DISCOVERING NEGLECTED SYNTHETIC GEOMETRY ON SOCIAL NETWORKS

Learning Maths as in the Historical Italian Academies

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ABSTRACT

In this paper, we present the work of an ongoing project, devoted to 13–15 years old students in southern Italy, aiming to improve their mathematical literacy.
Starting point of our research was the way to spread the culture in the sixteenth century Academies.
There were about 800 Academies in Italy in 1525-1700 years: they were fundamental for the development of intellectual networks and the dissemination of ideas in Europe. In these Academies, interdisciplinary paths were developed, including literature, arts, natural sciences, medicine and mathematics.
In Mathematics, there was a fervent discussion about “new geometry”, i.e. calculus vs “old” synthetic geometry. Because of calculus success, some very interesting results about synthetic geometry were neglected.
Aim of present work is planning and experimenting an interdisciplinary learning unit, focused on some hidden historical theorems of synthetic geometry developed for 1500-1800 years about “Cevian, Orthic and Pedal Triangles”, by simulating “old Italian Academies” in a modern key, also by using ITC.
These theorems are quite interesting from a Maths and Physics point of view, too. We studied, in particular, Ceva's theorem and Fagnano's problem.
Both Ceva and Fagnano theorems may be generalized, under some conditions, to orthic quadrilaterals of a convex quadrilateral and Fagnano's problem may be extended to “polygonal billiard” physics, too.
In order to re-discover these hidden theorems, blended learning and flipped lessons methodologies have been also used.
The planned learning unit has been experimented with about thirty students, attending the second year of a Secondary School in South of Italy. Italian Academies have been simulated in two ways: virtual and blended academies were founded, respecting main characteristics of ancient Academies. English has been chosen as universal language, instead of Latin used in the past. Nevertheless, some historical documents (e.g. Ceva theorem) have been studied in Latin and in archaic Italian, to critically analyze original fonts.
Information Technologies can provide significant advantages in this learning – teaching approach: they have been used to share and communicate main results, to create “virtual Academies” and to realize multimedia materials (a final video, summarizing all the activities), too, which are “modern fonts” to be archived on cloud software’s.
The experimented educational path shows that, although students generally consider Geometry as a difficult branch, they may excite if they study it in an innovative way and in an historical environment.

1 Introduction

A scientific revolution arose during the seventeenth centuries, thanks to the introduction of a new Maths branch with respect to traditional “Synthetic geometry”: the “calculus” (1666-1687), also known as “the new Geometry”, due to the mathematicians Isaac Newton (Newton, 1687) and Gottfried Leibniz, who independently invented it, with a different symbolism and approach. Because the success of calculus, some very interesting results about synthetic geometry were neglected.

This scientific revolution depended not only upon the introduction of new ideas, but also upon the development of new institutions for the acquisition and dissemination of knowledge. The most important of these new institutions were the princely court and the informal Academy (Ruscelli et al., 1984).

Over 800 academies flourished in Italy in the period 1525-1700, forming a significant and influential aspect of social and intellectual culture. Interdisciplinary in their interests, bridging literature, arts, medicine, and sciences, the Academies operated outside, but were often interconnected with official institutions like universities, courts, political and religious bodies, and offered a more flexible, apparently 'free' and 'equal' form of association. Members or affiliates could sometimes include socially marginal figures like women and artisans. Academies also attracted foreign intellectuals and their networks extended across Europe.

Scientific Academies were in all the Italian towns, first of all in Salerno and in Padua.

In scientific Academies, some "geometric problems" were also discussed, taking into account both calculus and synthetic geometry solutions. Few details may be found about these "discussions", also due to the general "academic rules": the members could only have access to all the research material developed inside the Academy.

In the present work, we show an interdisciplinary educational path, in which "hidden" (i.e. absent on the schoolbooks) synthetic geometry theorems have been re-discovered by simulating scientific Academies in a modern key, in a second class of a Secondary School, by using ITC (in particular social networks and GeoGebra), analyzing historical texts and using different maths approach. The class was subdivided into three Academies, where members were connected between them by using a social network. In these modern Academies, students also researched some information about ancient Italian Scientific Academies, evidencing the social connections between them, by using historical research methods in an intuitive way. Rarely, teacher scaffolding has been necessary.

