ENGAGING WITH PRIMARY SOURCES IN A MATHEMATICS FOR THE LIBERAL ARTS COURSE

Marcela CHIORESCU
Georgia College, 231 W Hancock St, Milledgeville, GA, USA
Marcela.chiorescu@gcsu.edu

ABSTRACT

There is evidence that integrating original sources in the mathematics classroom has significant pedagogical value, however, more empirical studies of implementing history of mathematics in teaching is needed. This paper describes a case study about replacing the textbook with primary historical sources projects for two topics: the Babylonian numeration system and the triangular numbers in a math for liberal arts course. Using history-as-a-tool in both projects, the author investigates whether the students can engage with primary sources to uncover math concepts and to develop their own understanding of these concepts, and obstacles and benefits of learning from primary sources.
Keywords: primary historical sources, student project, Babylonian numeration, triangular numbers.

1 Introduction

During the past four decades integrating the history of mathematics in the teaching and learning of mathematics attracted an increased amount of interest, see (Barbin, Jankvist and Kjeldsen, 2015). “Using history of mathematics in the classroom does not necessarily make students obtain higher scores in the subject overnight, but it can make learning mathematics a meaningful and lively experience, so that (hopefully) learning will come easier and will go deeper. The awareness of this evolutionary aspect of mathematics can make a teacher more patient, less dogmatic, more humane, less pedantic. It will urge a teacher to become more reflective, more eager to learn and to teach with an intellectual commitment.” (Siu, 1997/2000)

Tzanakis and Arcavi (2000) distinguished five main areas which can benefit from using the history of mathematics in mathematics teaching:

a) The learning of mathematics;
b) The development of views on the nature of mathematics and mathematical activity;
c) The didactical background of teachers and their pedagogical repertoire;
d) The affective predisposition towards mathematics; and
e) The appreciation of mathematics as a cultural-human endeavor.

Fauvel and van Maanen (2000) gave ideas and examples for thirteen possible ways to implement history of mathematics in the classroom:

1. Historical snippets;
2. Research projects on history texts;
3. Primary sources;
4. Worksheets;
5. Historical packages;

6. Taking advantage of errors, alternative conceptions, change of perspective, revision of implicit assumptions, intuitive arguments;
7. Historical problems;
8. Mechanical instruments;
9. Experiential mathematical activities;
10. Plays;
11. Films and other visual means;
12. Outdoors experience;
13. The world wide web.

The focus of this paper is on number three, namely on the use of primary sources. The use of primary sources in the classroom is "the most ambitious of ways in which history might be integrated into the teaching of mathematics" (Fauvel and van Maanen, 2000) but there is evidence that integrating primary historical sources in teaching and learning of mathematics has significant pedagogical value. (Furinghetti, Jahnke, & van Maanen, 2006; Pengelley, 2011; Jankvist, 2014, Barnett, Lodder and Pengelley, 2014). However, more empirical studies on incorporating primary sources in teaching and learning mathematics are needed as emphasized by Clark and Thoo (2014): "It is our hope that consumers of the contributions here (as well as from other publication outlets) will design empirical investigation of their own, analyze the results of those efforts, and critically reflect upon the experience in form of a conference paper, book chapter, or journal article." or by Clark, Otero and Scoville (2017): "There was scant focus on the use of primary sources as a classroom tool in the early work in the field of history in mathematics education. However, more recent work on the use of primary sources has been done in countries such as Denmark and Brazil, while such research has not yet been conducted with student populations in the United States"

The purpose of this paper is to share about the integration of two mini projects with primary sources in a mathematics for liberal arts course, Quantitative Skills & Reason, at Georgia College during spring 2017. After presenting the context of the study I will discuss the results and conclusion.

2 Context of the study

Georgia College (GC), a college located in Milledgeville, GA, about a two-hour drive from Atlanta, is Georgia's designated public liberal arts university. It has approximately 5,900 undergraduate students and 300 graduate students in 37 undergraduate programs and 25 graduate programs. The Department of Mathematics offers a Bachelor of Science in mathematics with an optional teaching concentration. It has about 80 major students instructed by 18 full time faculty. Mathematics, a cornerstone of a liberal arts education, is required at GC for every student. As part of their core courses, Georgia College's students must complete three hours in the Area A2 Quantitative Skills. The courses that satisfy this requirement are: Quantitative Skills & Reason, Introduction to Mathematical Modeling, College Algebra, College Trigonometry, Precalculus, and Calculus I.

