THE ART AND ARCHITECTURE OF MATHEMATICS EDUCATION

A Study in Metaphors

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ABSTRACT

This chapter presents the summary of a talk given at the Eighth European Summer University, held in Oslo in 2018. It attempts to show how art, literature, and history, can paint images of mathematics that are not only useful but relevant to learners as they can support their personal development as well as their appreciation of mathematics as a discipline. To achieve this goal, several metaphors about and of mathematics are explored.

1 Every story is a journey

This chapter is based on the talk given in Oslo in 2018, and as such it cannot and will not attempt to cover all that was said and not said. During questions after the talk also, much was discussed which would be unfair to summarise here, and so that will not be attempted.What is presented here is therefore a summary of what was said and at the same time a new piece of work. The aim of the chapter is to paint a picture of how history of mathematics can support learning in a personal way, and in the time in which it is becoming increasingly unclear what visions of the future we share – in Europe as well as the US.

The story that follows is that of a personal journey that I have shared with my students and colleagues, and the ways I used metaphors in mathematics education.

So let me begin. It was an honour to be invited to give the lecture on which this chapter is based, and it came at an important time for me personally. I had been, until the summer of 2018, in mathematics education for more than twenty years in various roles from working as a teacher, to being a teacher instructor and educator. Towards the end of that period, and leading up to this lecture, I have moved from teaching secondary to primary teachers. I also knew at the time that this period itself was coming to an end as I was looking to move to another position teaching students in STEM professions, and further away from educating teachers for reasons that will become imminently clear further bellow. This was therefore, a period in which my sense of purpose and the role(s) I had in mathematics education was moving from educating teachers to a wider educationalremit of using mathematics both as an applied discipline and, more importantly for me, as a tool by which my students can learn to think about and solve greater problems of their lives.

This was, therefore, an opportunityfor me to summarise a couple of decades' long experience and through some examples of my research show lessons that the history of mathematics can teach. To be able to do this in a relatively short time, and manifest the ways in whichmathematics history can help, support, and enhance learners' experience of mathematics education, requires a little bit of an art. Theart in question was therefore decided to rely on an underlying structure of a kind, and the use of some tools. I decided to paint some images of mathematics through metaphors that can be used for learning, and that I used in my work with teachers. Let us then, begin our journey of exploring such images.

1.1 Wise women and men (or paying the homage to mathematics and mathematicians)

A couple of years ago I did some experiments with my colleagues mathematicians: I asked a room fool of them to raise a hand if they were a mathematician. This may sound slightly illogical, but there was a preceding discussion to this experiment that made me realisethat some mathematicians don't consider themselves to be mathematicians. The answer to my question was intriguing – only three people out of about seventy admitted to being so. From this came a little research project (Lawrence, 2016) that showed the disparity of the views of mathematicians from within and without the profession. While pursuing the project, it became apparent to me that, as groups of professionals go, this one has been an incredibly *nice* one to do a research on. *Nice* in this case I use to mean unpresumptuous and unassuming, rather than, for example, superior, or rather I found almost no one in this group having images of themselves of being superior in any way.

It turned out that the status of mathematician was so highly rated by the group, that this was only bestowed by individuals from the group on those who changed the field in a major way or had a calling of a professor. To aim high, and to maintain such a discipline (however within the discipline) was I thought quite *nice*. It did not give much favours to the profession or the view of it from the outside, but it showed true humility.This paper is therefore, an homage to all those who do mathematics professionally, and are oriented towards education, pedagogy, and history, and are not even boasting that they do make a huge difference in the world.How? Let's hope what follows may explain that.

1.2 Desert of mathematical education landscape – where has the sea gone?

In 2004, a national enquiry about took place in England and Wales (Smith, 2004) that would have major consequences for mathematics education in this country. A question was posed to young people from around the country on what words describe their experience of mathematics education best. The two most commonly used words were ‘boring’ and ‘irrelevant’ (Lawrence & Ransom, 2011). Following this enquiry, the problem being identified, huge efforts have been made by successive UK governments to ensure this is rectified. Finally, from 2014, a large number of further studies, enquiries, and projects, led to a unified solution: to introduce Mathematics Mastery in state schools, therefore learning from the Chinese mathematics education experiences and successes (Mathematics Mastery).

To use a little metaphor, overnight the mathematics educational landscape became an unknown territory for hundreds of thousands of teachers entering the profession. The new system had no, or very little, history or familiarity with the existing. History of Chinese mathematics and history of its mathematics education is unfortunately, not a well-known field among mathematics teachers and educators in England. But there was a help at hand – a nation-wide network of consultancies and projects started to be funded by the government to introduce the system, a process still on-going across the country. Notwithstanding pluses and minuses of this novel system, it is worthwhile mentioning that schools must buy *into* and *subscribe to* it at a considerable expense (Benson, 2016; 2018).

In this context, it became even more important than before to remind how history can, apart from all other aspects of its benefits to mathematics education (Barbin et al., 2015), give teachers and learners opportunities to find their authentic pathways for learning and development.

The following chapter therefore is meant as a metaphorical travelogue from my last couple of years as a teacher educator in this new landscape of mathematics education.