Our research starts from these questions:

- May a "social" comparison between students improve their motivation in studying Mathematics?
- May an activity based on an historical, students self-managed, research intrigue and arouse interest in Mathematics, and especially in Geometry?
- May the use of an informal language encourage the students to discuss about Maths?

2 Italian Scientific Academies: main characteristics

In order to better simulate Italian Academies, main characteristics have to be outlined. Italian Academies of sixteenth and seventeenth centuries were a knowledge dissemination model, which was emulated in all the Europe.

Some common features may be found in all the Academies: an effective name, a logo, a motto, generally a noble or a "celebrity" who favored its growth, interdisciplinary discussions and freedom of thought.

Testa (2012), by analyzing these main characteristics, suggested that "Academies were the first intellectual networks of early modern Europe".

Their membership included women as well as men, and representatives of all social classes. In addition to their intellectual pursuits, the Academies had a more playful aspect, including the delivery of orations based on paradoxes, the performance of games, or the invention of amusing names for the Academy.

Members of Academies frequently published, for many different reasons, under their Academy nicknames, which often reflected the Academy name.

For 1525-1700 years, Academies became more specialized: some literary, artistic and scientific Academies were born in different Italian towns.

A study on academies in Bologna, Florence and Naples (Irace and Panzarelli Fratoni, 2011) analyses most relevant topics in the Academies: 7,7% is about science and maths.

Nevertheless, Scientific Academies were often in contrast with the Universities. This contributed to the University crisis. New topics and new methodologies were developed inside the Academies, which were absent in the University learning, in particular, an increasing freedom of thought characterized the Academic studies.

Mathematics discussions were mainly focused on Geometry, taking into account the "new branches" (calculus and projective Geometry) and "the old one" (synthetic geometry).

3 Educational Path "Academy 2.0"

An interdisciplinary educational path has been planned rounding about re-discovering of "hidden" geometry theorems and problems, which are missing in maths schoolbooks, but are largely diffused on the web.

Arts, Latin, English, History and Literature have been involved in addition to Maths, to effectively organize and realize this learning unit.

The present path has been experimented in a second class of scientific high school (about thirty students).

We simulated two different types of Academies:

a) Virtual Academy, in which members communicate between them only by using social networks;

b) Blended Academy, with both virtual and live meetings.

For each Academy we chose: a logo, a motto, some communication and sharing rules and a specific dress, for the blended Academies, too, as in the old Academies.

Students used their nicknames to login to social networks, in these modern Academies.

4 Methodologies and Information Communication Technologies

Integration between Flipped Classroom (Bergmann and Sams, 2012) and Blended Learning methodologies, with a *blended-on line* and in-class format approach (Novak et al. 1999) in a *Student-Centered Active Learning Environment* has been used.

We didn't choose a devoted platform, but we preferred social networks (*WhatsApp*) for a free learning materials exchange and for an effective communication between members. Main advantage of the chosen social network is that it is possible to create closed groups, doing their research "secret". Moreover, all the students daily use it.

An interactive and dynamical geometry software, GeoGebra, has been used, to verify main theorems. Students used their smart-phones and/or tablets, as in Bring Your Own Device (BYOD) practices, whereas a LIM and a PC have been used in the classroom, too. In order to verify geometry theorems, Information and Communication Technology (ICT) laboratory has been often used.

5 Experimental Activities

The educational path has been subdivided in three phases:
1) Learning activities in virtual Academies
2) Live meeting in Blended Academies
3) Final discussion and products realization.

5.1 Phases 1: Virtual Academies

The experimentation started by creating three virtual Academies (each one with about ten students as members), named as their specific research theme:

- Cevian Triangles;
- Pedal Triangles;
- Orthic Triangles.

