WHY BOTHER WITH ORIGINAL SOURCES?

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ABSTRACT

This workshop reports on a teaching experiment which was carried out in May 2017 in France, in the final year of elementary school (year 5) and in the first year of middle-school (year 6). Its starting is point the most famous passage of Plato's *Meno*, in which Socrates challenges a slave to construct a square twice as large (in area) as a given square. However rich Socrates' problem may be from a mathematical viewpoint (characterization of common quadrilaterals, distinction between length and area, comparison of areas, proportionality), its study does not require any explicit use of original sources. In this experiment, the challenge was not only to make students work on Socrates' *problem*, but on Plato's *text* as well. Before reporting on the outcome of the experiment, we will present tools and concepts which we found in research works bearing on literacy and reading comprehension, and which helped us design the experiment.

Keywords: original sources, area, proportionality, argumentation, literacy

1 Rationale

1.1 Mathematical content

In this paper, we report on a teaching experiment which was carried out in 2017 in Paris, with students in the final year of primary school (year 5, students aged 10) and the first year of middle-school (year 6). It was designed on the basis of one of the most famous problems in the history of mathematics; a problem which is discussed in one of the foundational texts of the Western philosophical tradition. In the dialogue entitled *Meno* (we will refer to the online edition of (Plato, 1967)), Plato (427-348 BCE) staged a discussion between two characters: Socrates – a philosopher, and Meno – a noble Athenian. About halfway through the dialogue, Socrates engages in a side-dialogue with one of Meno's slaves, who is referred to as "boy". Socrates starts from a square (A) (see Figure 1.1), and challenges the slave to determine or exhibit a square twice as large in area. The slave finds the question easy, and suggests doubling the side (diagram (B)). Socrates points out that this answer is incorrect: shape (B) is a square indeed, but its area is four times that of (A), as can be seen in diagram (C). Socrates then joins the four midpoints of the sides of the large square and claims that this new, tilted, shape is a square whose area is twice that of (A). He justifies this by counting the number of half squares: two in (A), four in the tilted square.

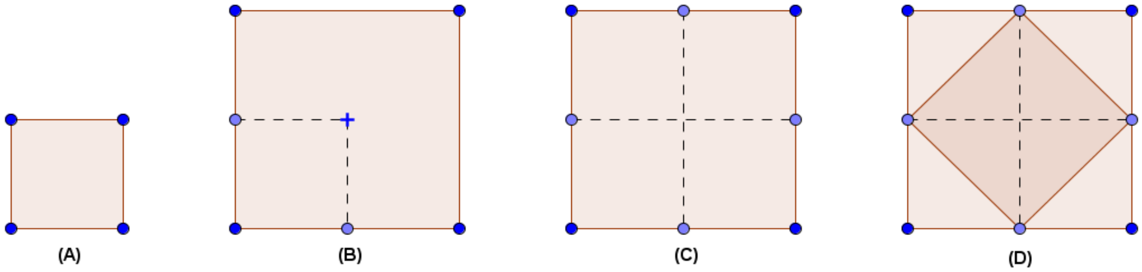

Figure 1.1: The diagrams referred to in Plato's text.

The mathematical content of this passage is rich but fairly elementary, and echoes standard curricular requirements for the end of primary school or the beginning of middle-school (depending on the country): describing rather complex plane geometrical shapes using with a precise vocabulary; characterizing common shapes (in particular when it comes to checking or proving that the tilted quadrilateral is a square); comparing areas, either through a cut-and-paste approach or by measuring (either with an *ad-hoc* or with a conventional unit); distinguishing between situations where proportionality holds, and situations where it does not (here: in squares, the area is *not* proportional to the side).

Of course, the mathematically trained reader would probably see more mathematics at stake in this problem, in particular Pythagoras' theorem. Indeed, Socrates shows that the area of the square on the hypotenuse of an isosceles right-angled triangle is equal to the sum of the areas of the two squares on the sides of the right angle of this triangle, which is a special case of the non-numerical version of the Pythagorean theorem. If we were to use the numerical version, a new feature of the problem would come up: it can be shown that the ratio between the length of the side of the first square and its diagonal (which is the side of the solution square) cannot be expressed numerically using only one whole number, or a ratio of whole numbers. There is probably an allusion to this mathematical fact in the dialogue, when Socrates acknowledges that, if the slave cannot "say" or "reckon" what the side of the solution square is, he can at least try to "show" it (Plato, 1967, 84a). The construction is indeed elementary, whereas the numerical determination is difficult, and depends what you consider to be legitimate "numbers". Since neither the Pythagorean property nor the irrationality of $\sqrt{2}$ are usually studied at this level of the educational system, we chose to leave this completely outside the scope of our experiment. Other choices would have been possible, for instance to engage in a numerical approximation of the measure of the side of the solution-square by trial and improvement, using decimals (Kosyvas & Baralis, 2010).

