THE GEOMETRY OF THE DAMBSUTERS

A cross-curricular approach using history in the mathematics classroom with students and teachers

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ABSTRACT

Since 1992 I have developed many lessons that use history in the mathematics classroom. These lessons are cross-curricular using events from history to enthuse students in their learning of mathematics. They show that the mathematics used today had, and continues to have, applications to everyday life. As well as being classroom lessons these episodes form mathematics master classes done on Saturday mornings around the UK. The lessons use old and new technology to provide students with an insight into how they are made and used. Recent developments with the STEM (science, technology, engineering and mathematics) initiative in the UK mean these lessons are even more relevant. One episode is described here; others have been presented at previous conferences.

1 The research impetus

I incorporate events from history into mathematics lessons because I find it very interesting to see the practical applications of mathematics set into the period when it was used. My research questions are qualitative and are concerned with whether students and teachers find the mathematics interesting and clear. I chose to write about this episode since it has been developed since 2003 and is very rich in both mathematics and history.
Tzanakis & Thomaidis (2011) classify the arguments and methodological schemes for integrating history in mathematics education and my episodes fit into the two-way table mainly as History-as-a-tool and Heritage though there are overlaps into the History-as-a-tool and History cell. The over-riding concept in my work is History-as-a-tool. The distinction between history and heritage was made by Grattan-Guinness (2004) where he writes in the abstract of his article:

> Their [mathematicians'] normal attention to history is concerned with heritage: that is, how did we get here? Old results are modernized in order to show their current place; but the historical context is ignored and thereby often distorted. By contrast, the historian is concerned with what happened in the past, whatever be the modern situation. Each approach is perfectly legitimate, but they are often confused.

The reliability of this research in the sense of reproducibility by someone else is impossible to quantify, since teachers use such episodes in different ways with different students and probably not in costume! Every session I do with students is different according to local conditions and the knowledge students bring to the sessions, so results will vary.

2 The evolution of the episode

Back in the 1990s I was introduced to the Royal Institution mathematics masterclasses for gifted and talented 13/14-year-old students. These took place (and still do) on Saturday mornings and last for 2½ hours. They are organised by local groups of interested teachers

and generally take place at prestigious places covering schools within travelling distance. They put on a programme of 4 to 8 sessions (though most places do 6) inviting staff at local schools to nominate a couple of students who would benefit from these master classes. Members of staff are also invited to attend and without these volunteers the sessions would be difficult to run since they attract between 30 and 60 students each session (I prefer to work with groups of about 30, because it is difficult making good contact with larger groups to ensure all remain focussed for the whole session.). These volunteers help when students are involved in the tasks since the sessions involve a lot of interactivity. The topics offered vary considerably and cover a wide range of mathematical topics. As the years progressed I felt that there was plenty of material that was suitable for general classroom work and so by developing the master classes I do in my own classroom I was able to trial the work with a wide variety of ages and abilities. In the 1990s I was inspired by John Fauvel to include various historical activities in my classroom and this was at a time when there was far more autonomy in the UK on what to teach and how to teach it: a description of the situation at the time can be found in Fauvel & van Maanen (2000) on page 16.

In my opinion the work should have relevance to the students, so I always try to include material that covers mathematics primarily and history secondly. However, science, engineering and geography also play significant parts and all these are brought together to provide a holistic coherent scenario in which to develop the mathematics.

Sometimes the opportunities arise by chance: a suitable anniversary, an interest or an opportunity can be the spur to spending many hours pursuing threads which gets sewn into a rich tapestry of mathematics. The STEM (science, technology, engineering and mathematics) initiative in the UK is aimed at developing an interest in those subjects since they have shown a fall in numbers at degree level over the years. By tackling these areas with scenarios that show how they were dealt with in the past I believe it gives students enthusiasm for mathematics and provides them with a rich background of general knowledge. The episode I mention here has been trialled with teachers and students in the classroom and then improved after feedback from them and my peers in the last year. The research questions used in this paper are based on three sessions with 107 13/14-year-old students in November 2017; 35 trainee teachers in August 2017 and August 2018 together with 46 similar trainees in January 2018; and with 16 participants at my ESU8 workshop in July 2018.