Whats App groups were the "virtual places", where students met and had their initial learning activities. The "ImageGroup" was the Logo, chosen in agreement with all the members (which were all administrators, so nobody was the leader). Teacher was also added to each Academy group, but only as a moderator.

The "virtual members" posted on the groups their web researches, video, images, idea, suggestions and comments.

Communication was asynchronous, students feel free to have their research in each place and time, so these *e-learning* activities were effective, and students were enthusiast. Virtual debates were very useful to "have a trace" about free discussions.

An introductive message was posted in each Academy/group by the teacher, suggesting main topic, as an example:

"*27/10/17, 18:14 Hi guys, now you are Pedal Academy members. Search on the web and post here all you can find about pedal triangles, both historical and geometric aspects*".

Informal language has been used by the teacher, too. No indications about websites to be used have been furnished by the teacher.

Dialogues posted by students prove the effectiveness of e-learning activity, because peer to peer education gave the opportunity to work in the zone of proximal development (Vigotsky, 1978).

Just as an example, let's analyze a brief virtual dialogue, between students and teacher, in the Pedal Academy Group:

27/10/17, 18:28 Mirko*: I found and interesting link*
http://web.mclink.it/MC2113/geometria/java/Tpedali.html

28/10/17, 19:02 Paolo*: I found definition of pedal triangle: in Geometry a pedal triangle of a point with respect a triangle is identified by the point projections on the triangle sides.*

[...]

05/11/17, 19:29 - Teacher: *Question: May I choose all the points I want?*

05/11/17, 19:42 Andrea: *Yes you can, orthocenter, incenter and circumcenter, too*

05/11/17, 19:43 Gabriele: *No, you can't choose all the points, they have to be inside the triangle.*

05/11/17, 19:47 Mirko: *I agree with Gabriele, just for an example, I think that the circumcenter of an obtuse triangle is out of the triangle, so I think it isn't OK.*

Andrea: *used an emoticon image to say "I LIKE"*

05/11/17, 20:05 Teacher: *Have you tried to represent it with some special points?*

Andrea posted 3 Geogebra files (as you can see in the Topics section)

05/11/17, 20:12 Teacher: *You have just studied circumference, look if there is a connection between pedal triangle and circumference.*

05/11/17, 20:14 Gabriele: *Prof, I'm trying... I draw the pedal triangle of the orthocenter (D), by Geogebra, I draw a circumference passing by D and a vertex C, CD is a diameter...*

05/11/17, 20:20 Andrea: *Gabriele, I put the Pedal of the Incenter, CD is always the diameter...*

05/11/17, 21:05 Giampietro: *Diameter is perpendicular to the cord, it is obvious*

Teacher: *Wonderful*

05/11/17, 21:50 Paolo: *I'm searching for historical origin of pedal word, but I didn't find anything...who does invent this word?*

By analyzing all the web debates, we may answer to some research questions; in particular, it is evident that:

- ✓ students discuss about Mathematics between them, without any fear to make a mistake;
- ✓ students use an informal language, also including “emoticon” to approve or disapprove, as in a friendly chat;
- ✓ in this free scheme framework, students have the opportunity to do mathematical and historical research and rediscover some results, which are not present in their schoolbooks, but are strictly connected to standard geometric topics;
- ✓ students are very interested in the topics, so they continue to study Maths also late in the evening.
- ✓ in the research/discovery phase, GeoGebra has a relevant role, as a flexible instrument to immediately verify some student’s hypothesis and some theorems they found on the web.

Moreover, for each group, teacher suggested an extra topic: to find some information about a specific Scientific Academy. For this historical purpose, students feel enthusiastic and posted their link late in the evening, too.

Nevertheless, we noted that, in this research phase, a great students’ astonishment was when they discovered that on the web all the historical texts concerning the studied theorems and numerous news concerning the Academies to be discovered could be consulted.

5.2 Phase 2: From Virtual to Blended Academies

After the “discovery phase”, students had some problems to virtually organize a coherent sub-unit about their own results: it was difficult, in a virtual discussion, to individuate main elements and synthesize them.

