DEVELOPING GEOMETRIC PROPORTIONAL THINKING TO 6TH GRADE STUDENTS WITH THE USE OF A HISTORICAL INSTRUMENT OF ERRARD DE BAR LE DUC

Triantafillos DELIGIANIDIS[1], Kostas NIKOLANTONAKIS[2]
University of Western Macedonia, Florina School of Education,
Department of Primary Education, P.O. Box 21, Florina, 53 100, Greece
[1]soflak@yahoo.gr, [2]knikolantonakis@uowm.gr

ABSTRACT

This paper describes a teaching experiment on the notion of Proportion to a Sixth Grade class in Greece. The teaching was based on the incorporation of History into Mathematics teaching through the study of a primary historical source and the use of a reconstructed 16th-century historical instrument of Errard de Bar-le-Duc (Errard, J., 1594). Prior to the intervention the pupils' personal geometric knowledge was investigated with the help of a pre-test, consisting of the ability to solve verbal and numerical proportions, to solve analogue tasks and to distinguish proportional and non-proportional situations. This was followed by a didactic intervention, which included a series of eight lessons. The collection of data during the series was done with field observations, recordings and worksheets. Upon completion, the pupils' personal geometric knowledge was rechecked through a post-test. Also, with the help of a questionnaire, the students' view of the teaching series and the integration of the History of Mathematics were explored. The results of our post-test showed that there was a clear improvement in the ability to formulate the students' mathematical proportional thinking. They improved their performance, used more strategies to solve the problems, gave examples of recognition and problem-solving, developed the capacity to analyze quantities in a given situation, and applied the multiplication relationship to their data. They have also been able to work together and harness the benefits of teamwork.

Keywords: Ratio and Proportion, History of Mathematics, historical tool, Primary Education

1 Introduction

Ratios and proportions are widely used in mathematics, science, and everyday life (Karplus, Pulos & Stage, 1983).

However, out of the fifteen chapters of the section dealing with Ratios and Proportions in the School Manual of the sixth grade, two activities only approach the subject geometrically.

Researchers such as Lawson & Chinnappan (2000) argue that Geometry offers the framework for resolving various problems and developing learning strategies to solve problems in other thematic areas.

On the other hand, the use of the History of Mathematics and its integration into teaching is almost non-existent in Greek reality. It is limited only to some historical notes at the beginning of the chapters, which are usually left unused in teaching.

At the same time, the new Curriculum (Pedagogical Institute, 2011) provides for the inclusion of handwritten and digital tools in the teaching of mathematics.

We therefore consider a geometric teaching approach of ratios and proportions useful and constructive, from the perspective of History and the use of a historical tool.

2 Research Methodology

The design of the interventions was based on the hypothesis that students can learn mathematical notions by following a history-based teaching approach that uses History as a “tool” (Jankvist, 2009). Our work follows the principles of the “modules approach”. They may consist of material focused on a particular subject of the curriculum, suitably adapted for short-term classroom intervention. However, they may have a larger extent, requiring more teaching hours, and are intended to study a subject that is not included in the curriculum. The use of History of Mathematics through teaching courses can be done in a variety of ways, such as studying texts from original sources or working with students with worksheets. The worksheets conduct students to work on the mathematical notions and they can contain also historical information, biographies and other material for the didactic situations. Finally, on a larger scale, this approach may consist of courses or a study of books, or extensive research work by students. The historical material is the object of a project, in our case the study of a primary source and reconstruction of the historical tool is the entry, which will ultimately lead to the discovery of its properties by the students.

Bussi (2000) argues that History of Mathematics can be applied to school activities by studying and constructing copies of ancient tools and other artifacts that are reconstructed with the help of historical sources. She believes that almost all the tools of ancient and modern technology incorporate a lot of mathematical knowledge that is accessible through a careful and targeted study. They can be used as a major mobilization for both young and old students, but also adults. Particularly students, who do not love Mathematics, enjoy working with physical objects that are closest to their everyday experiences. Cerquetti – Aberkane (1997) has already proposed some activities with a tool from Errard’s book.

The main research question was: *How effective in shaping the students’ Proportional thinking, can be an experimental geometric approach that uses the study of a primary historical text and a reconstructed historical instrument?*

Eight didactic interventions were designed aiming at studying translated primary sources and constructing the tool based on the historical text to discover the properties of Proportions incorporated in the use of the instrument and ultimately to implement these properties for measurements on the real space. During these interventions’ students worked alternately in small groups on outdoor activities and individually in classroom.