1.2 A difficult problem, in a difficult text

The study of this problem involves two well-known epistemological obstacles. The first one lies in the difficulty in distinguishing between two different magnitudes associated with one plane shape, namely the length of its border and the area of its surface. The second one lies in the force of the linear model, which leads most students (and probably most adults) to believe that when two magnitudes depend on one another, proportionality holds.

These two sources of difficulty do not play equivalent parts in our experiment, partly for curricular reasons. For students of that age (around 10), at least in France, the notion of area is a key target in the curriculum, and our teaching sessions are designed for students who are already aware of the following facts: a plane shape such as a polygon has both a length and an area; the procedures for comparing lengths and those for comparing areas differ; so do the conventional units for both magnitudes. Consequently, we assume that the students will be display some level of expertise when dealing with the length-area aspect of the problem; and that studying this problem will improve their command of these notions. By contrast, even if the French national curriculum requires that some situations where proportionality does not hold be studied, the main target for students of that age is to study situations where it does hold, and to solve linear problems using an ever growing range of techniques. For most of the students in our experiment, the study of Meno's problem was probably the first occasion to come across the fact that, in the enlargement of plane shapes, a scale factor of 2 leads to a multiplication of areas by 4 instead of 2. We do not claim that this isolated encounter with a counter-intuitive phenomenon will enable students to overcome this epistemological obstacle – should such a thing be possible at all. However, this encounter with a tricky and surprising phenomenon could be used later on in the year, in particular to justify the rules for changing units in areas (e.g. there are 100 cm in a meter, but there are 10 000 cm2 in a square meter).

Since we wanted students to study Plato's text and not only Socrates problem, the mathematical content along with its didactical and cognitive properties was not all we needed to pay attention to. The text is rather long – we used a 6-page excerpt – and of an argumentative nature (even if in a dialogical form). Moreover, this excerpt is a mathematical digression embedded in a philosophical dialogue whose main focus is not – at all – mathematics[1].The excerpt under study is structured by the interlacing of these two scales: main dialogue / digression, philosophical problems / mathematical problem. Indeed, the dialogue between Socrates and the slave – which bears on shapes and areas – is regularly interrupted by sibylline asides between Socrates and Meno; asides which bear on the teaching/learning process. Moreover, in the study of the mathematical problem, the situations of the three characters are asymmetrical: the slave (and the 10-year old reader) understands in the end that his intuitive solution is erroneous, whereas Socrates and Meno (and the teacher) know it from the start. For Meno and Socrates, what is a stake is not a geometrical problem, but rather the true meaning of "believe", "know" and "learn".

For these reasons, the text is not only long but objectively difficult to understand. Clearly, making sense of the text requires that shapes and magnitudes be studied; but it also calls for a continuous work of explicitation and reformulation of sibylline or ambiguous statements. For lack of some key information (what is really at stake for Meno and Socrates in their little "experiment" with the slave? What is the correct answer to the mathematical puzzle?), the reader has to continually make hypotheses as to what the various characters know and aim for. Many passages are rather obscure upon first reading, since the key to comprehend them is given in a later part of the text. For the reader, this has both a cognitive and an affective impact: one has to agree to go on reading without understanding everything. One has to accept the fact that, at different times along the reading process, the degree of understanding of different parts of the text will evolve.

Beyond these general features which make the reading experience a demanding one, two other specific aspects should be mentioned. First, Plato's manuscripts were transmitted without diagrams, and most contemporary editions chose not to provide visual help. There can be no doubt for the reader that Socrates is discussing and drawing diagrams, however it is left to the reader to sketch them along the way, which is not trivial since the text is occasionally ambiguous. In our design, we thought that making hypotheses about the diagrams mentioned, described, and discussed in the text was a task that could be fruitfully entrusted to students. Second, the mathematical vocabulary used by Plato is not ours. In particular, he used the same name for the units of length and of area (the foot). Although this fact was common in Ancient mathematics – in Greek mathematics but in Chinese or paleo-Babylonian mathematics just as well – it can be confusing for the reader. In our design, we regarded this feature of the text as providing an

[1] The main topics discussed in *Meno* are virtue/excellence (what is virtue/excellence? Are all men equally capable of virtue? Is it inbred or can it be taught?), and teaching/learning.

opportunity for the students to spot this ambiguity, discuss and criticize it, and maybe suggest ways to reduce it.