For this reason, all the students agree to change the “virtual Academies” in “blended” ones: members met at school, discussed about their topics and each Academy summarized its results in a poster, (see for example Fig.5.1 for Cevian Poster). No specific difficulties have been evidenced in the student's work.

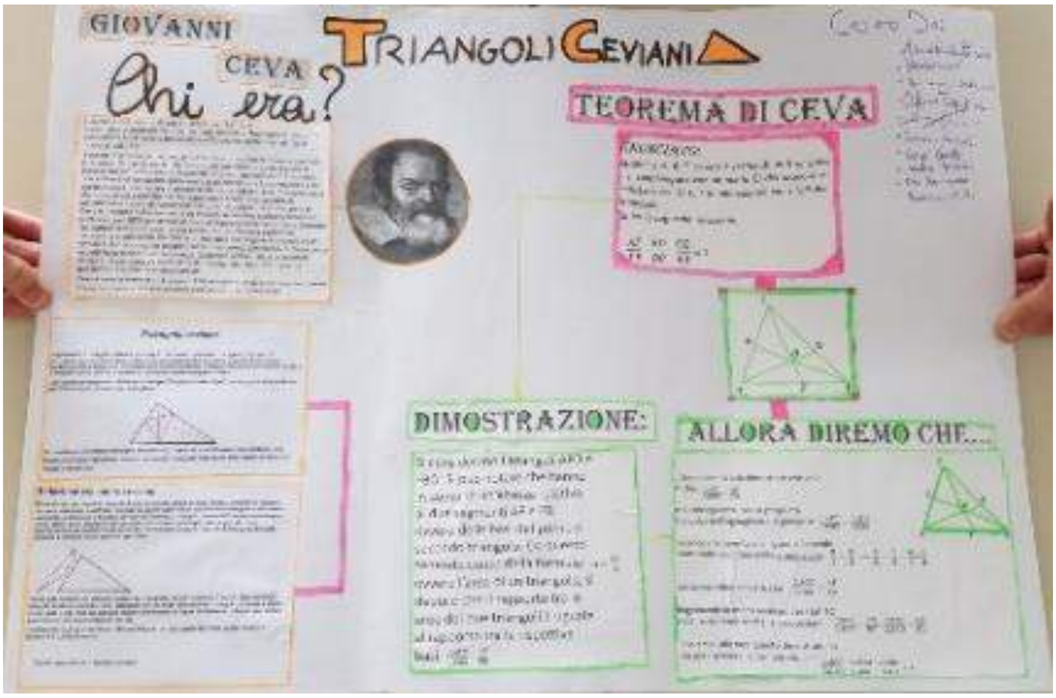

Figure 5.1: Posters realized by “Cevian, Pedal and Orthic Triangles Academies”

As you can see in Fig. 5.1, both historical and maths results have been organized in the posters, together with the GeoGebra verification's theorems. Some theorems proofs have been also posted, as the students learned, directly on the web.

5.3 Discussions and Final Product

In a final meeting, students belonging to each Academy, also wearing a specific uniform (with red, blue and white shirt, respectively for Cevian, Pedal and Orthic Academies), described their “research results” to all the “community”.

Literature, Physics and Mathematics teachers also were present at this meeting. They discussed about the realized posters and showed some geometric results by using GeoGebra. Some of them, in particular Cevian Group, discussed about historical documents they analyzed, in Latin language, from the book “*De lineis rectis se invicem secantibus, statica constructio (1678)*”, directly consulted on the web.

All the students were surprised about some common features in the analyzed topics, which will be outlined in the Topic paragraph.

At the end of this experience, students were invited to realize a multimedia product (video), summarizing their results and their own point of view about this adventure. The realized video (about 10 minutes long, that you can find on YouTube in the Italian version) was effective, too. They also included in it some comments, which are similar to “slogan”, i.e.:

– “From the past to the future: we worked as Academy members”
– “Our Academies 2.0: comparison, dialogues and research”
– “Learning all together is special!”