3 The implementation

The implementation started in the school yard, where groups of students were assigned a measuring distance assessment. The diversity of the results demonstrated the need to use an instrument to measure them with greater precision. Then, we presented a translated excerpt of the historical text, entitled *La géométrie et pratique générale d'icelle*, (Errard, J., 1594) which describes an instrument suitable for measuring inaccessible distances. The pupils constructed a similar instrument, according to the instructions of the text and used it during their experimental work to discover the properties of Proportions of the similar triangles formed via the use of the tool.

Finally, they used these properties to resolve problems of measuring inaccessible distances and moreover to design a scale plan of the school surroundings.

Our main path was to use and evaluate the idea of passing from the experiment in the real space into the paper-pencil classroom environment and from the real space drawing into geometrical figure. Via these interventions we have tried to see how our students developed their geometrical – proportional thinking by passing from experimental geometry (Natural geometry) into a deductive way of thinking (Natural axiomatic geometry-Euclidean geometry).

3.1 First Teaching Intervention

Students were divided into groups of 4-5 people. The teams went to the school yard, where they were given a worksheet. They were asked to work in whatever way they wanted and make estimations for the height of different objects from their own environment, such as the flagpole in the yard of the school or a tree in the courtyard. After initially locating objects in real space, students tried to estimate the heights visually, by using various objects of known length as a measure. Then they tried to describe how they worked to get the most accurate estimation. After completing the process and the worksheet, they returned to the classroom where they had to present their assessments and discuss their documentation. The diversity of the results demonstrated the need to use an instrument to measure them with greater precision:

Harris: Yes, but if we had something to measure it (a tree), we wouldn't have had so many variations.
Teacher: What do you mean?
Harris: Yeah, an instrument ...
George: Yes, but how? Would we go over the tree to measure?
Teacher: Do you think it is difficult to measure the tree because it is tall?
George: Yes.
Teacher: And if there was a tool that we could measure from a distance, would we have variations in our measurements again?
George: Yes, but fewer.
Teacher: Why is this?
Harris: Someone might hold it wrong, another slightly above ...
Teacher: Does it depend on the use of the tool
Harris: Yeah.

3.2 Second Teaching Intervention

In this phase we attempted to integrate the History of Mathematics into the teaching. For this purpose, an authentic 16th-century text by Jean Errard de Bar-le-Duc had been selected. This was the first and second chapter of his work "Géométrie et pratique générale d'icelle". The text was in French and it was accompanied by a translation in Greek as children did not know the language. For the sake of clarity, those parts that were considered to be particularly difficult for children were simplified. Using the projector and photocopies, Errard's historical text and tool was presented to students. The children showed great interest in Errard's book and began to ask questions about its authenticity, its origin and if there were any similar texts printed at that time in Greece:

Thomas: What does it write in the title, sir?

Teacher: "Geometry and General Practice"

Raphael: Is it printed in Paris? Is it authentic?

Teacher: Yes. In Paris and it is the authentic one.

Thomas: Did we (the Greeks) have such books back then?

Teacher: It would be very difficult, since we were in a period of Ottoman rule. But since you mention it, I want you to look at some of the references later in the text and discuss them.

LA GEOMETRIE
ET
PRACTIQVE GENERALLE
D'ICELLE.
Au Tres-Chrestien Roy de France
& de Nauarre,
Par I. Errard de Bar-le-duc, Ingenieur ordinaire
de sa Maiesté.
SECONDE EDITION.

A PARIS,
Chez GVILLAVME AVVRAY au haut de la ruë
S. Iean de Beauuais, au Bellerophon couronné.
CI϶. I϶CII.
Auec Priuilege du Roy.

Figure 3.1: The cover page of the Book's 2nd edition

Figure 3.2: One image of tool's use

The presentation aimed at students to find out the purpose for which the text was written, and that similar concerns about measuring inaccessible distances have occupied man from very old age. It also helped pupils to realize that the use of such a tool could solve the problem they faced in the previous phase and stimulated their interest in the construction of this tool:

Spyros: The tool in the first image looks different than that in the second image.

Teacher: It's the same tool. Just the first one shows how to use it.

Panagiotis: It measures from the one side of the river to the other.

Teacher: Right. It targets a particular point, the tree.

Eleni: Some triangles are formed ...

Teacher: Yes, not really. They are conceivable. But for these we will talk below.

