three equal squares

فأما المهندس فإنه إ ذا سئل عن عمل مربع من مربعات قلت أو كثرت فإنه يحدس الخط الذي هو على تلك المربعات ولا يبالي بتقطيع المربعات كيفما كانت.

وذلك أنه إذا سئل عن عمل مربع من ثلاث مربعات فإنه يوصل قطر أحد المربعات ويقيم على احد طرفي القطر خطا يكون عمودا عليه مساويا لضلع المربع المؤلف من ثلاث مربعات

مثال ذلك

فإذا أردنا أن نعمل مربعا واحدا مساويا لثلاث مربعات كل واحد منها مساو لمربع أ ب جـ د , أخرجنا قطر ا جـ فيكون أ جـ ضلع المربع المركب من مربعين ثم أقمنا على نقطة أ من خط أ جـ عمود أ هـ مساويا لخط ا جـ ووصلنا هـ جـ فيكون خط هـ جـ ضلع المربع المساوي لثلاث مربعات كل واحد منها مساو لمربع أ ب جـ د (فإذا حصل عند المهندس هذا الخط لم يبال بعد ذلك كيف كان تقطيع المربعات , وقال أنه متى عمل على خط هـ جـ مربعا كان مساويا للمربعات الثلاثة)

If we want to construct a square from three equal squares *ABGD;EWZH; TIKL*, we bisect two of the squares at their diagonals, by means of lines *AG, EH*, and we transport [them] to the sides of the [third] square. Then we join the right angles of the triangles by lines *BZ, ZW, WD, DB:* On either side [of the straight line], a small triangle has now been produced from the sides of the [two big] triangles. That [empty position of the triangle] is equal to the triangle which has been cut off from the big triangle. Thus triangle *BGM* is equal to triangle *MZH*, since angle *G* is half a right angle, angle *H* is half a right angle, the two opposite angles of the triangles at *M* are equal, and side *BG* is equal to side *ZH*. Therefore, the remaining sides of the triangles [*BGM, MZH*], and the triangles are equal. Thus, if we take triangle *BGM* and put it in the position of triangle *MZH*,18 line *BZ* is the side of the square constructed from three squares. This is a correct method, easier than what was constructed [by the artisans], and the proof of it has been established. This is the figure for it (translated by Alpay. O, in Alpay, 2000, p. 183)