1.3 Metaphors and their role in the world of education

The role of a metaphor has long been recognised in mathematics education (Latterell & Wilson, 2017; Bishop, 2012; Erdogan et al., 2014; Gibson, 1994; Presmeg, 1997).To present the types and uses of such metaphors, I will first begin with a metaphor about the pillars of wisdom – something I picked when researching the history of geometry in England in the early modern period (Lawrence, 2002). In this tradition, there is a reference to Euclid, for example, and the two pillars of wisdom are mentioned as means through which geometrical knowledge was transferred during and beyond the great flood as described in the Bible (Lawrence, 2002).

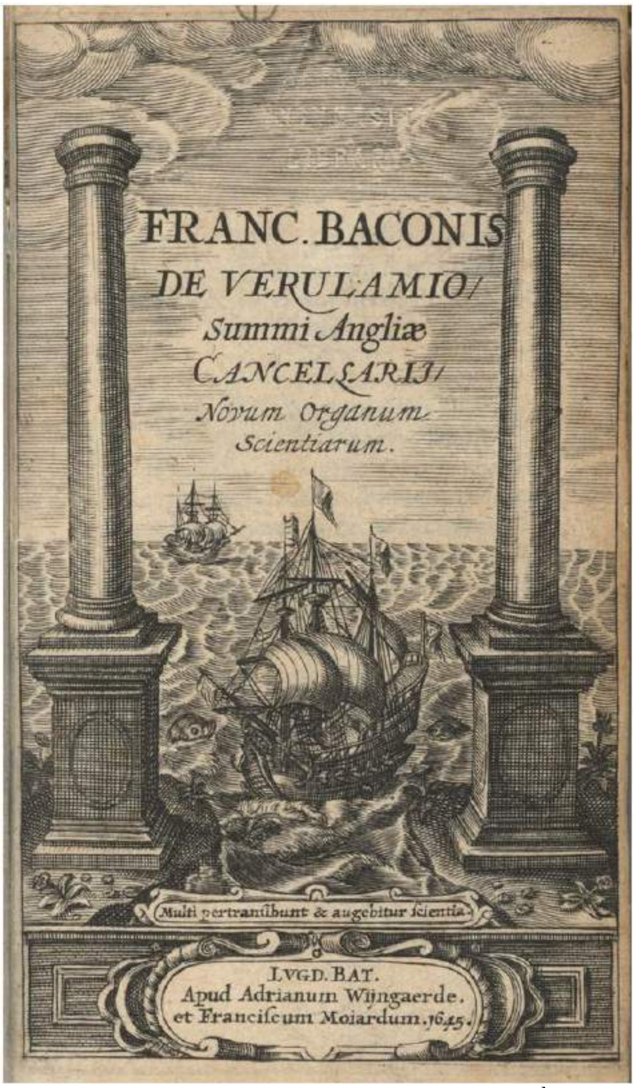

Figure 1.1: Two wise pillars of *Novum Organum* (1645, 2nd edition), London, Francis Bacon

But we can also think of twopillars of wisdom as central symbols of Francis Bacon's (1561-1626) *Novum Organum* (1620) which were about finding the essence of a thing'. This, Bacon suggested, is done by the use of inductive reasoning, which is illustrated by the two mythical pillars of Hercules that stand at the either side of the Straight of Gibraltar, and should be smashed through to open up possibilities to explore new world of exploration.

What relevance is there of a metaphor of the 'pillars of wisdom' to mathematics education? I will explore thismetaphor and use it so we can see how a construction of an edifice that mathematics education can be seen as,could withstand the destructive forces (whatever these may be) and could even be erected in our newly developed metaphorical desert.

2 Solving or resolving: a dichotomy of the current system in mathematics education

In the introduction I mentioned why the history of mathematics is becoming increasingly, in my opinion, more important to mathematics education in the UK. Here I want to identify and name what I believe is going on currently: the moving of the focus from something that teachers have free access to (known sources and resources of mathematics and mathematical cultures) to something that can be done only via an agent (sometimes the state assuming this role) and through controlled exchanges and publishing projects, is a *de facto* process of de-contextualising mathematics education. But how is this detrimental to the outcomes of mathematics education? The question can indeed be discussed for much longer than I intend and am able to here, but we will concentrate on one simple focal point: by de-contextualising mathematics a whole field of appreciation of the discipline, its history, methods, and traditions are lost.

Let us look now at two divergent but perhaps equally important aims of education in any discipline: proficiency and skill on the one hand, and the appreciation of the discipline in question on the other.

At this point it is important to make clear that, in my opinion, appreciation of a discipline could not be the *sole* purpose of education in general, and mathematics education in particular; here I agree with Paul Sally (Sally, 2008). And equally, the other 'pillar of wisdom' of the discipline, the teaching of appreciation, cannot be replaced by a process which leads to acquisition of skills and utility only. Arguments have been made around this question for centuries: let us then through them examine whether we can say that two aims, skill and appreciation, are equally important in mathematics education. In other words, can we use the metaphor of two 'pillars of wisdom' for mathematics education in this way?