3 Description of the episode

Full details of this can be found in Ransom (2004). The workshop presented at ESU8 focuses on fewer activities due to time constrictions. The mathematics involved stems from an event that took place on the night of 16 May 1943 when a significant raid took place during World War II. Full details of this can be found in Sweetman (2003).

The workshops are always presented by Flight Sergeant ‘Kidnap’ Ransom in 1940’s Royal Air Force uniform and participants work in twos. They hear about the Dambusters raid of 1943 and how mathematics was used to navigate the Lancaster aeroplanes over Germany. Participants plot the route on A3 maps, working with scales and bearings then look at how the lights were arranged on the plane so that it travels at 60 feet above the surface of the water, using geometrical constructions to find the angles at which the lights were inclined. They also problem solve by finding the position of the plane when the

bomb needs to be released and the time taken for the plane to pass over the edge of the dam. These two worksheets are illustrated in Figures 3.1 & 3.2:

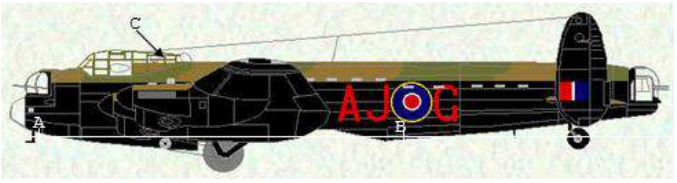
Pilot's Sheet

Your job is to fly the Lancaster at 60 feet above the ground.

To do this you have to adjust the angle of the Aldis lamps (at A and B) so that their light meets 60 feet below the front edge of the wing so the navigator could look out of window C and see the where they meet.

The scale of the Lancaster is 1 cm to 5 feet.

Draw in the ground first.



The Lancaster is …… feet long.

The tail fins are …… feet high.

The Aldis lamps are …… feet apart.

Lamp A is angled at ……° to the line.

Lamp B his angled at ……° to the line.

Use trigonometry to check if you know how.

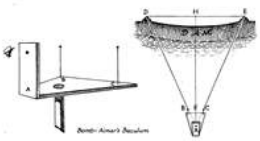
Figure 3.1: Working with scales and construction of a perpendicular to a line from a point

Bomb aimer's Sheet

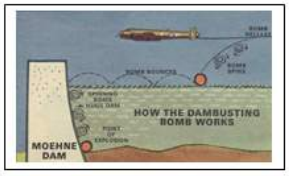
Your job is to draw a plan of the position of the Lancaster when Upkeep is to be dropped. It should be dropped 450 yards from the dam and on the perpendicular bisector of the towers. The Lancaster should be travelling at 220 mph when Upkeep is dropped.

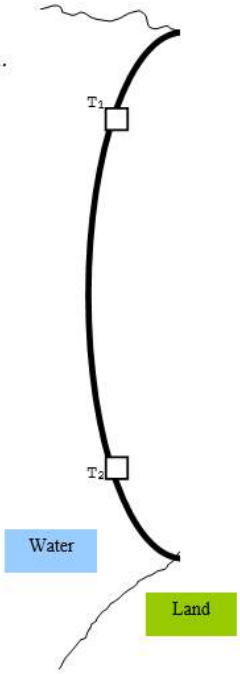
1 yard = 3 feet
1 mile = 1760 yards

The scale of this plan is 1 cm to 100 feet.



This is the device used to ensure Upkeep was released at the right distance from the dam.





The distance between the towers (T_1T_2) is …… feet.

The dam is …… feet wide.