MATH 1001 Quantitative Skills & Reason is a mathematics for liberal arts course and is not intended to supply sufficient algebraic background for students who intend to take Precalculus or the calculus sequences for mathematics and science majors. This course places quantitative skills and reasoning in the context of experiences that students will be likely to encounter. It covers a variety of topics such as logic, counting methods and basic probability, data analysis, and modeling from data. For my two sections of MATH 1001 in spring 2017, I decided to explore a couple of these topics with primary sources, namely the Babylonian numeration system and triangular numbers.

"Many topics in the modern undergraduate mathematics curriculum are presented as a fast-paced newsreel of fact and formulas with little discussion of the motivating problems or intellectual struggle behind the textbook definitions, lemmata or algorithms. We deprive our students of knowledge about the origin of the subject and reveal little about the initial motivation or its study, leaving our students to believe that the subject emerged as a logically precise edifice with the present version of the textbook". (Jerry Lodder, 2014) With primary sources we can create authentic math inquiry activities, where students work like real scientists. Original sources expose students to "mathematics-in-the-making" as opposed to "mathematics-as-an-end-product" they found in their textbooks. (Siu and Siu, 1979). "Through observation, analysis, interpretation, synthesis and evaluation, students discover clues and integrate new information into their knowledge base" (Carlston, 2009).

Jankvist (2009) describes a way of organizing and structuring the discussion of why and how to use history in mathematics education. He is proposing two approaches for the 'whys' and different approaches for the 'hows'. He distinguishes two big categories for the 'whys': history-as-a tool and history-as-a-goal. In both my projects with primary sources I used history-as-a-tool, especially playing the role of a cognitive tool in supporting the learning of mathematics.

Jankvist (2009) also presents three major categories in which history can be used in the teaching and learning of mathematics: illumination approaches, the modules approaches and the history-based approaches. In my case study, I used the modules approaches which are similar to the "guided reinvention" activities of Freudenthal (2002) who strongly believes that "learners should be allowed to find their own levels and explore the paths leading there with as much and as little guidance as each particular case requires. There are sound pedagogical arguments in favor of this policy. First, knowledge and ability, when acquired by one's own activity, stick better and are more readily available than when imposed by others. Second, discovery can be enjoyable and so learning by reinvention may be motivating. Third, it fosters the attitude of experiencing mathematics as a human activity." I will refer to my projects with primary sources as *guided exploration modules* (GEMs), since my idea of using primary sources in the classroom resonates with Freudenthal's didactical principles in (2002).

2.1 Guided exploration modules

My GEMs are inspired from two primary sources projects (PSPs) of the Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS) project: Babylonian Numeration was developed by Dominic Klyve (2017) and is about the Babylonian numeration system. The second is about triangular numbers and is a part of the Construction of the Figurate Numbers project designed by Jerry Lodder (2017).I used these two GEMs in two of my sections of MATH 1001 in spring 2017. With one of them I met for 50 minutes three times per week (Monday, Wednesday and Friday) and with the other one for 75 minutes twice per week (Tuesday and Thursday).

2.2 Babylonian numeration

The GEM on the Babylonian numeration system was based only on a picture of a tablet discovered in

Figure 2.1: Cuneiform Texts, ed. Hilprecht, Vol. XX, Part 1. (1906). It's on Plate 16, No. 27.

the Sumerian city of Nippur (in modern-day Iraq) and dates to around 1500 BCE (figure 2.1). The purpose of this module was to introduce the Babylonian numeration to students. Students' task was to discover by themselves this numeration system. For my two sections I designed this GEM to be integrated in only one class meeting. I implemented it after I discussed with my students other numeration systems such as, the Hindu-Arabic numeration system and the Mayan numeration system but mentioned nothing about the Babylonian numeration system. For both classes, students were divided in groups of size 2-3. The groups were formed from the first day of class. At the beginning of the class students were asked to close their notebooks, textbook and turn off their electronic devices. After a very short introduction of the project I asked my students to study the image of the tablet and to answer the following questions written by Klyve (2017):

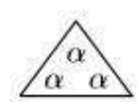
(1) How do Babylonian numerals work?
(2) Describe the mathematics on this tablet.
(3) Write the number 72 in Babylonian numerals.