1.3 Why bother with the original text?

This experiment was designed in the context of a larger research programme on the use of original sources in the classroom. An outline of its theoretical background can be found in (Chorlay, 2016, pp.9-14). To put it in a nutshell, we choose not to focus on history of mathematics in the classroom, but on the use of historical "documents" – be they texts, diagrams, or instruments – as a means to entrust students with tasks of a reflective nature; tasks which bear on a sample of mathematics. These tasks – which we collectively denoted as *meta*-tasks – are usually referred to by verbs such as: reformulate, translate, make explicit, disambiguate; assess, criticize; justify, prove, spot a missing argument and provide one; generalize, assess the generality. With its rather long and sometimes oddly worded list of arguments bearing on reasonably basic mathematical notions, Plato's text seems to lend itself particularly well to this type classroom work.

As a consequence, the classroom sessions reported upon in this paper are to be regarded as part of a research programme, and not as a teaching resource which we would claim should or could be used widely in more ordinary contexts. One reason for this is that the three sessions were designed – over a rather long period of time – by a group of three: the researcher, one primary school teacher2, and one secondary school teacher3. Hence, those who actually implemented the sessions should be considered as associate researchers. A second reason is that, for research purposes, we decided to keep the tasks as difficult, demanding, and challenging as we deemed possible, at the risk of facing classes of nonplussed students supplying irrelevant, senseless or random answers; or no answers at all. This highly demanding format is in keeping with our goal, which is to probe and try to delineate the thin line between the fruitfully demanding, and the altogether impossible (for students of a given age); or, to put it differently, between productive and unproductive struggles.

This general perspective has to be kept in mind in order to understand the many specific choices reported below in the description of the sessions. For now, let us mention three consequences. First, in order to collect data showing what students managed to do when working on their own, we put the emphasis on written tasks, even in cases when we think it would not be necessary or even useful in ordinary teaching conditions. Of course, we also audio-recorded the sessions in order to study the collective phases as well as the interactions with the teacher. Second, the investigation is of a qualitative nature, not only because of the size of the samples (two classes) but also because the aim is to study what is possible in a given educational context; hence we take one instance as a proof of possibility; hence, we focus on the analysis of the qualitative variations in the range of actual answers rather than on their relative frequencies. Third, since we were ready to face "failure" – suggesting a relevant research-result of "impossibility" – we were also ready to let some students fail. At this point, there is tension between the goals of the researcher and that of the teacher.

This report provides an opportunity to discuss a key element of our research programme which was only mentioned in passing in (Chorlay, 2016). Then, the focus was

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on a first characterization of meta-tasks. Working on Plato’s text made it necessary to take into account didactical problems which are not directly related to mathematics, but to reading comprehension and, more generally, literacy.

For this purpose, we drew mainly on the work of the research team of Roland Goigoux. With a theoretical background in the didactic of literacy and in textual linguistics, these experts developed a research-based teaching resource entitled Lector & Lectrix, which was designed to improve the reading comprehension skills of students from year 3 to year 9 (Cèbe & Goigoux, 2009). According to them, “understanding a text” rests on the interplay between several skills: decoding the written code (the basic meaning of “being able to read”); linguistic and textual decoding skills regarding syntax, lexicon, punctuation, connectors; availability of referential knowledge (knowledge about the world: in this specific case, about shapes and geometric magnitudes), and strategic skills (regulation, control, and assessment – by the student – of his/her reading activity). Following Umberto Eco’s Lector in Fabula, they highlight the importance of the latter skills:

> To understand a text, the reader has to simultaneously use all these skills so as to carry out a twofold processing activity: some local processing – which gives access to the meaning of groups of words and of sentences – and some more global processing – allowing for the construction of a coherent mental representation of the whole. (…) The latter process – called semantic integration – is of a cyclic nature: each new input leads the reader to reorganize the representation which he/she constructs step by step, along the way (…). This means that the reader should be flexible enough to be able to acknowledge that his/her first representations are provisional, hence revisable. (Cèbe & Goigoux, 2009, 7. Our trans.)