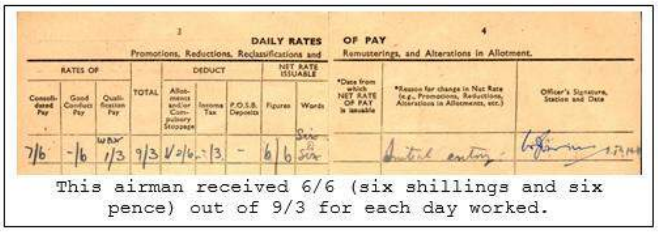
The angle at the bomb aimer's eye is ……°

The Lancaster takes ……seconds to reach the dam after releasing Upkeep.

Figure 3.2: Problem solving, using construction of a perpendicular bisector

These two activities involve ruler and compass constructions and are excellent problem-solving activities. They develop their mental geometry with a short exercise that should improve their 3D coordination. With clips from the Dambusters film they then make a bomb-aimer's sight and test it. This activity involves creativity in the mathematics classroom. As far as possible, the workshops recreate, in the form of mathematical theatre, what happened with Squadron X (later known as Squadron 617, the Dambusters squadron) in 1943. There is also a civilians' sheet (Figure 3.3) which deals with the coinage of the time and how that relates to the decimal system used today. That sheet is illustrated here.

Civilian's Sheet: The coinage of 1943: see if you can fill in the table!


DAILY RATES OF PAY

This airman received 6/6 (six shillings and six pence) out of 9/3 for each day worked.

Coin	Number of old pence	Number of coins/notes in £1	Number of new pence
farthing	¼		
halfpenny			
penny	1	240	
threepenny	3		
sixpenny			
shilling		20	
florin	24		10
half crown	30		
10 bob note		2	
pound note			£1

What do you notice if you multiply the number of old pence by the number of coins in £1?

.....

Figure 3.3: Working with historical objects (coins) and inverse proportion

Figure 3.4: Teachers test their bomb-aimer's sights with Flight Sergeant 'Kidnap' Ransom

The other benefits to this class' learning included the following: working cooperatively and developing their creative skills.

Did the historical aspect make a difference? I have no evidence to say 'yes' or 'no' at this point since no question was directed at this aspect. (This links in with the History side of the two-way table mentioned in Tzanakis & Thomaidis (2011)). However, there are some comments about the history in the feedback analysis section of this paper.

When trialling new work with my students I never know what will happen, but with over 30 years' experience of the 11-18-year-old classroom I think I can judge reasonably well what will be accepted. In the plenary at the end of a session students are asked about what they learnt and it is only the comments that students offer at the end of the session that give me an insight into whether these sessions are successful or not. Comments such as 'The part I enjoyed least was going home' and 'It was interesting because we were given a realistic scenario for the maths.' encourage me to continue with such sessions adapting them for different groups. Comments such as 'The civilian's sheet wasn't very clear'; 'I didn't enjoy the mental geometry at the start' means I make amendments to the work if something is not enjoyed by most students.

4 Sensitivity issues

The UK is a multi-cultural society with students from many different nationalities and this subject is dealt with as sensitively as possible. War sets nation against nation and is not to be treated lightly or jingoistically, which is why I explain to the students the fact that although the raid on the dams was seen as a success at the time, most of the casualties were civilians, rather than militia. I also mention that this had an effect on the Geneva Convention, resulting in an amendment that said dams, being a non-military target, were not allowed to be attacked under the articles of war.

It is unfortunate that mathematics and science both make progress during wartime. We see how the mathematics of fortification improved during the 17^{th} and 18^{th} centuries in France when Vauban played a major part in fortifying strategic towns and cities. The study of projectiles during this time also developed. Then we see the development of the computer under Alan Turing during WW2, with the original intention of cracking the Enigma Code. This is a great benefit, less so the development of nuclear warfare.