2.1 Arguments for appreciation of mathematics in education

Dewey's argument for appreciation is this: aesthetic experiences are ones that promote growth of an individual, and this goal must be central to the conception of the goals of education (Dewey, 1934). Wilson's example is similarly related to the creation of community of individuals, the cornerstone of any peaceful and successful society: he points that the universally shared aesthetic preferences contribute to building of such societies (Wilson, 1998). Žižek sheds some further light on thisparticular aspect (building of society) from a political and ideological stance of a philosopher: and says that there is no movement without a poet (Žižek, 2001). This is, Žižek says, because poetry ‘continues to give us artistic pleasure long after its historical context disappeared; this universal appeal is based in its very ideological function of enabling us to abstract from our concrete ideological-political constellation by way of taking refuge in the “universal” content’(Žižek, 2001: 2). So these are views that relate in general to the importance of aesthetics in societies.

We can further analyse and see how this can actually work in educational practice. Le Lionnais, for example, gives us a classification scheme of facts and methods and points us to another vantage point from which we can see mathematics education.Facts can be seen as artefacts, methods as proofs or techniques, and our choice of study is based on subjectivity of aesthetic experience (Le Lionnais, 1948).

If we think of how important it is that our pupils become individuals who can think for themselves too, Pinker and Dissanayake’s argument can be useful. It goes to tell us of the consequential dimension of human behaviour, the way in which ‘‘making special’’ or ‘‘registering enabling acts’’ forms the basis for aesthetic sensibility as a necessary tool in education (Pinker, 1997; Dissanayake, 1992).

2.2 Aesthetics and elitism

For a book relating to mathematics education though, Poincaré’s view must certainly be somewhere there at the top of the important views. It is quite well known that he promoted the view that the aesthetic dimension of mathematics is the defining characteristic of mathematics, and not the logical. This was met with the charges of elitism as most learners will not have the opportunity to do (real) mathematics, whilst others will not be capable of experiencing the aesthetics of it as they may not know enough of it (Papert, 1978). But does this charge mean that we may not try? And not try to teach the teachers how to find such aesthetic dimension in mathematics either?

2.3 Value of aesthetics in mathematics education

To go back in history, I found a couple of views which were interesting. Alexander Baumgarten (Baumgarten, 1735) emphasised aesthetics as the experience of art, and this subsequently as a means of knowing. In the same vein, Ehrenfels (Ehrenfels, 1897) explored the meaning of value in education. Value, he said, is attributed to something, contingent through desiring it: “We desire things not because we comprehend some ineffable quality ‘value’ in them but we ascribe value to them because we desire them” (Ehrenfels, 1897, p. 44). Certainly this type of thinking has made consumer society flourish. But let us stick to mathematics education: there are many papers and books that have been written on this, too numerous to mention, bar the ones given to begin investigating such claims (Sinclair, 2008; Betts, 2003).

The aesthetics is generally then accepted as an important aspect of social life, and of educational experience. How can then mathematics education flourish and inspire if it is stripped of such experience and searching for it in various examples rich in different contexts, some of which would surely be historical?

3 Metaphors about mathematics

In this section I will first look at some examples of metaphors that can be used in mathematics education.

There are a few metaphors which are often used for mathematics. An example is ‘mathematics is a universal language’. Like all metaphors, this is partly true, depending on the possible interpretation. The quote is a take on the original by Galileo:

> This book (i.e. the universe) is written in the mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth…(Galileo, 1623, p. 178).

Then there are other famous metaphors about mathematics: ‘mathematics is a universal language’ is about what mathematics is or can be. ‘Structures are the weapons of the mathematician’ is a metaphor also often used that tells us about how that is possible, a famous quote by Bourbaki (Bourbaki, unknown date). Luckily we have a particular example of this ‘weapon’ in Euclid: “Reductio ad absurdum, which Euclid loved so much, is one of a mathematician’s finest weapons. It is a far finer gambit than any chess play: a chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers the game” (Hardy, 1941, p. 12). In the same book, Hardy of course offers a few more metaphors – but perhaps in our context of mathematics and aesthetics, it is most pertinent to mention his view that “mathematician’s patterns, like the painter’s or the poet’s must be beautiful; the ideas, like the colours or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics” (Hardy, 1941, p. 84).

In a very short space, we have therefore seen three metaphors: about what mathematics can be described as being, about what tools it has, and about how mathematicians act and use mathematical tools.

3.1 Grounding metaphors

The three previous metaphors about mathematics require some experience and knowledge of what mathematics is like, but are in effect not too complex. They provide the ground upon which mathematics can be further explored through the study of metaphors related to its pursuit and history. Let me however give an example of a complex metaphor – one which can only be understood by having some knowledge of mathematics (in this case geometry).

3.2 A hope from Flatland

The value of such a complex metaphor is that it can provide a basis upon which to explore the mathematical content in education. An example that I have often used in work with teachers was that of *Flatland* (Abbott, 1880; Lawrence, 2015). This mathematical novella uses mathematics to provide opportunity to consider other aspects of life of an individual and society: “I exist in the hope that these memoirs… may find their way to the minds of humanity in Some Dimensions, and may stir up a race of rebels who shall refuse to be confined to limited Dimensionality” (Abbott, 1884, p. 3).