6 Topics

Analyzed topics were geometry theorems about triangles, discovered after 1500, which are not classified, not well dated, not always organized, not present in maths school books, but they are very useful, interesting and easy to be studied at High School and directly connected with curricula.

We focused on three sub-units (corresponding to the three virtual Academies). Just some results, which have been individuated by the students, are shown here.

6.1 Cevian Triangles

A Cevian is any line segment in a triangle with one endpoint on a vertex of the triangle and any other endpoint on the opposite side.

Given a point G, interior of a triangle ABC, the Cevian triangle A'B'C' is defined as the triangle composed of the endpoints of the cevians, being G the Cevian Point (see Fig.6.1).

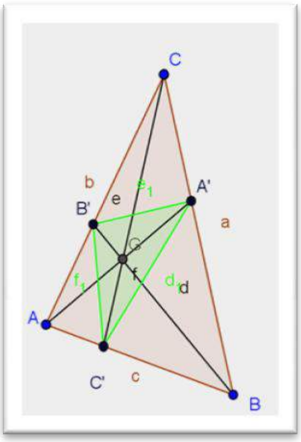

Figure 6.1: A'B'C': Cevian triangle of the main ABC triangle, G = Cevian Point.

The condition for three general Cevians from the three vertices of a triangle to concur is known as Ceva's theorem**:**

In a triangle ABC, three lines AQ, BR and CP intersect at a single point G (i.e. they are concurrent) if and only if:

$$\frac{AP}{PB}\cdot\frac{BQ}{QC}\cdot\frac{CR}{RA}=1$$

As laboratorial activities, students verified it by using GeoGebra (See Fig. 6.2), in the ICT laboratory.

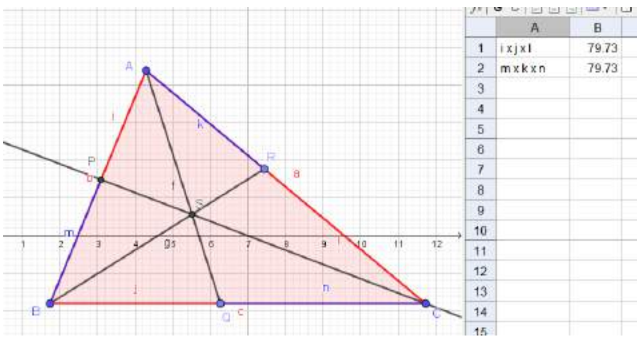

Figure 6.2: Test of “Ceva's theorem” by using GeoGebra

The use of Ceva's theorem is an effective way to introduce remarkable points (barycenter, incenter and orthocenter points), in a nontraditional way: medians, angle-bisectors and heights are cevians, all concurrent in a specific point, as we show here:

Theorem 1 (Existence of Barycenter) - Medians are cevian, concurrent in a point named Barycenter.

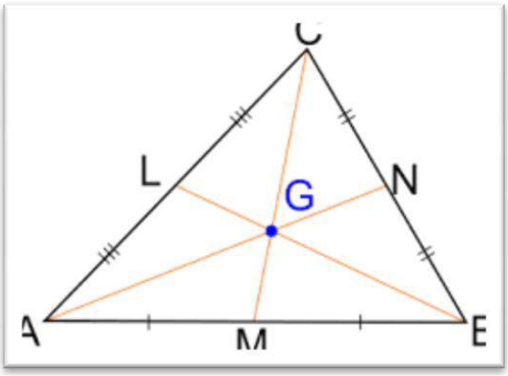

Figure 6.3: Medians concur in the Barycenter

Proof:

Medians connect vertices with the midpoints of the opposite sides.

Therefore,

$$AF/FB = BD/DC = CE/EA = 1.$$

Each of the ratios is 1 and so is their product.

Theorem 2 (Existence of Incenter) - Angle bisectors are cevian, concurrent in a point named Incenter.