On this theoretical basis, Goigoux points out that these strategic skills are not often taught and trained explicitly, and that this could account for the persistence of a significant proportion of low-achieving students who can decode written texts but not actually read them as soon as their length or level of complexity exceeds the very basic. To address this issue, his team wrote a series of textbook specifically aiming for an explicitly training of students in reading comprehension. Let us mention some of their “guiding principles”:

- Make students more active and able to regulate their own reading activity: avoid long lists of detailed questions; ask students to assess their own degree of understanding (“I’m sure of this”, “I’m quite sure”, “I’m not so sure”…)
- Ask students to fill the “blanks” of the text: one has to cooperate with the text to go a little beyond what it says explicitly. One should teach the distinction between what the text says, and what it leaves for the reader to infer (and inferring is not the same as imagining or inventing); everyone has his own “way of understanding”, but a socially shared understanding is to be aimed for.
- Ask students to reflect on the characters’ thoughts, in terms of goals (for the future), of motives (in connection to the past), but also in terms of feelings and emotions; of knowledge and reasoning.
- Learn to memorize and make sense by constantly reformulating and paraphrasing.
- Learn to adjust the reading strategies to a specific goal: Reading strategies are goal-dependent, and there are many possible reading goals; the teacher should point to this variety, and make the current reading goal explicit.

- Pay a constant attention to the lexicon: The meaning of a word can be explained by the teacher before or during the reading. Students should also realize that a reader can make hypotheses as to the meaning of a new word.

In summary, beyond the specific content-related goals (shapes, area, proportionality), we wanted students to experience argumentation in a mathematical context, by reading – but also by reformulating, complementing, assessing, or providing – arguments. Since the opportunity to do this was provided by a long and difficult text, these *meta*-tasks were intertwined with less specific – but just as challenging – text-reading tasks. We thus drew on the guiding principles of the *Lector & Lectrix* teaching programme to design our experiment.

Before describing the three teaching sessions, we need to mention two negative choices. First, we decided not to expatiate on the philosophical meaning of the text; we touch on it when we feel it is necessary to make sense of some passages in the dialogue. This may be frustrating to the educated readers who knows how deep Plato's text is, and who have experienced other ways of using it in their teaching – in particular in teacher-training contexts. Second, we chose not to expatiate on the historical context. However, as preparatory work for the sessions, students were asked to read and summarize basic background information (location of Athens on a map of Europe, short biography of Plato etc.).

2 Outline of the teaching sessions. Samples of students' worksheets

The three 1-hour teaching sessions designed with the two teachers were taught in May 2017 in Paris, in two "ordinary" classes, one in the final year of primary school, and one in the first year of middle-school. All students' worksheets were collected, and the sessions were audio-recorded. For lack of space, we will only present the outline of the sessions (in particular the list of tasks entrusted to students), and discuss a few samples of students' individual worksheet (for a more detailed account, see (Chorlay, 2018)). This implies that this report will be biased, since we will mention only in passing what happened during the collective discussion phases, either among students or with the teacher. We will also focus on the mathematical tasks, at the expense of the reading tasks; lack of space is not the only reason: we also need more time – and probably need to collaborate with researchers working on reading-comprehension – to be able to analyze these aspects at research level.

The outline of the three sessions is the following:

Session 1: Discovery of the text (up to "And might there not be another figure twice the size of this, but of the same sort, with all its sides equal like this one?") ; discovery of the main problem ; questions on the characterization of the square by its sides only ; questions on the meaning(s) of "foot".

Session 2: Reformulation of the duplication problem; comparison between solutions suggested by students and the solution of the slave; assessment of Socrates' criticism of the slave's answer.

Session 3: Discovery and assessment of Socrates' solution. Final look back, and reflection on the meaning of the whole dialogue, in particular with respect to the asides between Socrates and Meno.

2.1 Session 1

Outline of session 1 (we italicized the questions students were asked directly). The text referred to is (Plato, 1967, 82a-82d); the first diagram drawn by Socrates is a square with sides of two feet each (diagram A of Figure 1.1):

- Collective work: correction of the preparatory homework: Plato, citizenship and slavery in Ancient Greece, location of Greece and of Athens on a map of Europe. Short presentation of the goal and format of the three sessions.
- Silent reading of the beginning of the dialogue
- Second silent reading. *Use three colours to sort the sentences or words into three categories: “This I understand” “This, I understand a little” “This, I don’t understand*”
- Collective discussion: *Why is this text difficult?*
- *We all agree that part of the difficulty stems from the fact that the characters are discussing diagrams that are not available in our edition of the text. Take five minutes to draw, in the margin, what you think the diagrams are.*
- *The sentence “*The space is twice two feet” *is very important:*
    - *Can you explain what Socrates means? (you can write, draw …)*
    - *Do you agree with him?*
- Collective discussion
- *In the text, Socrates seems to be saying that a shape with four equal sides has to be a square. Do you agree with him?*

The answers students gave to the final question were not surprising: many remembered that a quadrilateral with equal sides can be a (non-square) rhombus; those who said it had to be a squared were quickly convinced by the collective discussion. The two questions about the contention that “the space is twice two feet” were less standard, and elicited a variety of responses. In figure 2.1A, the student found a geometrical interpretation of the values in the text (since a side is 2 feet, two sides are 4 feet) which does not involve areas.