3.3 Third Teaching Intervention

The aim here was to carry out the construction of the tool by the students themselves. For this purpose the teacher ensured that each group had the materials for the construction of the instrument: three numbered bars, two by one meter and one by hundred and twenty centimeters, a movable connector for the vertical bar and a foldable joint with a viewfinder for the moving rod. We also used a photocopy of the historical text for each child, with translated instructions for constructing the instrument:

"So I would like the construction to be as follows: two brass bars completely straight be linked to diabetes as AB, AC, and each one and a half each foot approximately, and one inch wide, moving and rotating in center A. Next, on the AB bar (which we will call a

position), there is an engraving on which will apply a semicircle labeled here as D, on which one stands another EF bar (called base) which will be of similar length but equal width with the others, or a little narrower, and applied with the semicircle against length of AB, and in a way that in the center of E (which will be right on the straight line AB), we can close and make the angle we want with rod AB, along and joining the surfaces of the rods of the seat and its movable AC bar, and which, however, can be stabilized through a screw applied to the so-called semi-ring. Being so made, and the binoculars are focused on the ABC points (as we usually do in all instruments) each of the so-called three bars will be divided into 300 equal parts or in any other number we would like".

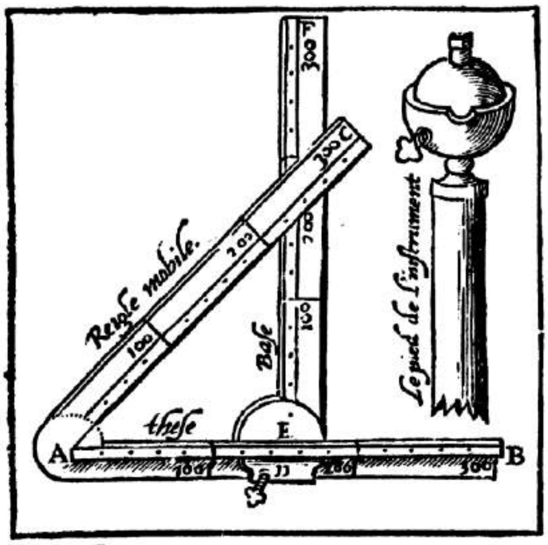

Figure 3.3: The picture for the description of the tool

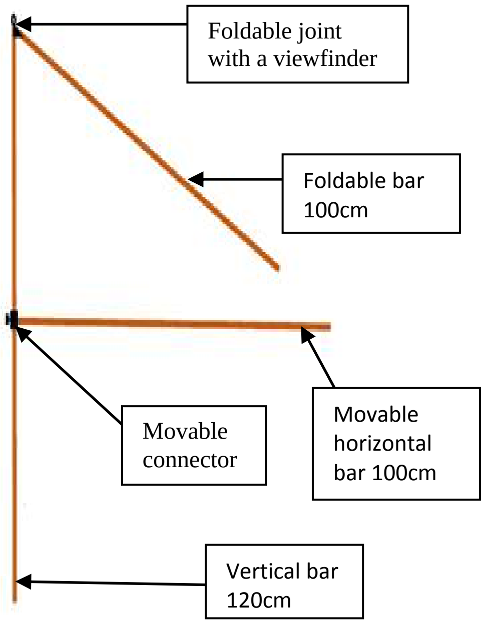

Figure 3.4: The parts of Errard's tool in modern scheme

The children's attention was greatly gained by the images of the historical text, which in addition to their artistic value, were able to provide a wealth of information on the construction material and how it was assembled:

Konstantinos: It's made of metal and it has a screw on it... it looks like a machine.

Helen: This ball, here, what is it?

Teacher: It's the joint. He presents it in detail in the text below.

The whole construction was generally considered simple and it was completed in four simple steps. The assembly of the tool caused great satisfaction to students and stimulated their creative mood:

Thomas: This is the same as in the picture!

Teacher: But of course, and you made it yourself!

Marianna: When will we use it?

Teacher: From the next lesson.

At the same time, teamwork and interactions between team members created the appropriate conditions for exploring the attributes of the tool provided in the next interventions.

Figure 3.5: Picture of the tool assembled by the students

3.4 Fourth Teaching Intervention

At this point we tried to train students to use the Errard's tool, by measuring accessible distances in the schoolyard, transfer their measurements onto the paper, find that two rectangular triangles are formed and that the distance requested is the basis of the big triangle.