2.3 Triangular numbers

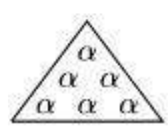
The main goal of the second GEM was for students to explore one category of figurate numbers, namely triangular numbers. This module was integrated after we discussed some elements of logic and some counting techniques as an example of inductive reasoning. This GEM is based on the Lodder's project (2017) and has as starting point excerpts from Book Two of Nicomachus' ancient Greek text *Introduction to Arithmetic* (1926), namely Chapter VIII. This module was also implemented in only one class meeting for both sections and,similar to the first one, students were not allowed to consult any other materials. After a short introduction of the project, students were asked to read the English translation of Nicomachus' text and to answer the following questions. Some were identical with the ones developed by Lodder (2017) and some were a slightly modified version of these:

Exercise 1: In this exercise we introduce the **triangular numbers**, which count the number of dots in regularly shaped, equilateral triangles. Although Nicomachus is somewhat reluctant to state that the value of the first triangular number is 1, since the triangle drawn around one alpha, is only potentially the first triangle,  we will begin the triangular numbers with the value of one. Let T_1 denote the first triangular number. Then $T_1 = 1$.

a) Let T_2 denote the second triangular number. Compute the numerical value of T_2 by counting the number of alphas (α) in the triangle:



b) Let T_3 denote the third triangular number. Compute the numerical value of T_3 by counting the number of alphas in the triangle:



c) Let T_4 denote the fourth triangular number, T_5 the fifth triangular number,…, and let T_n denote the nth triangular number for a natural number n. Fill in the following table for the first eight triangular numbers:

n	1	2	3	4	5	6	7	8
T_n								

d) Carefully explain how T8 is computed. Does Nicomachus include a picture of this eighth triangle in his writing?

Exercise 2: Nicomachus writes that "The triangular number is produced from the natural series of number…by the continued addition of successive terms, one by one, from the beginning…"

a) Write T_2, the second triangular number, as the sum of two successive whole numbers by using the number of alphas in each row of the second triangle.

Thus, $T_2 = \square + \square$.

b) Write T_3, the third triangular number, as the sum of three successive whole numbers by using the number of alphas in each row of the triangle.

Thus, T_3= ☐ + ☐ + ☐

c) Following the above format, write T_4 as the sum of four successive whole numbers so that

T_4 = ☐ + ☐ + ☐ + ☐.

d) What ten successive whole numbers would be needed to be added together to produce T_{10}, the 10th triangular number?

Exercise 3 Arrange two copies of T_n to produce a rectangle with n rows and (n+1) columns. Use this to answer the following:

a) Find a simple formula for T_n in terms of n. What geometric idea does this formula represent?

b) Find a simple formula for 1+2+3+…+n using part (a).

3 Results

As a teacher my main goal has always been to create a rigorous, supportive learning environment, which fosters and encourages the ability to learn, to reason, and to communicate with proficiency. The teaching strategies that I implement to reach my goal focus on methods that allow actual participation in mathematical activities, such as: learning as discovery using group work, discussions among peers and between students and teacher, and presentations of problems at the blackboard either from their homework or from in-class material.

Using history-as-a-tool in my projects, especially playing the role of a cognitive tool in supporting the learning of mathematics, my research study was framed around the following questions:

- Are the students able to engage with primary sources to uncover math concepts and to develop their own understanding of these concepts?
- What do students identify as obstacles and benefits of learning from primary sources?

To answer these questions, I analyzed my students' work from their projects and their answers to a post-project reflection survey.