I also mention the fact that Barnes Wallis, the inventor of the 'bouncing bomb' that allowed the mine to bounce along the surface of the water and run down the inner face of the dam, felt that the success of the raid was far out-weighed by the loss of eight Lancasters and their crew – a total of 56 young men of the RAF.

5 Feedback analysis from 13/14-year-old students

No formal survey is conducted after every workshop, but for ESU8 I thought it would be useful to collect some simple data in a format that was easy for students to complete. Bearing in mind that students who have survived 150 minutes (with a 10-minute break) of working with a partner they have not met before do not want to spend more than a couple of minutes filling in a feedback form, this is kept simple.

Dambusters Evaluation Form

Thank you for attending a Dambusters masterclass. I hope that you found it interesting.

Your feedback helps me to develop the session so I would be grateful if you could complete my evaluation form. Please include any additional comments on the back of the form.

Sussex masterclass – Saturday 11 November 2017

Please rate the session	**Poor**	**Average**	**Good**	**Very good**	**Excellent**
How interesting was the session?					
How clear was the work?					

	Too low	**Just right**	**Too high**
The level of the session was …			
What part did you enjoy most?			
What part did you enjoy least?			
Please circle your gender	**Female**	**Male**	

Figure 5.1: Feedback form used with students

It is important to me that the mathematics I present is accessible by both genders, hence the last part of the form. This was used with 107 students over two sessions in November 2017 (one with 45, the other two with 32 and 30) and the data gathered (not everyone returned a feedback form) are shown in the following tables:

Female results

Please rate the session	Poor	Average	Good	Very good	Excellent
How interesting was the session?			6	24	12
How clear was the work?		4	13	21	4
	Too low		Just right	Too high	
The level of the session was …	1		41	0	

Male results

Please rate the session	Poor	Average	Good	Very good	Excellent
How interesting was the session?		2	10	29	10
How clear was the work?		4	13	25	9
	Too low		Just right	Too high	
The level of the session was …	3		48	0	

So, overall, 95% of the replies indicated that the level of the work was 'just right', which indicates to me that the work is most suitable for the ability of the students. This is a 10% difference in the genders between how interesting they found the session: 86% of the females found it very good or excellent, compared to 76% of the males. I thought that if there would be any difference it would be the opposite way around. There is a 7% difference in the genders between how clear they found the work: 60% of the females found it very good or excellent, compared to 67% of the males.

As a further check on whether there is any significant difference in the interest between the genders, I started with the null hypothesis that there would be no difference in the interest of the material between the genders and used Fisher's exact test to see whether this hypothesis should be accepted or not. This produced the following two-way table:

	Male	Female	*Row total*
Very good / Excellent	39	36	*75*
Poor / Good	12	6	*18*
Column total	*51*	*42*	*93*

This gives a probability of 0.270, which is well above the probability of 0.05, below which the null hypothesis would be rejected. Therefore, I accept the null hypothesis that there is no difference in the interest of the material between the genders.

To keep the wording at a minimum on the worksheets I keep the instructions as concise as I can because these tasks are the type of problem-solving activities that is being promoted in the UK. Students are expected to tackle problems using the mathematics they have learnt and for sessions like this where students are working in pairs with others from different schools it encourages them to communicate mathematically and suggest approaches to the problems that they have not seen before.

I found it interesting that 44% overall (48% of the females and 41% of the males) enjoyed the making of the bomb aiming sight the most. This was by far the most enjoyable part that was mentioned in the feedback form. However, there were six comments (6.5%) about the history (or history related) being the most enjoyable part: here are some of their replies to the question 'Which part did you enjoy most?' with the gender of the responder.