Now this is a complex metaphor – to understand and appreciate Abbot’s text, both an understanding and knowledge of context and history are needed, as well as the understanding of some mathematical ideas (Lawrence, 2015). In fact Abbott used mathematics here to give a message about society and equality.

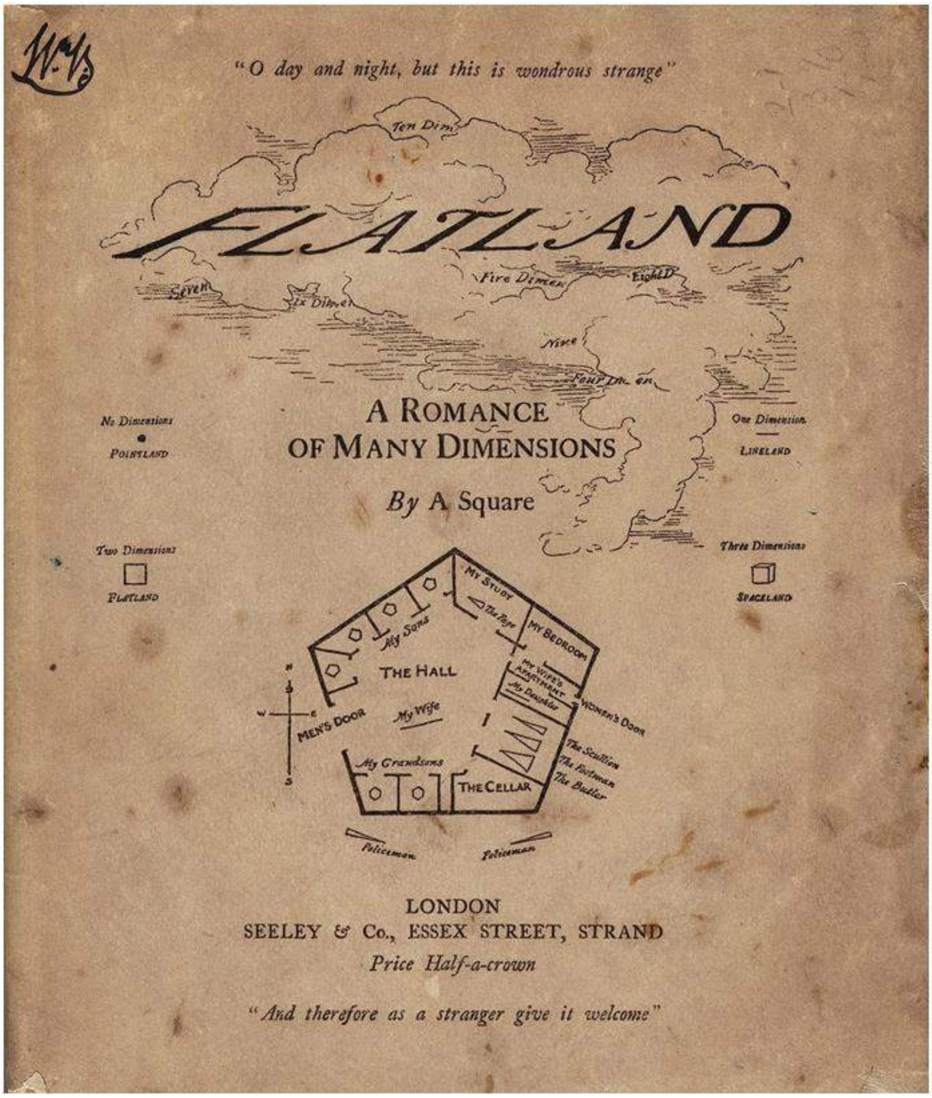

Figure 3.1: Front page of Flatland, 1884, a novel by Abbott. Abbott is a good example of complex metaphor using mathematics to promote a message about equality between people.

Such metaphors are complex to recognise, deconstruct, and use in mathematics education. They are a good pathway to introduce the learning the mathematics and its history which are needed in order to understand the meaning of the text. This in turn offers opportunities not only for the learning of our discipline, but for examining and considering the values of societies of which we are part.

These are then the types of metaphors that I have used and developed in my work with teachers in the past decade. Whilst I have here explained what *kinds* of metaphors can be used, I will now explore two of my projects to show *how* that can be done.

4 Mathematics is a space

One of my projects was about searching for an aesthetically based experience based on a historical piece of art, which would lead students towards learning some new mathematics. The aim of this particular approach was to take my students away from an increasing pressure of (often and increasingly, state-) prescribed approaches and the utilitarian view of mathematics, and to remind them of our ‘second pillar of wisdom’, where they would explore how to teach appreciation of beauty contained in mathematics, and how to attempt to facilitate such experience in aprocess of learning mathematics.

I have describedmy project on Euclid already in other publications (Lawrence, 2016a; Lawrence 2018), so will not go into details about this project, but will need to describe what has been the new lawyer to this existing base.