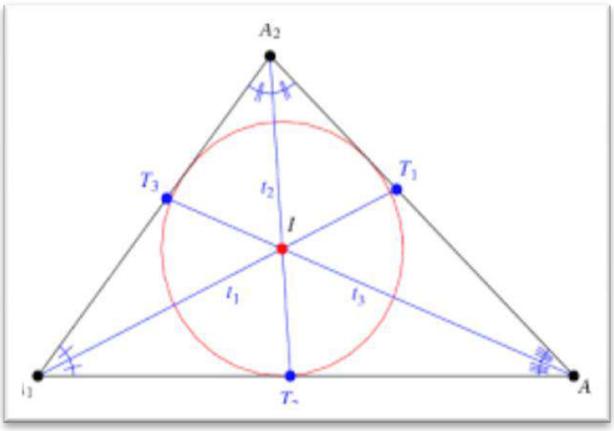

Figure 6.4: Angle bisectors concur in the Incenter

Proof:

For angle bisectors theorem,

$$AF/FB = AC/BC, BD/DC = AB/AC, CE/EA = BC/AB.$$

Multiplying the three yields 1.

Theorem 3 (Existence of Orthocenter) - Heights are cevian, concurrent in a point named Orthocenter.

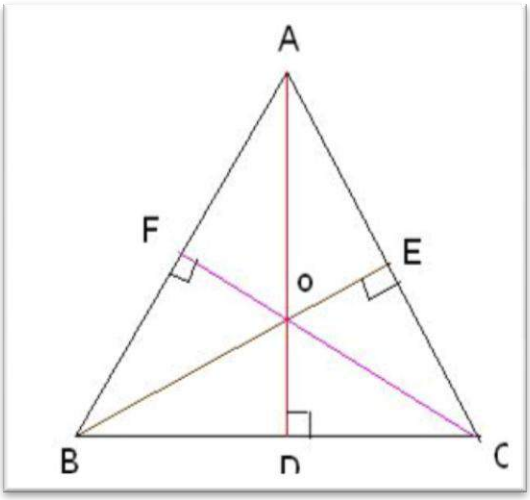

Figure 6.5: Heights concur in the Orthocenter

Proof:
Indeed, right-angled triangles ACD and BCE are similar.
Therefore $\frac{CE}{DC} = \frac{BE}{AD}$

Analogously, $\frac{AF}{EA} = \frac{CF}{BE}$

and $\frac{BD}{FB} = \frac{AD}{CF}$

Indeed: $AF/FB \cdot BD/DC \cdot CE/EA = CE/DC \cdot AF/EA \cdot BD/FB = BE/AD \cdot CF/BE \cdot AD/CF = 1$

6.2 Pedal Triangles

A pedal triangle is obtained by projecting a point onto the sides of a triangle.

More specifically, let's consider a triangle *ABC*, and a point *P* which is not one of the vertices *A*, *B*, *C*. Let's drop perpendiculars from *P* to the three sides of the triangle (these may need to be produced, i.e., extended) and label *L*, *M*, *N* the intersections of the lines from *P* with the sides *BC*, *AC*, *AB*. The pedal triangle is the *LMN* one (see Fig. 6.6)

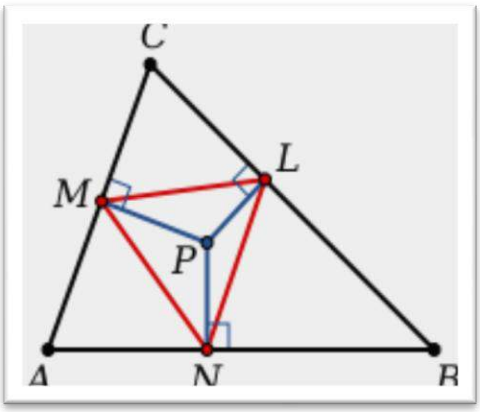

Figure 6.6: LMN: Pedal Triangle of main triangle ABC

Several properties can be proved about pedal triangles, all starting from a “main property”:

Main Pedal Property: “Given an ABC triangle and an $A_1B_1C_1$ pedal triangle respect to P point, then A, C_1, P and B_1 belong to a same circumference, with AP as a diameter” (Fig.6.7).