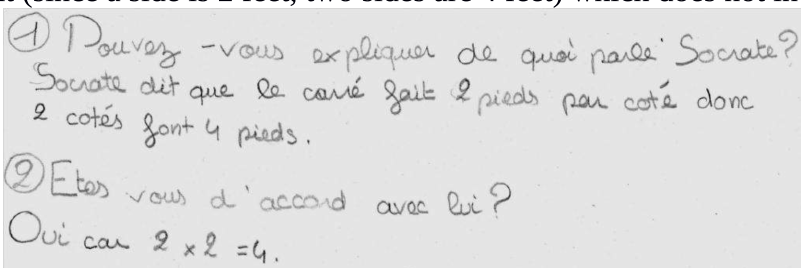

① Pouvez -vous expliquer de quoi parle Socrate?
Socrate dit que le carré fait 2 pieds par coté donc
2 cotés font 4 pieds.
② Etes vous d'accord avec lui?
Oui car 2 × 2 = 4.

Figure 2.1A[4]

[4] “(1) Can you explain what Socrates is talking about? Socrates says the square is of 2 feet per side, so 2 sides are 4 feet.(2) Do you agree with him? Yes, since 2×2 = 4.”

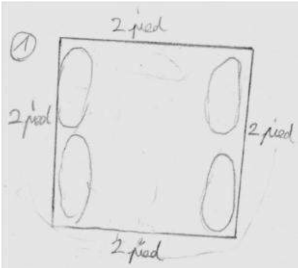


Figure 2.1B

In figure 2.1B, the student drew a clever diagram which accommodates the ambiguity of the text: the schematic feet denote both a unit length, and the location of the four unit-squares. Figures 2.1C and 2.2D show symmetrical successes/shortcomings: student 2C identified multiplication as away to work out the area of the square, but did not spot the need to change units for areas. Student 2D did not clearly mention areas, but chose to express the result in square centimetres. The written trace “two times two cm2” is ambiguous, since it can be interpreted either as “(2×2) cm2” or as “2 × 2cm2”. Both answers are correct, the first one being closer to the standard formula, the second one being closer to Socrates explanation (in which he decomposes the square in two rectangles of 2 square-feet each).

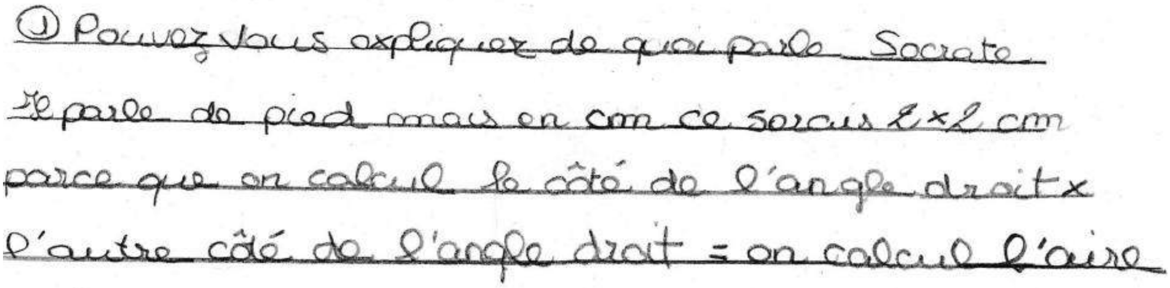


Figure 2.1C[5]

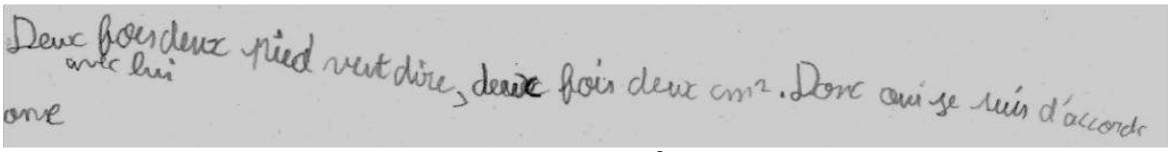


Figure 2.1D[6]

On the basis of the individual answers of the students, it was not difficult for the collective discussion to lead to a consensus on two points: First, in the text “4” refers to an area and not a length; second, given the fact that “foot” is a unit of length, maybe Socrates should have used expressions such as “square foot” or “foot of area” to avoid confusion between length and area.