4.1 Nature of mathematical space

The image on which my *Euclid* project was based was Raphael’s *The School of Athens*, a fresco in the Vatican, painted between 1509-1511. The image portrays most famous scholars from the Greek tradition: Aristotle and Plato walk together and are surrounded by some of the finest minds of their civilisation, finding themselves in an imaginary space in which they discuss their work.

Figure 4.1: Rafael’s *The Academy of Athens*, Vatican, 1509 - 1511

In this painting, I first concentrated (Lawrence, 2018) on the figure of Euclid, and his demonstration of a theorem on a little black-board. From there, the journey began to find which theorem exactly was being demonstrated. This led to consider mathematics from the Euclid’s *Elements* and the late antiquity contributions by pseudo-Euclid, believed to be mathematicians from late antiquity, Hypiscles and Isidore of Miletus (Lawrence, 2018). The journey further continued by the examination of Proclus’ claim that the *Elements* may have been writtenin order to present all mathematics one would need to understand regular polyhedra:

Euclid belonged to the persuasion of Plato and was at home in this philosophy; and this is why he thought the goal of the *Elements* as a whole to be the construction of the so-called Platonic figures (Proclus, p. 57; Rosán, 1949: pp. 11-35; Sanders, 1990).

Although that was refuted by Heath, (Heath, 1956: I, p.2) it is an interesting and possibly truthful view, and one that is important in the contact of the discovery and re-discovery of semi-regular polyedra in Antiquity and then during the Italian and European Renaissance (Field, 1997).

In this, probably best described as the second leg of the research journey, we worked on Platonic and Archimedean solids and conversed about artists and mathematicians such as Piero della Francesca, Dürer, Luca Pacioli, Barbaro, and later Kepler. An exciting result was to realise that the rhombicuboctahedron, the translucent object subtended from the ceiling behind Pacioli, painted (c. 1500) by de'Barbari (c. 1460-1516) was actually for the first time visualised, ordrawn and recorded, in this image. This is interesting as there have been some charges of plagiarism, by Pacioli, of Piero della Francesca (Stakhov, 2009). The proof that this may not quite be so was to be found (not in the pudding this time, but) in this painting of Pacioli. The project ended up by considering mathematical knowledge needed for such a painting to be made, and the importance of the supremacy of invention in mathematics. This in turn is an important aspect of both the history of mathematics and the understanding of the concept of originality in mathematics, and how these can be explored in education.

Figure 4.2: *Luca Pacioli*, portrait of the famous mathematician by Jacopo de'Barbari, 1500.

One can take a view that the nature of mathematical space is such that it has to be described and constructed in some way. A step towards that would be to be able to construct regular and semi-regular three-dimensional solids which can populate such a space (Sanders, 1990). And that is where the lesson of that particular project, designed for secondary (therefore specialist) mathematics teachers rested. So what could possibly be done further with the primary teachers?

4.2 Euclid and his friends

Whilst I could go into the details of mathematics contained in the original image (Euclid's diagram on Rafael's painting) and subsequent research with specialist mathematics teachers, the primary mathematics teachers in training could not access that mathematical content due to their lack of pre-entrance qualifications in mathematics. The majority of my work with them was therefore to a) enable them in their functional skills in mathematics and b) venture with them into the world of mathematics history to offer views of mathematics that would draw them into further study. In this way, I hope, they would be able to build their own subject-knowledge in years to come. The focus therefore changed slightly. Whilst with the secondary teachers my focus was on the history of mathematics, by the way of investigating mathematical examples, with the primary teachers my focus was shifted onto the method by which they could learn some mathematics through historical research.

The work therefore focused on re-creating the journey I had undertaken in previous couple of years with secondary teachers as if creating a travelogue for the primary school teachers. Avoiding mathematical detail, there was much left from the research that could still be explored. The emphasis turned from understanding mathematics to understanding the relationships between those who create mathematics, whether they are contemporaries or somehow grouped by their interest or their 'pursuit of truth' through mathematics.

An interesting outcome was the exploration of other sources as a consequence. One student mentioned at some point that Pacioli, Rafael, Dürer, could be put in a friendship'group'. There ensued my re-reading of the *Euclid and His Modern Rivals* with the students of this 'year group', which then made a connection between literature (easily accessed by primary school teachers) and mathematics. I of course refer to the author of both *Euclid and His Modern Rivals,* as well as novels *Alice in Wonderland*, and *Alice Through the Looking-Glass*, Charles Dodgson (1832-1898). From here on we could explore many aspects of correspondence between mathematics, logic, and literature. We followedsome of Dodgson's dialogues that are topical in the current global rise of fake news, and imagined where and when we could produce arguments such as these:

> "Alice laughed: 'There's no use trying," she said; "one can't believe impossible things.'
> 'I dare say you haven't had much practice,' said the Queen. 'When I was younger, I always did it for half an hour a day. Why, sometimes I've believed as many as six impossible things before breakfast'" (Dodgson, 1871: 24).

And so Euclid was able to again take us from 300BC to 2018AD, and from finding solutions to mathematical problems to those of contemporary every-day life. What new lessons could we find in Euclid's *Elements,* and how can the history of mathematics support general goals of education? I will come back to this point in the conclusion.