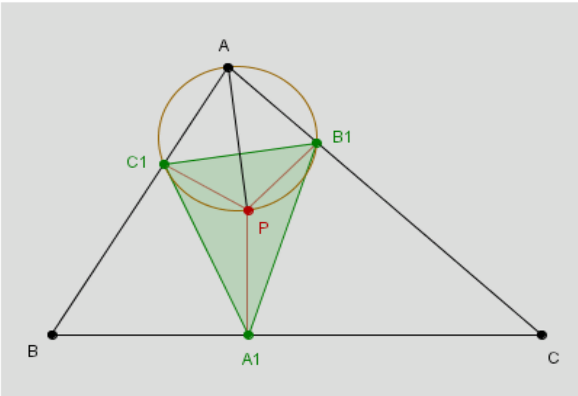

Figure 6.7: Pedal Triangles Main property: A, C1, P and B1 belong to a same circumference, being AP the diameter

6.3 Orthic Triangles

Given a triangle A, B, C, the triangle $H_AH_BH_C$, whose vertices are endpoints of the altitudes from each of the vertices of ABC is the orthic triangle. Orthic triangle is both the pedal and the cevian triangle of a specific point, the orthocenter.

An important property of “orthic triangle” is the following one:

Theorem 1. If DEF is the orthic triangle of ABC, then ABC heights are DEF angle bisectors, i.e. ABC Orthocenter is the Orthic triangle (DEF) Incenter.

Students also verified this theorem in the ITC laboratory, by using GeoGebra, as shown in Fig. 6.8.

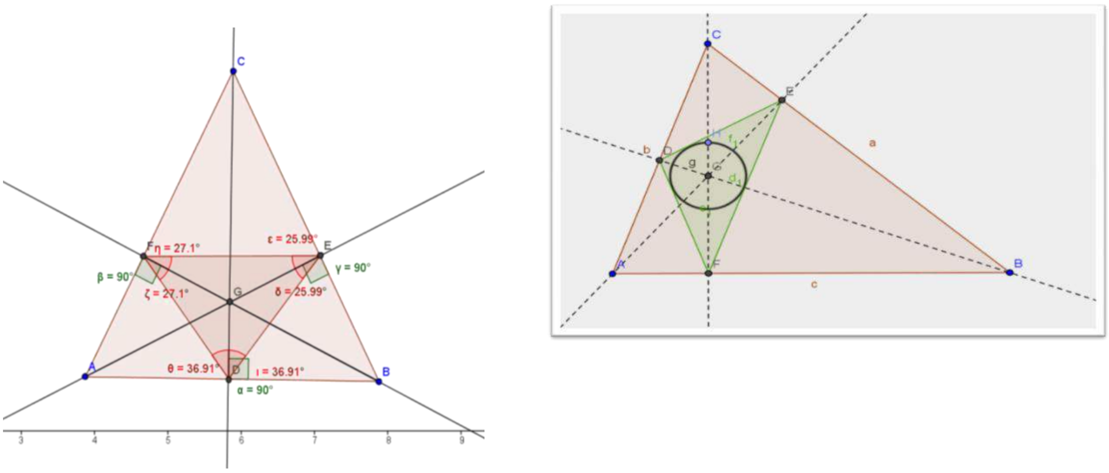

Figure 6.8: Test of the property “ABC orthocenter is the Orthic Triangle incenter”

During 1700s, because of calculus inception, main discussions were about minimum and maximum problems.

A known minimum problem about these topics was introduced by Giovanni Fagnano (Giulio's son) in 1775:

“For a given acute triangle determine the inscribed triangle of minimal perimeter”. The solution is “the orthic triangle”.

Fagnano' solution used the “calculus”, whereas L. Fej´er and While H. A. Schwarz gave a proof by using synthetic geometry (axial symmetries), in an independently way.

Let's notice that Theorem1 does not immediately follow from Fagnano's Problem, since Theorem 1 is valid both if main triangle is an acute-angled and an obtuse-angled triangle, whereas the Fagnano's problem is true only in the acute-angled triangle case.