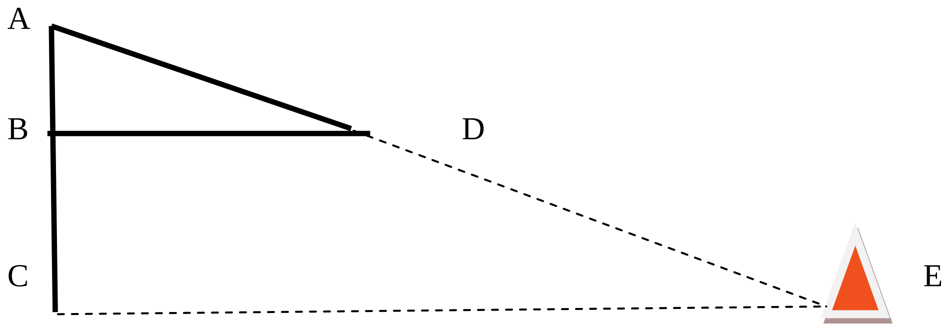

Figure e 3.6: Using the tool to measure accessible distances in the schoolyard

Students worked with the Errard's tool using a worksheet, confirmed their measurements with a tape measure and transferred the triangles formed on a sheet of paper. They kept a record of all the measurements done in the yard. In order to simplify the process, we took AB=20cm. in all measurements:

	segment	1st measurement in cm	2nd measurement in cm	3rd measurement in cm
Group A	AB	20	20	20
	AC	120	120	120
	AD	50	65	70
	CE	300	400	450
Group	AB	20	20	20
	AC	120	120	120

B	AD	60	77	62
	CE	420	520	450
Group C	AB	20	20	20
	AC	120	120	120
	AD	85	75	100
	CE	330	540	480
Group D	AB	20	20	20
	AC	120	120	120
	AD	50	35	65
	CE	300	250	370

Table 3.1: The results of the measurements in the yard

Although students used the tool for their measurements, its properties were yet completely revealed. They will be discovered in the next lessons.

3.5 Fifth Teaching Intervention

This time pupils worked in the classroom, using their notes and schedules from their previous session. They presented their findings in class and discussed the results. The comparison led to the conclusion that the sides of the triangles are associated with a proportional relationship.

At this point, we looked back at the historical text to find references about the property of Proportion that we had just discovered. It was a real surprise for students to realize that some other people, centuries ago relied on this piece of knowledge of mathematics.

3.6 Sixth Teaching Intervention

The knowledge just gained from the previous activity had to be verified in practice and applied to a multitude of cases so as to be generalized and formalized in the consciousness of students.

So, we asked the children to work in groups, alternating roles to get the proper familiarity with using the tool. They placed a variety of objects (cones) at accessible distances in space and then targeted them with the tool and tried to measure their distance. Then they measured the distances by a tape measure and confirmed their calculations**:**

Group	Distance measured with Errard tool	Distance measured with tape measure
A	4,60m.	4,80m.
B	2,40m.	2,40m.
C	3m.	3m.
D	3,90m.	3,80m.

Table 3.2: The divergence in pupils' measurements

When recording the results, it was found that there was little variation in the measurement results between the instrument and the confirmation with the tape measure. The students were able to give some good explanations for this:

Marianna: Maybe we did not just target correctly the cones (objects).

Panagiotis: The horizontal bar is divided by 5cm. Our measurement can be between those spaces…

Angelina: The instrument has to be upright, otherwise you get a false measurement!

3.7 Seventh Teaching Intervention

The students at this lesson worked in groups in the yard of the school. They placed various objects, as indicators, on the four corners of the building, as well as other selected parts of the courtyard. Then they made the measurements with Errard's the tool and recorded their results. The teams took different points and worked alternately.

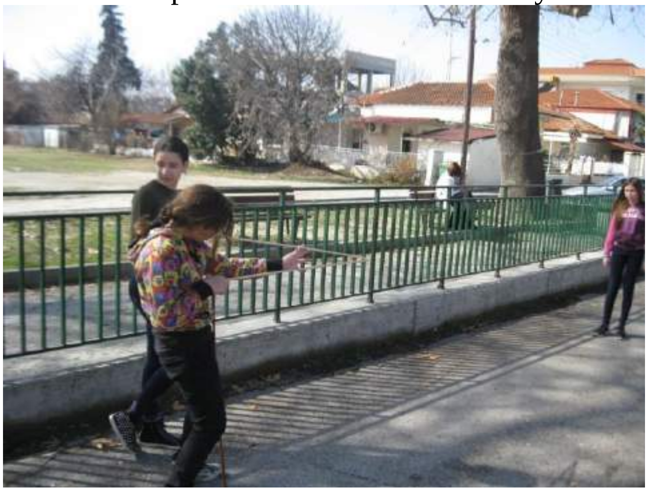

Figure 3.7: The students working with the tool in the yard

The results of the measurements were not any longer confirmed by metric measurements, as the distances were considered to be inaccessible and the functionality of the tool to measure such distances had also been confirmed:

Group	Type of measurement	Measurement result
A	Width of the building	28m.
B	Width of the schoolyard	85m.
C	Length of the building	54m.
D	Length of the schoolyard	130m.