4. 3 How is mathematics 'space'?

As metaphors go, 'mathematics is a space' is not perhaps the most 'catchy' one. But notwithstanding its lack of appeal at first sight, let us now examine what it refers to. When I used that metaphor with my students towards the end of the work with them in both cases when the project run (with secondary and then primary teachers) I was referring to mathematics that transcended cultures, eras, and personal connections and that had made possible the creation of a universal space. This space, which may well be imagined as *The Academy of Athens*, is where ideas can be exchanged, tested, and discussed, just as in the Rafael's painting.

Against this backdrop we may look at some images of modern Academies in the UK. If you conduct search on the Internet for such images (one local to me, Thomas Clarkson Academy for example), you will see fantastic new architecture being made to house the new state-private enterprises that schools are becoming, which are changing not only the landscape of school architecture, but the landscape of educational values too. Within that landscape however, there is not much change in terms of providing 'space' for children, teachers, their supporters – sponsors and business partners of such academies, and parents, to *discuss* mathematics, challenge each others' views, or simply try to agree on their aims of education.

Mathematics education, I propose, and ideas that are contained within mathematics, require space, and offer space, to do just that: discuss, challenge, find common ground. But not, it seems, just now (West and Wolfe, 2018).

My final thought on that for this chapter is that the metaphor is a valuable tool for all involved in mathematics education. *Mathematics is a Space* as a metaphor points to an ideal space where people from different times and backgrounds can test validity of their ideas, so that they are empowered as well as educated, to make their own decisions based on facts and consensus.

5 Mathematics is an ocean

The metaphor of mathematics as an ocean is linked to a project I did some years ago with the students working on the Shakespeare's *Tempest*, as part of the cross-curricular project in the schools in South-West of England. The aim of the project was to enable students, teachers in training, to build framework of cross-curricular explorations, look beyond the curriculum, and design activities that would inspire their pupils of middle-school age(years nine to twelve). The original hurdle was to find how to connect *Tempest* with mathematics. *Tempest is* a story about how a disposed duke, with his daughter, finds himself in a tempest (tempest itself here being a metaphor for both a real-life event and a political turmoil) but manages to return to his rightful place through dialogue, forgiveness, and some magic (Shakespeare, 1611).

Figure 5.1: Prospero disarming Ferdinant, an illustration of the scene from the *Tempest*, Shakespeare's play with same name. Henry W. Banbury and Francesco Bartolozzi, 1793, London.

5.1 Dee's one book for which he is most famous

The problem of how to connect such a story with mathematics for the middle school children and their teachers was luckily short lived. It transpired that John Dee (1527-1608) was most probably the inspiration for the central character of the play, Prospero (Grant 1976, Lawrence, 2011). Dee was a mathematician, astronomer, philosopher, and advisor and code maker for Queen Elizabeth I. Most famous for his introduction to the first English-language translation of Euclid's *Elements* by Henry Billingsley (d. 1606), Dee's most famous work is his *Mathematical Preface*. This preface is a treasure-trove to discuss, with mathematics teachers and students alike, a classification of mathematical sciences, including some 'jabberwocky' disciplines such as 'traumaturgike' (Lawrence, 2011). Discussions about views of whether such disciplines belong to contemporary mathematics can shine a light on the understanding of what constitutes mathematics, and its importance for a society as well as individual.

Dee spent six years journeying Eastern Europe, visiting Poland, and Bohemia, and staying in Prague for some years. He was not only an avid traveller but also had a special interest in scientific exploration of the New World. He gave instruction and advice to pilots and navigators of the English travellers conducting exploratory voyages to North America, and even conjured angels to help them on their voyage. The angels also suggested to Dee to name the new colony 'Atlantis', which, as we know, was not to be.

During his travel to Poland in 1583 however, his library was ransacked, with most books and objects stolen, never to be found in one place again.

5.2 Bibliotheca Mortlacensis

It is known that Dee had amassed a large library of books, manuscripts and curious objects. We know this as he compiled the inventory of this collection of about 3000 items before he set on his journey, and listed books on mathematics, mechanics, astronomy, optics, military and naval sciences, philosophy and magic. Among these books was a copy of Apollonius of Perga's (late 3rd – early 2nd centuries BC) *Conics* (Apollonij, 1537).

This book had an interesting page written in Dee's hand facing the title-page: it was Dee's attempt at classifying the mathematical sciences, a sketch that would serve as a basis for his later diagram in *Mathematical Preface,* his *Ground plat*.

5.3 To survive a tempest

This book survived the destruction of Dee's library (at the time of writing it is on sale with a New York antiquary for half a million US dollars). Strangely enough, Dee had a premonition that his library will be destroyed and wrote about this in his diary:

> I dremed that I was dead, and after my bowels were taken out I walked and talked with diverse, and among other the Lord Thresorer who was come to my house to burn my bokes… (Dee, 1582, as reported by Sherman, 1995, p. 51).