- *Learning the back story* (female)
- *It was interesting because we were given a realistic scenario for the maths* (female)
- *History behind lesson* (female)
- *Learning about the history of it* (male)
- *Historical math*(male)
- *It being maths + history*(male)

6 Feedback analysis from trainee teachers

This analysis consists of feedback from three groups. These consisted of 35 Quantum Scholars (trainee teachers who have come from different countries to teach in the UK,

some of whom have taught for a small number of years) in two cohorts, one of 13 in August 2017 and the other of 22 in August 2018. The third group was 46 Mathematics Scholars in January 2018. These are trainee teachers who have been through a rigorous selection process to gain the prestigious title of Scholar. They are given no extra money, but receive two years' free membership of The Mathematical Association, the London Mathematical Society and the Royal Statistical Society as access to various resources and a community entitled to attend various events for their benefit without charge. The Quantum Scholars sessions lasted 2½ hours, like the 13/14-year-olds, the Mathematics Scholars had a shorter 1¼ hour session.

The data gathered from these groups (again, not everyone returned a feedback form) are summarised in the following tables. This time, because there was little difference between the results for each gender (and two sheets were returned without any gender identification), I have not separated out the results based on gender, but on the different types of trainee since they had sessions of different lengths.

Quantum Scholars results

Please rate the session	Poor	Average	Good	Very good	Excellent
How interesting was the session?			1	12	22
How clear was the work?		1	6	20	8
	Too low		Just right	Too high	
The level of the session was …	0		27	0	
Are you likely to use any of this material in the future?	No	2	Yes	33	

Mathematics Scholars results

Please rate the session	Poor	Average	Good	Very good	Excellent
How interesting was the session?			8	23	15
How clear was the work?		3	17	17	9
	Too low		Just right	Too high	
The level of the session was …	0		45	1	
Are you likely to use any of this material in the future?	No	3	Yes	43	

For these trainee teachers the level of the session was 'just right' for all bar one, which indicates to me that the work is most suitable for the ability of the trainee teachers! This is a 14% difference in the two groups between how interesting they found the session: 97% of the Quantum Scholars found it very good or excellent, compared to 83% of the Mathematics Scholars. I thought this difference was probably due to the different time spent with each group. This is important feedback for me as it indicates that one should not try to rush to include as much as possible in the session, but to allow people more time to think and discuss the work with others. There is a 23% difference in the groups between how clear they found the work: 80% of the Quantum Scholars found it very good or excellent, compared to 57% of the Mathematics Scholars.

As a further check on whether there is any significant difference in the interest between the genders, I started with the null hypothesis that there would be no difference in the clarity of the material between the two groups and used Fisher's exact test to see whether this hypothesis should be accepted or not. This produced the following two-way table:

	Quantum scholars	Mathematics scholars	*Row total*
Very good / Excellent	7	28	*35*
Poor / Good	20	26	*46*
Column total	*27*	*54*	*81*

This gives a probability of 0.016, which is well below the probability of 0.05, below which the null hypothesis would be rejected. Therefore, I reject the null hypothesis that there is no difference in the clarity of the material between the two groups and accept that the Mathematics Scholars found the work less clear than the Quantum Scholars. As mentioned in the previous paragraph, this is probably due to the shorter time the Mathematics Scholars had to digest the material.

To keep the wording at a minimum on the worksheets I keep the instructions as concise as I can because these tasks are the type of problem-solving activities that is being promoted in the UK. Again, I think this is due to the time spent and lack of the discussion time.

The bomb aiming sight was not made with the Mathematics Scholars due the time restriction. An extra question "Are you likely to use any of this material in the future?" that was not used with the 13/14-year-old students was added to the feedback form as I wanted to know whether any of the trainee teachers would find the materials useful when they are teaching full-time. 94% of each group said they would use some of the material in the future, which I find most encouraging as I always hope to pass on useful material.

There were 31 comments (38%) about the history (or history related) being the most enjoyable or interesting part, with 34 comments (42%) about it being real-life applications of mathematics. Here are some of their feedback comments that I enjoyed reading most!