Table 3.3: The measurement results of the building

3.8 Eighth Teaching Intervention

In this last session the students worked individually in the classroom. They had a draft of the school from the previous activity with its dimensions and now they had to transfer their measurements to the paper and to scale the ground plan of the building and the yard of the school using their geometric instruments.

4 The evaluation

The results of the post-test gave us an insight into the configuration of the pupils' personal space as it was formed after the didactic intervention. Overall, the results showed a significant variation in pupil's performance compared to the pre-test results.

If we look at the results of the post-test in general and compare it to the pre-test, we can say that there is clearly an improvement in both quantitative and qualitative results.

The tests were based on four main axes: a) the distinction between proportional and non-proportional situations; b) the ability to solve analogue projects; c) the ability to solve numerical and verbal proportions and d) the ability to compare ratios.

The post-test gave us useful information. In the first axis, students had to distinguish proportional from non-proportional situations such as, for example, the price and the quantity of a product. Here we had no significant improvement since the correct answers increased from the pre-test from 110 to 114.

In the second axis where we examined the ability to resolve proportional projects, we had a significant improvement. Here, the right answers grew in the post-test from 21 to 38, the wrong answers decreased from 32 to 24 and the students who did not respond at all, also dropped from 16 to 7. Apart from the quantitative data, we also had a significant improvement in the qualitative characteristics of the pupils' abilities. Thus, in most cases, students used multiple heuristic methods to solve the exercises, adopted the multiplicative reasoning to solve the problems, motivated their choices and tried to solve even those problems that could not control. Characteristic is the case of Konstantinos, who, after seeing a tree with a height of 1.5 m makes a shade of 3m and the height of another tree shading 9m is being sought, says: "*The height of the tree is half, i.e. 4.5m.*"

We also had the same improvement in the third group of exercises where we looked at the ability to solve numerical and verbal proportions, where for example pupils had to complete the fourth term of a proportion with a number or a word. Here the correct answers to the post-test increased from 45 to 58, the errors dropped from 33 to 30, while students who did not respond at all dropped from 16 to 4.

The last group of exercises that examined students' ability to compare ratios gave us the following comparative results. The correct answers to the post-test increased from 19 to 27, the errors decreased from 18 to 13 and the students who did not respond at all dropped from 9 to 6.

Improving performance can be seen in the diagram below, which compares student performance in both pre-test and post-test:

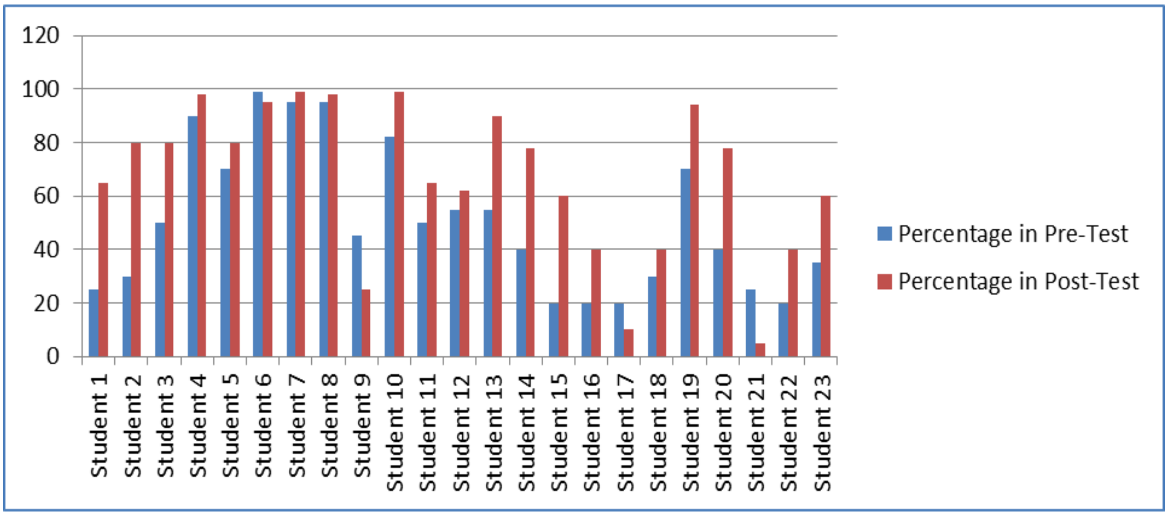

Diagram 4.1: Students' performance in pre- and post-test

Thus, we see that most students after the intervention have made significant progress, with about one third of them approaching excellent performance.