2.2 Session 2

Session two ran very smoothly, so we will not report on it in any detail. Since our goal was not to design a problem solving session, but a session in which students were to make sense of, and assess arguments from the dialogue, we only gave them 1 minute to show us

[5] “(1) Can you explain what Socrates is talking about? He is talking about feet, but in cm it would be 2 × 2 cm since we work out the side of the right angle × other side of the right angle = we work out the area.”

[6] “Twice two feet means, twice two cm². So yes, I agree with him.”

their intuitive answer to the main problem. As was expected, many suggested doubling the side. Some suggested drawing a congruent square next to the first one, which clearly doubles the area, but results in a non-square rectangle. In one answer-sheet, the student drew arrows to denote the enlargement process in a way which is reminiscent of the use of touch-screens on computers or cell-phones. The outline of the session is:

- (Collective work, without the text) Reminiscing and reformulating Socrates' problem: "And might there not be another figure twice the size of this, but of the same sort, with all its sides equal like this one?"
- Spontaneous answers of students (1 min).
- Silent reading of Socrates' explanation of the incorrectness of the slave's answer. Students are asked to draw the missing diagrams.
- Collective discussion, consensus on the incorrectness of the slave's answer.
- Collective discussion on the meaning of the aside between Meno and Socrates:

  Boy : Clearly, Socrates, double.

  Socrates: Do you observe, Meno, that I am not teaching the boy anything, but merely asking him each time? And now he supposes that he knows about the line required to make a figure of eight [square] feet; or do you not think he does?

  Meno: I do.

  Socrates: Well, does he know?

  Meno: Certainly not.

As far as the philosophical aside between Meno and Plato is concerned, once students have come to the conclusion that the slave's answer is incorrect, they can be asked to spot the verbs in the excerpt. Clearly, Socrates and Meno mean to distinguish between, on the one hand "thinking you know", and on the other hand, "knowing". Thus, they are only willing to use the word "know" in cases where the conviction bears on a true statement.

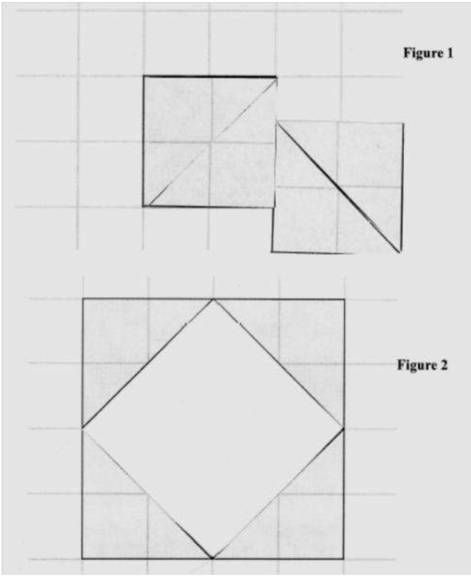
2.3 Session 3

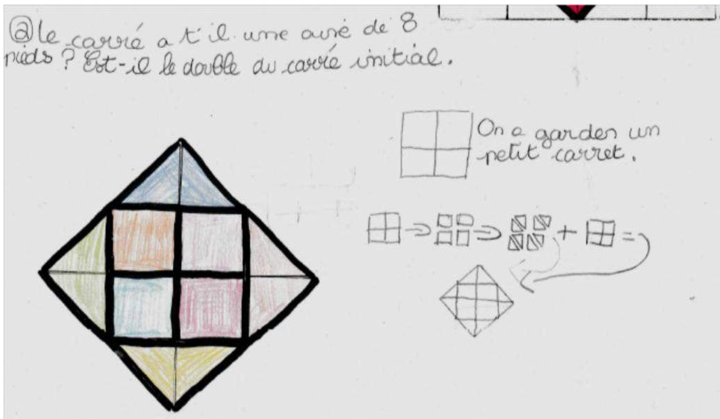
In session 3, Socrates solution was first read by the teacher, who supplemented the missing diagrams along the way (Figure 1.1). The students were then asked to assess Socrates' proposal, without having the text at their disposal. The outline of the session is:

- Reading Socrates' solution. The teacher draws the corresponding diagram on the blackboard.
- *To see if Socrates' answer is correct or incorrect, we need to check two things:*
  - *That the tilted shape in diagram* (D) *is, indeed, a square. Write down what geometrical instruments you need to use to check this*.
  - *That its area is twice that of the square* (A) *from which we started.*
    To do this, you can use either: (1) shape (A)*, a marked ruler and a calculator; or (2) two shapes of type* (A) *and one of type* (D)*, with scissors and glue; or (3) shape* (D).
- Final look back. Collective discussion on the meaning of the text.
  - *Do you think the goal of Meno and Socrates was to make fun of the slave?*
  - *Does the dialogue between Meno and Socrates bear on squares and areas? If not, what is it about?*

Let us focus on the answers to the area question. We thought that many middle-school students would rather measure lengths on the diagram and use multiplication to find an approximate value of the measure of the area of the tilted square, since the use of formulae is the standard procedure to deal with areas in middle school. It so happens that in the context of this problem, no students did that; all used the cut-and paste approach, usually in very clever and convincing ways.