From both sections, I had in total 40 students who engaged in the two GEMs. They tackled these in groups of 2-3 with little or no help from me. My students were graded on these projects on participation only, since my goal for them was to leave them to explore math without the stress of a grade, which implicitly leads to the stress of making mistakes.

Table 3.1

Are the students able to engage with primary sources to uncover math concepts and to develop their own understanding of these concepts?

Babylonian numeration	Question 1:Describe the mathematics of this tablet	Question 2: How do Babylonian numerals work?	Question 3:Write the number 72 in Babylonian numerals
Percentage of students who answered correctly	83%	70%	70%

Most of the students (83%) discovered immediately that the first two columns of the tablet represent the numbers from 1-13 and the third one looks like the product of the first two. Also, 70 % of the students gave (a fairly) complete response with reasonable explanations to the second and third question, showing understanding of the concepts. Figures 3.1 and 3.2 are two samples of student work.

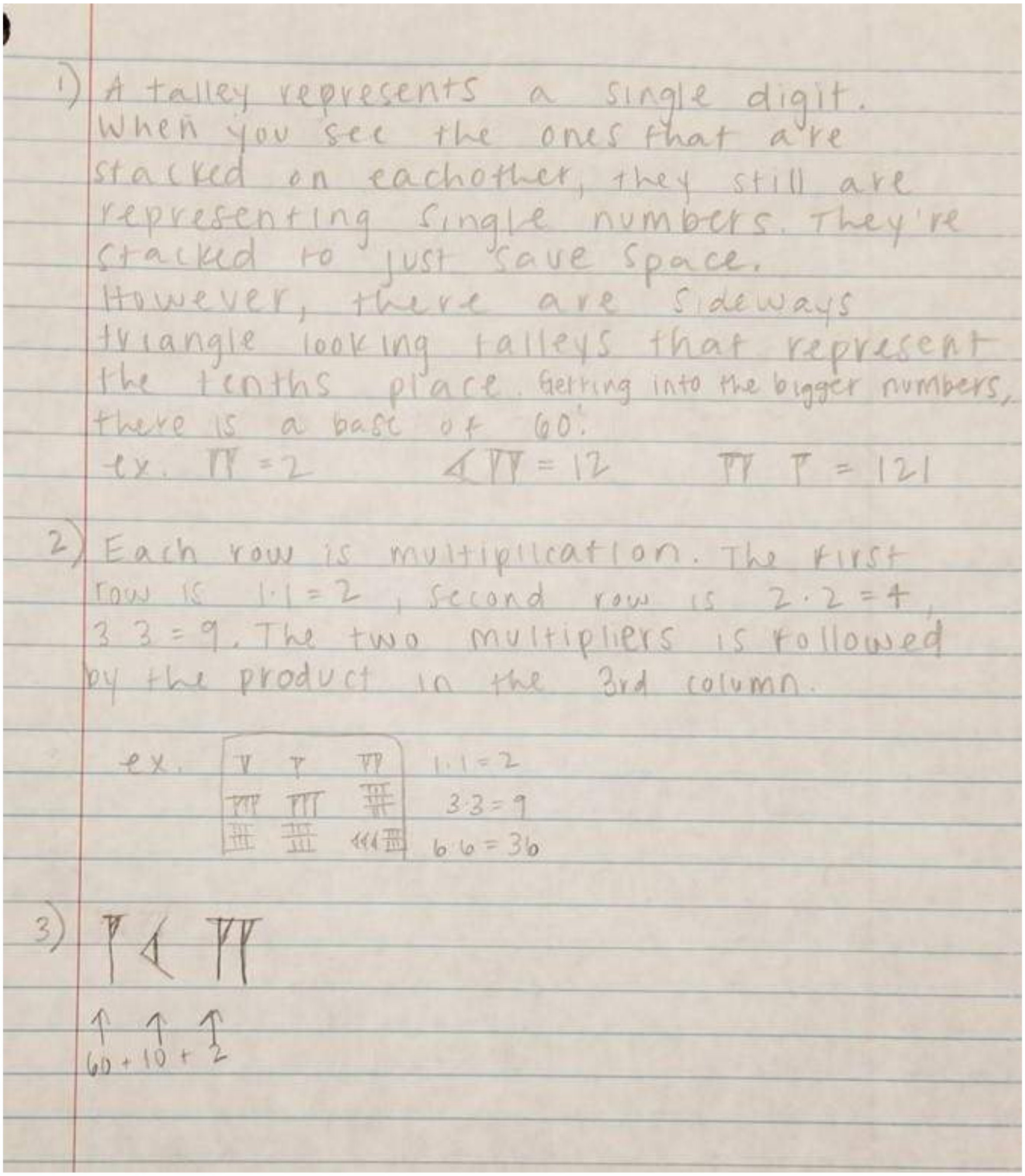
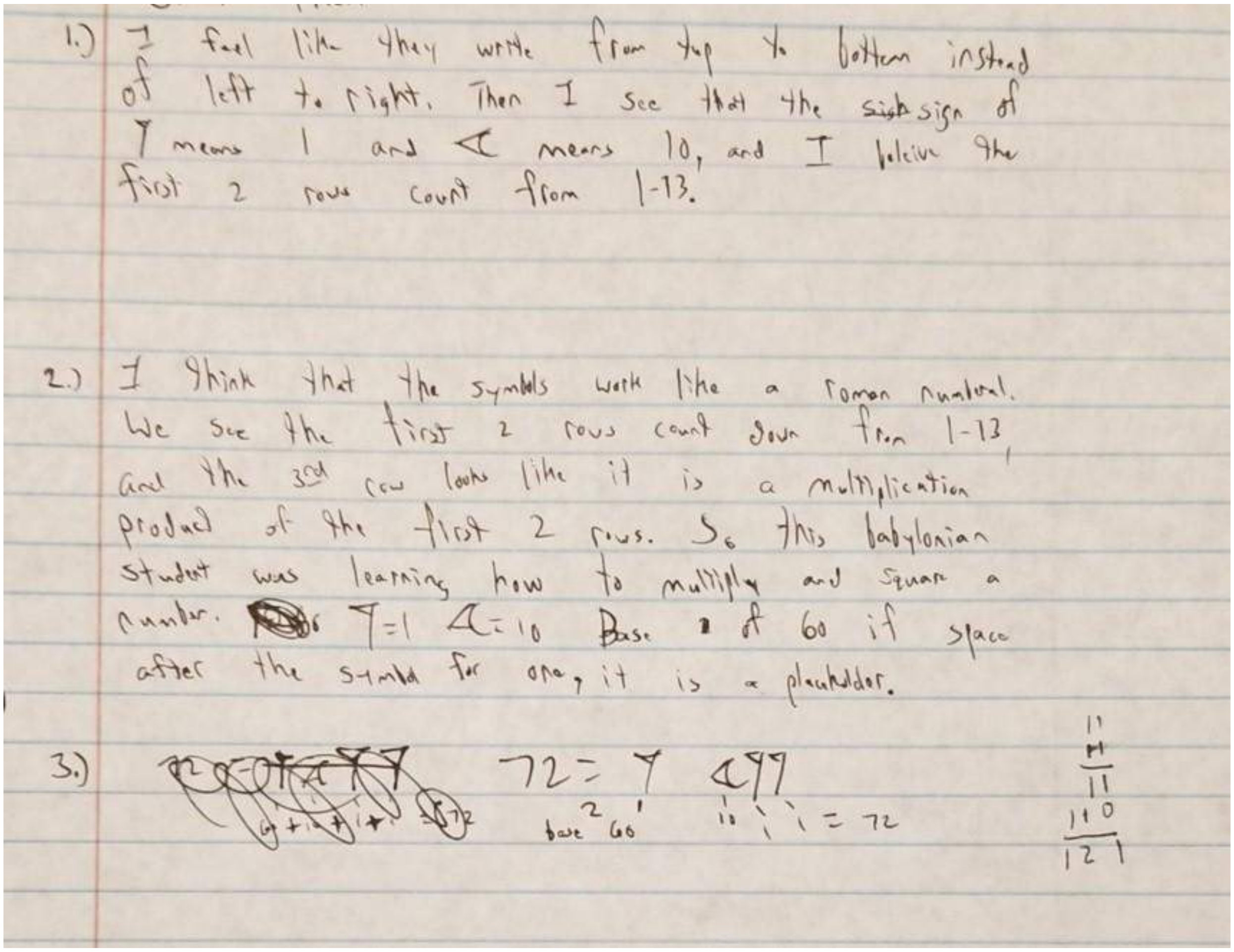


Figure 3.1


1.) I feel like they write from top to bottom instead of left to right. Then I see that the sigh sign of Y means 1 and < means 10, and I believe the first 2 rows count from 1-13.