It is possible to extend these triangle results to “orthic quadrilaterals” (Mammana et al., 2010) and to billiard physics, too (E. Gutkin, 1997).

7 Extended Topics: Scientific Academies

In each Academy, students were invited to find some information about a Scientific Italian Academy. In particular, their researches were focused on Academia Secreta, Academia degli Infiammati and Accademia dei Lincei. Some news about these Academies are here summarized, as organized by the students in a final booklet.

The first Scientific Academies in the world were: Academia Secreta, probably in Salerno, and Accademia degli Infiammati in Padua, both founded in 1540.

Academia Secreta was founded by Girolamo Ruscelli; scientific interests were mainly about chemistry, alchemy and medicine. Few news can be found about this Academy (Ruscelli et al., 1984), the only reference book being “*Secreti Nuovi di Maravigliosa Virtù*”, written by Ruscelli, under the nickname “*Reverendo Alexis Piemontese*”, containing all the “recipes” about experiments they practiced in the Academy. The *Proemio* to “*Secreti Nuovi*” contains a description of Academy members and place where they met can be found, with no specific references.

Accademia degli Infiammati was founded by Leone Orsini, Frejus’ bishop, Ugolino Martelli and Daniele Barbaro (6 June 1540).

Its motto was “*Arso il mortale, al ciel n'andrà l'eterno*”, referring about Hercules in the fire on the Oeta mountain, as also shown in the Academy logo (Fig.7.1).

Figure 7.1: Accademia degli Infiammati, Logo

Some members of this Academy were: Giovanni Corner, Galeazzo Gonzaga, Alessandro Piccolomini, Sperone Speroni, Bembo, Lodovico Dolce, Torquato Tasso.

Aim of Accademia degli Infiammati was writing about phylosofical (including scientifical) and literary topics, both in prose and verse, by using the vernacular*: «vera et natural idea» di scrivere «compiutamente», in prosa e versi in volgare, su argomenti filosofici e letterari.*

An important, both scientifical and literary, contribution was due to “Sperone Speroni”, in its “*I Discorsi del modo di studiare, La difesa del volgare*”. Speroni outlined the importance of Mathematics (mainly Geometry) to be happy:

“Solamente le discipline matematiche (come la geometria) abbiana una utilità, perché servono come exercitio. La matematica allora, insieme alla logica, è fondamentale per il raggiungimento della felicità costituita dall’unione di sapienza e eloquenza: sono due quasi prohemii, o previe dispositioni alla felicità de’ mortali”

In 1603, the first European Scientific Academy was founded by Federico Cesi, Accademia dei Lincei.

Some members were: Giambattista Della Porta, Galileo Galilei, Francesco Stelluti, Anastasio De Filiis, Johannes van Heeck. .

Generally, Academies were ephemeral, often without programs and specific organizations. Nevertheless, Accademia dei Lincei had a specific program and admission rules, but it didn't survive when its prince, Cesi, died (1630). Nowadays, a renovated “Accademia dei Lincei” is going on, with specific scientific interests, this year under direction of a famous physician, Giorgio Parisi.

Scientific contributions of this Academy were mainly in “Astronomy”, thanks to Galileo Galilei.

8 Conclusions

An interdisciplinary learning unit has been planned and experimented in a second class of a secondary school, which turns around “synthetic geometry theorems, absent in the Italian maths schoolbooks”. This unit has been realized by simulating sixteenth-seventeenth century Scientific Italian Academies. About 800 Academies were founded in Italy in 1525-1700 years, being a worldwide, very important phenomenon that introduced a new way to acquire and disseminate knowledge.

Simulating an Academy also gave a considerable boost to the “Maths social use”, which is very important to increase motivation to study scientific matters, and in particular the Mathematics.

Flipped lessons and Blended Learning were effective methodologies to better organize and share research results.

Students feel enthusiastic about this new learning way and their feelings were also evidenced in all their dialogues and in a final video they realized to summarize all the activities about this learning unit.

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