Figures 2.2A and 2.2B show two solutions using (or alluding to) scissors and glue:



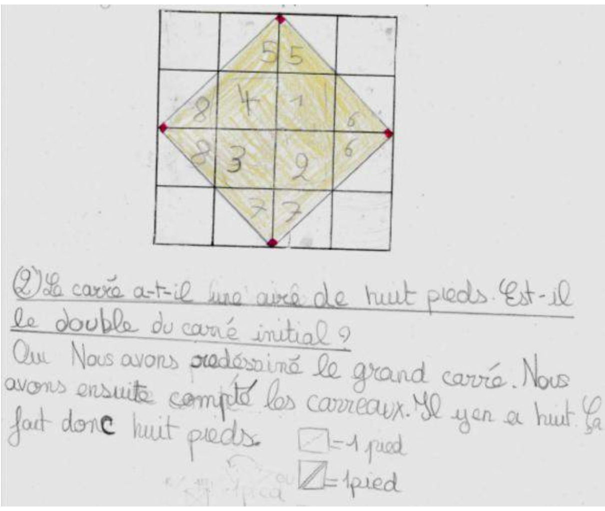


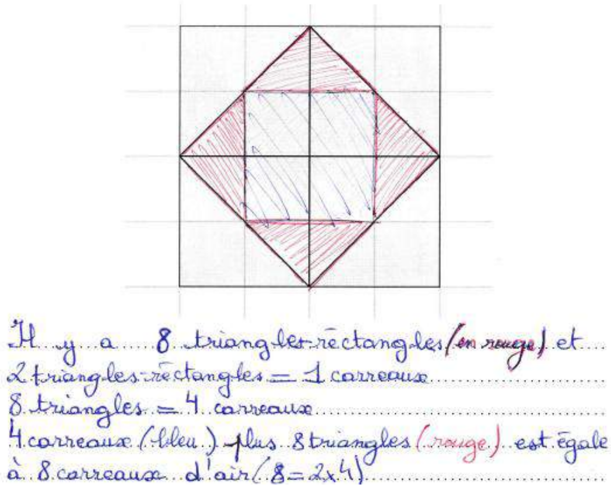
Figures 2.2A (left) and 2.2B[7] (right).

In 3A, the student cut out the tilted square from the diagram at the bottom of the page, cut it in four isosceles right-angled triangles, and used them to make up two copies of the original square. In 3B, the student explained with a mixture of diagrams and words that the tilted square can be decomposed into the original square plus one copy of the original square decomposed into eight halves of the unit-square.

Other students managed to validate Socrates' answer without any instruments:

[7] "(a) Does the square have an area of 8 feet? Is it twice as large as the initial square?" [teacher's questions]. "You can keep a small square."





Figures 2.3A[8] (left) and 2.3B[9] (right).

In 4A, the student counted the number of unit-squares in the tilted square, and added a small diagram to show that the area of one (square)-foot could be found either in a square or in a pair of half squares. 4B shows a variant of this reasoning.

One student provided a correct answer that we had not anticipated:

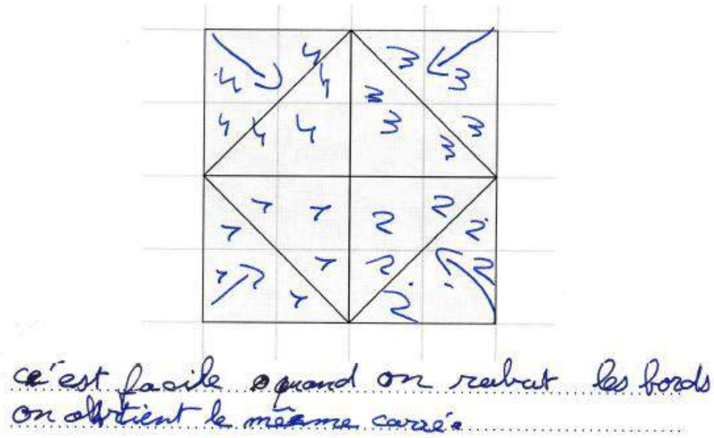


Figure 2.4[10]

In figure 2.4, the tilted square is seen as half the large square (you just need to "fold the corners to get the same [i.e. tilted] square").