When talking about the burning of his books, Dee's dream, and ransacking of his library, a reality that was to befall him (and by all accounts contribute to his demise), is a point to note in its own right. The 'burning of the books' as a metaphor in dangerous times may turn into action. Dee was aware of this possibility - and we would do well to also remember and teach this - the power of knowledge contained in such books and the desire to destroy it by those who fear knowledge is an archetypal energy and event that is repeated throughout history.

There is of course a happy ending for the mentioned copy of Apollonius' *Conics*. The book was bought by John Winthrop the Younger, in the year when he was to leave for America to join his father, the first governor of the Massachusetts Bay Colony, in 1631. Through Winthrop Jr., Dee's work spread to Puritan New England (Woodward, 2011; Calis, et al. 2018), and hence some of Dee's library survived the tempest of his life and was passed on to the new land of which he cared so much and which he wished to call Atlantis.

5.4 How is mathematics an ocean?

History teaches us not only of the successes, but also of lost and burnt libraries and books, of suffering and tempests. But mathematics survives such events, and knowing of such historical instances can give us not only hope but structure. Like in Shakespeare's story about tempest, it is good to know that keeping a clear mind can get us out of a big trouble. In mathematics education, teaching how to do that is a valuable lesson.

6 Conclusion

To complete my chapter, I chose to mention finally our pillars of wisdom again. In this case though, I want to mention my namesake's famous *Seven Pillars of Wisdom* Lawrence, 1935). For anyone who has read this book, the first question that comes after reading close to some seven hundred pages, must surely be: where are these seven pillars to be found? There really are no pillars to speak of that are explicitly mentioned in Lawrence's book.

Could we really describe the pillars of wisdom that mathematics is (using a metaphor), and for that matter what is then mathematics education? I would suggest that we could certainly try, and by using metaphors, history, art, and literature, we can attempt to paint a picture of mathematics that is enchanting, inspiring, and gives hope to our young people. It also offers some concrete instructions, through mathematical history as well as through learning how to solve mathematical problems, on how to use mathematical thinking and make it a safe place, even a harbour in the troubled, even tempestuous times. Such education gives examples and practices the mind on how to conduct a conversation in a mathematical and logical space that the peoples of our era share with many who have explored similar ideas before us. We can paint the pictures of mathematics as a space to think and live in, and through which one can communicate and make friendships.

One of the threads that connect these metaphors that I did not have the time to go into detail here but had discussed in my talk in Oslo, is also one that is about the overarching purpose of education, and of mathematics. Love of a discipline, of learning, and of truth:

> *I loved you, so I drew these tides of men into my hands*
> *And wrote my will across the sky in stars*
> *To earn you Freedom, the seven-pillaried worthy house,*
> *That your eyes might be shining for me…* (Lawrence, 1935: dedication).

REFERENCES

Apollonii Pergei (1537). *Conica (translated in Latin by* Giovanni Battista Memo). Venice: Bernardino Bindoni.

Barbin, É., Jankvist, U. T., & Kjeldsen, T. H. (Eds.) (2015). *History and Epistemology in Mathematics Education: Proceedings of the 7th ESU*. Copenhagen: Danish School of Education, Aarhus University.

Baumgarten, A. G. (1735). *Meditationes philosophicae de nonnullis ad poema pertinentibus*.Venezia (original text and translation by K. Aschenbrenner & W. B. Holther, University of California Press, Berkley, 1954).

Benson, D. (2016). Maths mastery: The Key to pedagogical liberation? In *Mathematics Teaching*, vol 254. Association of Teachers of Mathematics. (https://derby.openrepository.com/handle/10545/622916)

Benson, D. (2018). *Maths mastery: a blessing or a curse?* https://researchschool.org.uk/derby/blog/guest-blog-maths-mastery-a-blessing-or-a-curse, (accessed 3rd April 2019).

Bishop, J. (2012). "She's always been the smart one. I've always been the dumb one": Identities in the mathematics classroom.*Journal for Research in Mathematics Education*, *43*(1), 34–74.

Calis, R., Clark, F., Flow, C., Grafton, A., McMahon, M., & Rampling, J. M. (2018). Passing The Book: Cultures of Reading in the Winthrop Family, 1580–1730. *Past & Present*, *241*(1), 69–141.

Dewey, J. (1934). *Art as Experience*. New York: Penguin.

Dissanayake, E. (1992). *Homo Aestheticus: Where Art Comes from and Why*. Seattle: University of Washington Press.

Dodgson, C. L. (1871).*Alice Through the Looking-Glass*. London: Macmillan.

Ehrenfels, C. (1897). *System der Wertlehre*. Leipzig: Reisland.

Erdogan, A., Yazlik, O. D., and Erdik, C. (2014). Mathematics teacher candidates' metaphors about the concept of ‘mathematics’. *International Journal of Education in Mathematics, Science and Technology*, *2*(4), 289–299.

Field, J. V. (1997). Rediscovering the Archimedean Polyhedra: Pieroella Francesca, Luca Pacioli, Leonardo da Vinci, Albrecht Durer, Daniele Barbaro, and Johannes Kepler. *Arch. Hist. Exact Sci. 50*, 241–289.