2.) I think that the symbols work like a roman numeral. We see the first 2 rows count down from 1-13, and the 3rd row looks like it is a multiplication product of the first 2 rows. So this babylonian student was learning how to multiply and square a number. Y=1 <=10 Base of 60 if space after the symbol for one, it is a placeholder.

3.) 72 = ... = 72

Figure 3.2

After students finished their first GEM they need to complete at home a reflective survey about their experience with this project: "Using original sources in the teaching of mathematics makes it possible to contextualize the mathematics in ways that many textbooks cannot afford. Describe your experience using the mini project Babylonian numeration as a way to explore the benefits and obstacles you identified". For the first GEM, students identified as challenges:

- "difficult to read at first"
- "none"
- "difficult at first"

and as benefits:

- "made me use my critical thinking skills and apply my math knowledge to historical artifacts"
- "made me think outside the box, learning something new"
- "much more interesting than simply reading the textbook; allows to apply what we have learned previously about numerals and other ancient math system; I enjoyed it; using alternate methods like this help me learn, understand and retain the knowledge better"
- "forced you to think beyond what is in your notes and try to seek out your answer".

For the second GEM, the Triangular numbers, when examining Table 2, we notice that most students answered correctly the first two exercises and they had some problems with the third one.

Table 3.2

Are the students able to engage with primary sources to uncover math concepts and to develop their own understanding of these concepts?

Triangular numbers	Exercise 1:				Exercise 2	Exercise 3		
	a)	b)	c)	d)		Drew apicture	a)	b)
Percentage of students who answered correctly	81%	81%	93%	81%	70%	69%	50%	50%

Figures 3.3 and 3.4 are samples of student work for exercise 3.