Summing up the above and attempting to answer our research question, we would say that our experimental geometric approach to Ratio and Proportion has succeeded in bringing positive results to the development of the proportional thinking of our students.

5 The evaluation of the teaching series by the students

The evaluation of the teaching series was based on a questionnaire and aimed at the following axes: the general evaluation, the historical text, the tool and the usefulness of the series.

Summarizing the children's answers to the general assessment of the teaching series, all students responded positively. The answers they gave to justify their point of view were quite interesting and seem to converge to the following:

Eleni: They (the lessons) were useful, educational and entertaining.

Harris: It gave me an incentive.

Angelina: I like to spend my time with Mathematics

Chronis: I liked that we all worked together

Konstantinos: It helped us in Mathematics.

We see, therefore, that all students have positively evaluated the possibility of other similar practical teaching interventions because they believe that they have a different benefit in their learning work.

In the second part of the questions, where the text was evaluated, students were given the opportunity to give multiple answers. The first question asked students to characterize the text. According to their answers, all 23 students in our sample, found the historical text interesting, 21 students found it pleasant, 18 found it comprehensible, while one added that he found it detailed. In the second question, the children had to answer if the text was related to Mathematics. All students responded positively and completed what concepts were included in the text. The most typical answers were: “rectangular and equal triangles, angles of the triangles, ratios and proportions of triangle sides, distance measurement, straight lines and Euclidean Geometry”.

We see that most students have responded positively to the text, since they considered it interesting, enjoyable and comprehensible, while everyone was able to understand the relationship of the text with mathematics and to distinguish the mathematical concepts included in it.

The next category of questions involved the evaluation of the tool itself. Generally, their experience of building and using the tool seems to be positively evaluated by the students themselves, since most of them found the construction and use of the tool easy, they feel it helped them and they would like to construct similar tools in the future:

Stefanos: I like to see things I've construct with my classmates.

Angelina: Nice experience and helps you learn.

Panagiotis: Tools are interesting and you learn a lot from their use

Harris: I want to discover new things and go deep into the magic of mathematics.

Vasiliki: It was fun.

Nikos: I want to learn about other tools.

Great impression also made to the students that a 16th century tool was used to teach mathematics. Thus, 7 students were surprised that with a historical tool they were able to measure inaccessible distances, 5 others that they had the experience to see in practice a 16th century idea and other 2 that they did not know the existence of such a tool.

The last question in the questionnaire attempted to assess the pupils' view of the usefulness of the teaching series and its practical use. Here, 19 students estimated that they will use what they have learned in their daily lives.

The following responses of the children, who responded positively, stand out:

Spyros: I will need them if I become a mathematician, engineer or topographer.

Sophia: I can appreciate distances better.

Harris: I can measure distances without a measure.

Marianna: I can calculate the actual distance of two areas on a map.

Raphael: To calculate inaccessible distances.

6 Conclusion

By making a general assessment of the eight didactic interventions, we would say that the goals were largely achieved. The pupils worked together, discovered the benefits of incorporating the History of Mathematics into their work, construct a historical tool with their own hands, used it, discovered the properties of similar triangles, and led to the discovery and use of scales. They therefore met, in an experiential way, the ratios and the proportions our research sought.

It is true that the use of History in this work was relatively limited. This happened because we had chosen to approach the subject in a way that History is mainly used as a "tool". Also, the extent of the teaching intervention was limited and there was always the risk of getting away from our original goal which was, developing geometric proportional thinking to 6th Grade Students.

Nevertheless, a descent attempt was made to rebuild the critical steps of the historical development of the tool, by leading a sequence of mathematical problems of scalar difficulty in order for the student to build his/her knowledge on the previous experience (Tzanakis et. al 2000).

There is, of course, much space for a deeper and more detailed discussion on the key points of the historical evolution of the subject we are examining and the way they have affected mathematical knowledge. We hope, however, that we will have the opportunity to develop them further into some other project.

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