Galileo, G. (1623). *Il Saggiatore.* Rome: Giacomo Mascardi. English translation: *The Assayer* (translated by S. Drake in *Discoveries and opinions of Galileo*(pp. 230-280). New York: Anchor Books, Doubleday (first English translation by Thomas Salusbury in *Mathematical Collections and translations*, London 1661).

Gibson, H. (1994). Math is like a used car. In D. Buerk (Ed.) *Empowering students by promoting active learning in mathematics: Teachers speak to teachers*(pp. 7–12). Reston, VA: National Council of Teachers of Mathematics.

Hardy G. H. (1941). *A Mathematician’s Apology*. London: Cambridge University Press.

Heath, T. L. (1956). *Euclid: The Thirteen Books of Elements*, 3 volumes. New York: Dover.

Latterell, C. M., & Wilson, J. L. (2017). Metaphors and Mathematical Identity: Math is Like a Tornado in Kansas. *Journal of Humanistic Mathematics*, *7*(1), 46-61.

Lawrence, S. (2002). Geometry of Architecture and Free masonry in 19th century England. PhD thesis, Open University, UK.

Lawrence, S. (2015). Life, architecture, mathematics, and the Fourth Dimension. In *Nexus Network Journal*, *17*, 587–604.

Lawrence, S. (2016). What are we like… In B. Larvor (Ed.), *Mathematical Cultures: the London meetings 2012-2014, Trends in the history of science* (pp. 111-125). London: Birkhauser/Springer.

Lawrence, S. (2016a). The Old Teacher Euclid and his Science in the Art of Finding One’s Mathematical Voice. *MENON* ©online *Journal of Educational Research*, 2nd Thematic Issue - The Use of History of Mathematics in Mathematics Education (pp. 146-158).

Lawrence, S. (2018). Learning New Mathematics from Old – Euclid’s Art after Bath. In K. M. Clark, T. H. Kjeldsen, S. Schorcht, & C. Tzanakis (Eds.), *Mathematics, Education and History: Towards a harmonious partnership* (pp. 367-382). Cham: Springer.

Lawrence, T. E. (1935). *Seven Pillars of Wisdom*. Garden City, NY: Doubleday, Doran & Co.

Le Lionnais, F. (1948). *Le grands Courants de la Pensée Mathématique*. Marseille: Éditions des “Cahiers du Sud”. English translation: *Great currents of mathematical thought* (Vol. I, translated by R. A. Hall & H. G. Hermann; Vol. II, translated by C. Pinter, H. Kline). Mineola, N.Y.: Dover, 1971.

Mathematics Mastery (accessed 4th April 2019) https://www.mathematicsmastery.org/

Papert, S. A. (1978). The mathematical unconscious. In J. Wechsler (Ed.), *On Aesthetics in Science*, Cambridge, MA: MIT Press.

Betts, P., & McNaughton, K. (2003). Adding an Aesthetic Image to Mathematics Education. *International Journal for Mathematics Teaching and Learning*. (CIMT - Centre for Improvement of Mathematics Teaching, Plymouth, University of Plymouth), April 2003 http://www.cimt.org.uk/journal/index.htm (accessed 7th August 2019).

Pinker, S. (1997). *How the mind works*. New York: W. W. Norton & Co.

Presmeg, N. C. (1997). Reasoning with metaphors and metonymies in mathematical learning. In L. D. English (Ed.) *Mathematics reasoning: Analogies, metaphors, and images* (pp. 267–280). Mahwah, NJ: Erlbaum.

Proclus. (1970). *A Commentary on the First Book of Euclid’s Elements*. Translated by G. R. Morrow. Princeton: Princeton University Press.

Rosán, L. J. (1949). *The Philosophy of Proclus*. New York: Cosmos.

Sally, Jr. P. J. (2008). *Tools of the Trade: Introduction to Advanced Mathematics*. Providence, RI: AMS.

Sanders, P. M. (1990). *The Regular Polyhedra in Renaissance Science and Philosophy*. PhD Thesis, University of London, Warburg Institute.

Sherman, W. H. (1995). *John Dee: the politics of reading and writing in the English Renaissance*. Amherst, MA: University of Massachusetts Press.

Sinclair, N. (2009). Aesthetics as a liberating force in mathematics education. *ZDM Mathematics Education*. *41*(no 1-2), 45-60.

Stakhov, A. P. (2009). *Mathematics and Harmony From Euclid to Contemporary Mathema*tics. London: World Scientific.

West, A., & Wolfe, D. (2018). *Academies, the School System in England, and a Vision for the Future*. LSE Academic Publishing. Available from http://www.lse.ac.uk/social-policy/Assets/Documents/PDF/Research-reports/Academies-Vision-Report.pdf (accessed 12th April 2019).

Wilson, E. O. (1998). *Consilience, The Unity of Knowledge*. New York: Vintage Books.

Woodward, W. W. (2011). *Prospero's America, John Winthrop, Jr., Alchemy, and the Creation of New England Culture*, 1606-1676. Chapel Hill: The University of North Carolina Press.

Žižek, S. (2001).*Repeting Lenin*. Zagreb: Arkzin.