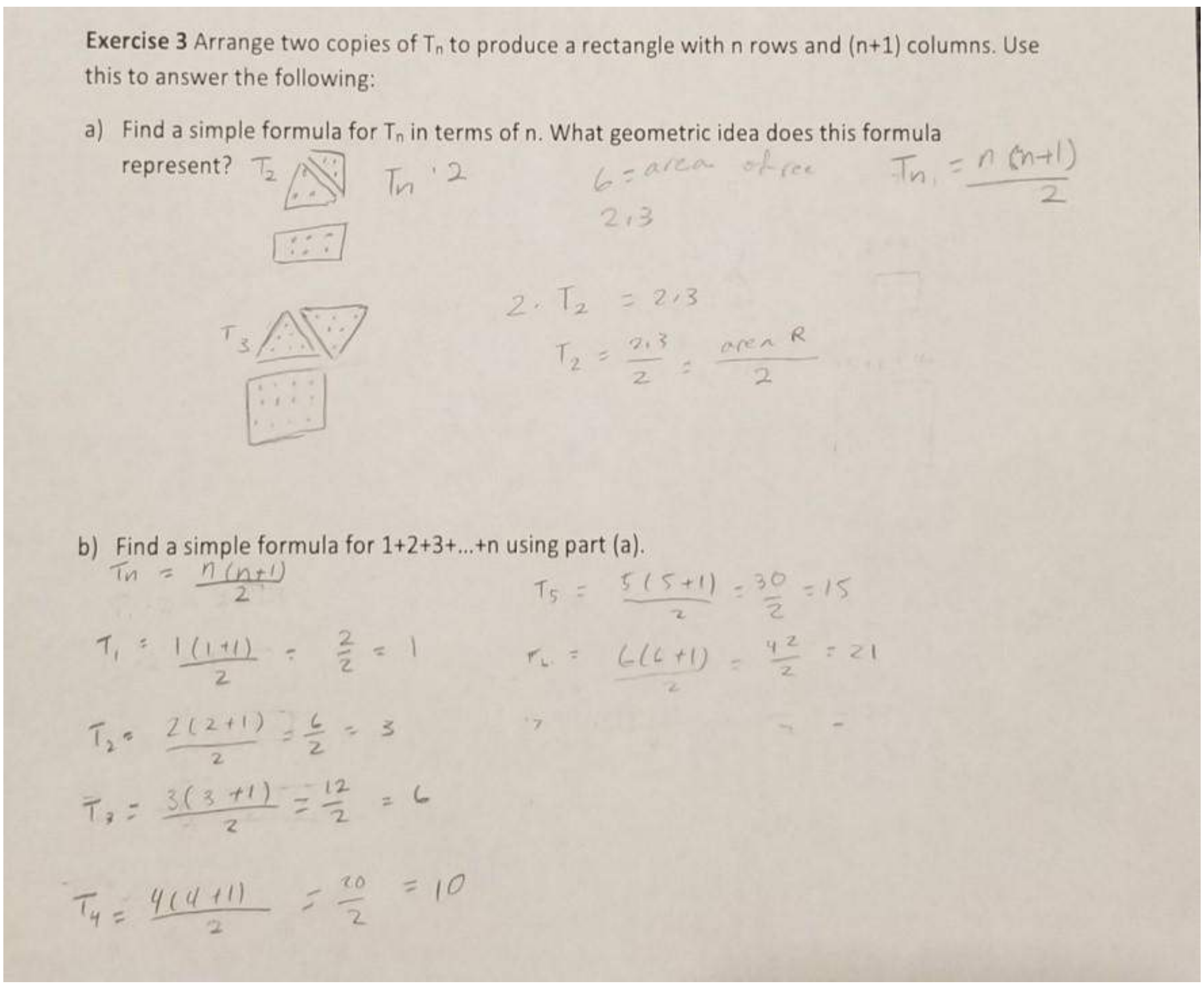


Figure 3.3

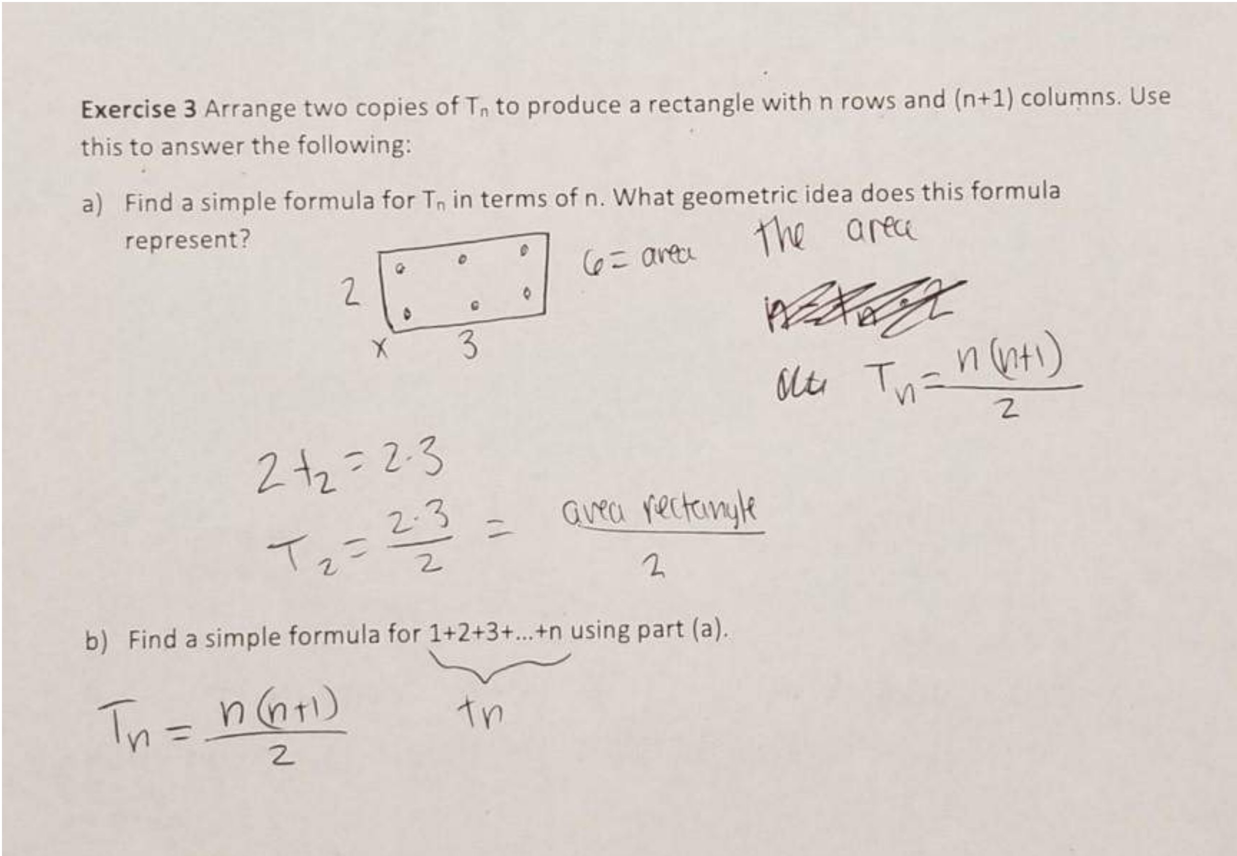


Figure 3.4

On their reflective survey, students identified for the second GEM the following challenges:

- “it does not provide as much explanation as a textbook might”
- “first I didn’t understand but then it was fun”
- “challenging to create the formula”
- “last question was a bit difficult”
- “hard to understand what he said”

and the following benefits:

- “it was interesting to act as a mathematician and discuss the pattern”
- “work together to figure it out instead of just being told; better understand and retain the info”
- “allows you to see differences and patterns; beneficial in every day situation as bowling; bowling pins are set up in a triangular numeral of 5”
- “makes think more critically”
- “remember better; figure it out by myself”.

4 Discussion and conclusion

Each of the two GEM’s was designed to engage students in the discovery of mathematics. The goal of the first GEM was to allow students to discover by themselves the Babylonian numeration system. I was very thrilled to observe the excitement of most of the students when they were deciphering the tablet. Most of the students recognized with no help from me that the first two columns represent the numbers 1-13 and that the third column represents the multiplication of the first two columns. They also found immediately the symbols used for 1 and 10. Some students had a bit of trouble identifying the base for this numeration system, so I gave them a hint to look at the product of 11 and 11. For most of them this was enough. For the second GEM, my goal was to have students discover some interesting relations between numbers, especially triangular numbers. With this project students had most difficulty with the third question. One of the possible reasons students had difficulties with the last question, is that in their previous math courses, they were rarely engaged in creative mathematical activities,it was more about doing mathematics as a rigid process by following fixed and predeterminated procedures.

Overall, I was happy with the results of this study.I found that using primary historical sources in the classroom has the potential to enrich students learning experiences,prompting them to develop their own understanding of concepts, bringing them close to the experience of mathematics creation. I was also pleasantly surprised to read in their post-GEMs reflections such as:

> "The experience of using this activity to further my knowledge of how Babylonian mathematics and numerals work was much more interesting and captivating than simply reading the textbook. The mini project allowed us to apply what we have previously learned about numerals and other ancient mathematics systems to understand the Babylonians. The obstacle of having no prior knowledge about this particular civilization contributed because we had to use basic observations and math skills to understand their numerals. I enjoyed this activity and I feel that using alternate methods like this help me learn, understand, and retain the knowledge better than a lecture or reading form textbook would."

> "We have used original sources in multiple mini projects now. It helps us as students understand where the math originated from. It blows my mind how we have evidence from thousands of years ago to show how math was formed like the triangle and rectangle method that we learned Tuesday. I enjoyed it even though it took a lot of thinking and coming up with ideas."

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