- *How the women were involved and how maths made a difference in the war.*
- *Learning about something I had no idea about.*

- *The nice mix of maths topics.*
- *Really nice contextualisation of a few topics.*
- *History of it really brought it [the mathematics] to life.*
- *Presentation of material – very good delivery. The chap was a character.*
- *The way Peter created learning without noticing – hands on with real life connections.*
- *Hands on, real world content – highly engaging and collaborative.*

7 Feedback analysis from ESU-8 participants

Feedback was received from 16 participants from various countries at my workshop on July 21, 2018. This was timed for 1½ hours, but since it was timed against other 2-hour workshops most (if not all) participants stayed for 2 hours. The feedback here is far more varied than for the previous two sets of feedback as those present have generally spent far more time in different branches of mathematical education. Therefore, I have listed more comments as I find these very helpful in developing my teaching. Due to some incomplete feedback forms the numbers do not always add up to 16.

Participants' results

Please rate the session	Poor	Average	Good	Very good	Excellent
How interesting was the session?				4	12
How clear was the work?			3	6	7

	Too low	Just right	Too high
The level of the session was …	0	14	0

Are you likely to use any of this material in the future?	No	3	Yes	11

So, overall, 100% of the replies indicated that the level of the work was 'just right', which tallies well with the feedback from previous sectors. A higher proportion of this group (21%) were unlikely to use any of this material in the future, which is what I expected, given that few of this group are teaching 13/14-year-olds. Here are their replies on what they found most useful:

- *The connection between mathematics and military (the real problem).*
- *The stories in the history. They can be used to teach mathematics.*
- *Change of perspective, transformation geometry, estimation.*
- *Range of mathematical tasks the context applied to.*
- *The turning card. That was nice and surprising. I think this is especially adequate for teacher training.*

- *The use of simple, even primitive material, to solve significant and interesting problems.*
- *To see how the approach was a hands-on approach. It was a wonderful performance! Thank you!*
- *Mathematics teaching is teaching a story. The more authentic we can make the story, the more students appreciate its utility.*
- *I am no more teaching mathematics, but it will inspire me with my grandchildren!*

Here are some of their replies on what they found most interesting:

- *The use of mathematics to solve the actual problem.*
- *Measure and draw the bomb aimer's sheet.*
- *Different kind of math concept applied in one lesson.*
- *The whole story and the connection with real sources as well as the personal bonding.*
- *The authentic setting was both interesting in itself and quite relevant to the kinds of problem needed to solve.*
- *I liked the concrete calculations and it inspires me to do similar things in other contexts.*
- *All the different applications of math (trig, data analysis, spatial reasoning, constructions) all in one thematic context.*

8 Development

I am always looking for new material to include in these sessions and since doing the research in this paper I have come across a navigator's mapping board, shown below.

Figure 8.1: Navigator's mapping table with movable arm allowing a course to be plotted and detail of the North pointing arrow (circled) and movable protractor

The board itself is made of wood and is hidden underneath the map. The metal arm allows the movable protractor to be put on any position on the map and is a very interesting linkage, consisting of two sets of parallel rulers, joined by a right-angled isosceles triangle. Thus, since the sides of the triangle always remain parallel to the sides

of the board, the small metal arrow always points North and the course can be read from the protractor. So, this board will be shown at future student workshops and students will be allowed to use it, if time permits.

9 Conclusion

My experience with using history in the mathematics classroom is in relating the mathematics students do with episodes from history when it was used. I believe students need to see where mathematics has been used to appreciate its importance. I imagine that teachers will not run a similar session in its entirety as few mathematics lessons are long enough to do that. However, some of the activities are stand-alone and can be used when appropriate. This activity was developed for use in the classroom and master classes, not as a research project, so the evidence of their success (or failure) is mainly ephemeral through comments received by participants after each session.

Finally, I wish to express my thanks to all the students and colleagues who have provided the feedback on the sessions, and my reviewers whose comments (especially with regard to using Fisher's exact test) on this paper have been invaluable and very much appreciated.

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