When it eventually came to discussing the general meaning of the text, it was not difficult for students to say that Socrates and Meno are clearly not doing this to make fun or humiliate the slave. Students usually interpreted Socrates lengthy explanations as a sign of benevolence, and suggested that, for Meno and Socrates, the dialogue bears on what a good teacher is: someone who explains patiently and in detail; someone with whom even an uneducated slave can learn. They also mentioned the fact that making mistakes is not shameful, and that it is sometimes necessary to make mistakes, in particular if it helps you

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[8] " (2) Does the large square have an area of eight feet? Is it twice the initial square? Yes. We redrew the larger square. Then we counted the squares. There are eight of them. So it makes eight feet."

[9] "There are 8 right-angled triangles (in red) and 2 right-angled triangles = 1 small square. 8 triangles = 4 small squares. 4 small squares (blue) plus 8 triangles (red) is equal to 8 small squares of area (8 = 2×4)"

[10] "It's easy, when you fold the edges you get the same square."

realize that what you think is wrong. We are not claiming that is a deep, or even accurate rendition of the philosophical content of the text. However, for lack of additional information on Plato's philosophy and on the content of the rest of the dialogue, we take these elements to be indicative of students' ability to make reasonable hypotheses on the meaning of this excerpt as a whole, and to acknowledge the fact that something is at stake here beyond squares and areas.

3 Conclusion

Even though this experiment centred on a famous mathematical problem, the sessions we designed were not problem-solving sessions in the ordinary sense. Rather, students were to study the problem along with a list of answers (some incorrect, some correct), and the three sessions were designed so that the tasks which were most regularly entrusted to students were: reformulate, disambiguate, make explicit; assess an answer, assess an argument, complement a justification. Consequently, three levels can be distinguished: 1/ the level of the mathematical problem (shapes, areas, proportionality) 2/ the level of argumentation (making sense of the arguments, assessing their validity, their clarity), and 3/ the level of semantic integration (ability to go on reading even if everything is not clear or even makes perfect sense, ability to make hypotheses as to what the characters know and want, ability to revise these hypotheses as the text unfolds). If we were to give *one* answer to the "why bother with the original source?" question, we would say that our initial target was level 2: we regarded the study of this exchange of arguments about a geometrical problem as providing opportunities for *meta*-tasks, and as a means of enculturation into argumentation in mathematics, at a stage of the educational system where argumentation does not usually play a prominent part (if any). However, as we designed this project, we began to take level 3 into account for both practical and theoretical reasons. From a practical viewpoint, we did not want the three sessions to be complete failures because this long and difficult text made no sense to the students! Objective properties of the text – in particular, the interlacing of two dialogues: a mathematical dialogue between Socrates and the slave, and a philosophical dialogue between Socrates and Meno – made it necessary for the design to include scaffolding strategies. From a theoretical viewpoint, the fact that our research programme on the use of original sources in the teaching of mathematics called for reflection on reading-comprehension was highlighted in (Chorlay, 2016, 10), but, then, we gave no indications as to how to do it. The *Meno* experiment gave us a first opportunity to attempt to make use of inputs from research on literacy in the design of a teaching sequence.

As usual, whether or not this experiment was successful depends on the criteria for success. Some global indicators are positive: the engagement of students in the sessions was fair or good, as the written productions and the recordings show. Also, the two teachers found the sessions intense but rewarding, and included them in their teaching in 2018. As far as level 1 is concerned, the mathematical notions at stake in the text are relevant for students of this age: on some occasions, most students gave correct and sometimes creative answers (as for the assessment of Socrates' answer); on other occasions, as had been anticipated, some provided correct but oddly-worded answers, and some made standard mistakes. In the latter case, the variety of answers among students was sufficient for the collective exchange of arguments to lead to a consensual correct answer, under the guidance of the teacher. As far as level 2 is concerned, the extent to which this experiment contributed to an enculturation into mathematical argumentation cannot be assessed, since such a process can only take place over a long period of time. As far as level 3 is concerned, we probably need to work with researchers in literacy so as to specify research questions and methods.

A report on this experiment was published in (Moyon & Tournès, 2018), among a selection of experiments in using historical documents in the mathematics classroom for what the new French national curriculum calls "cycle 3" (final two years of primary school + first year of middle-school). This book is circulated by two professional associations, that of secondary school maths teachers (APMEP), and that of primary school teacher-educators (ARPEME). Thus, it is presented not only as an experiment in the context of a research programme, but also as a resource for training teachers and teaching children. Needless to say, a study of the reception of this resource by educators and teachers who were not associated to its design would greatly contribute to the reflection on the use of HPM in teaching and training, from a different perspective than that of task-design.

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